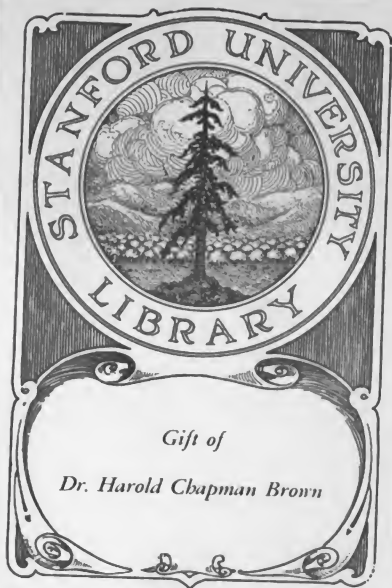


Logic

Christoph Sigwart



Gift of

Dr. Harold Chapman Brown

AUTHOR'S PREFACE TO THE FIRST GERMAN EDITION

As the need of a clear consciousness of the starting points, methods and aims of scientific knowledge has made itself felt within particular provinces of investigation ; and, on the other hand, as the reform of logic has been demanded in high quarters from the side of philosophy itself, as by Prantl and Harms, and started in a work so important as that of Lotze, I have felt it my duty to investigate and exhibit as completely as was possible for me to do the presuppositions from which scientific endeavour starts, the most general concepts employed in its researches, and the fundamental forms of the methods of which it makes use. It has given me special satisfaction that in one main part of methodology, the theory of induction, W. Stanley Jevons, in his important work *Principles of Science*, agrees on essential points with my own view. I have myself been more sparing than he has with illustrations from the history of science, and have selected as far as possible elementary examples from the region of ordinary knowledge. In doing so I have been guided partly by the desire for brevity, partly by the conviction that abstractions required by logic can be most easily made in connexion with what is most familiar to us.

August, 1878.

Thord Chapman Brown

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LOGIC

LOGIC

BY

DR. CHRISTOPH SIGWART

Professor of Philosophy at the University of Tübingen

VOL. II

LOGICAL METHODS

SECOND EDITION, REVISED AND ENLARGED

TRANSLATED BY HELEN DENDY



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August, 1878.

AUTHOR'S PREFACE TO THE SECOND GERMAN EDITION

What I have said in the preface to the second edition of the first volume, as to the considerations by which I have been guided in the revision of the work, applies also to the second volume. To a still greater extent than there I have endeavoured, on the one hand, to give as accurate and definite a form as possible to my propositions, and on the other to treat more fully of particular points. More especially have the questions concerning the methodological presuppositions of psychology and its dependent science of history received fuller discussion. I do not conceal from myself that the position which I here take will appear to many to be antiquated and to have been relinquished by the later developments of the science; this could not withhold me from giving expression to my conviction. I wish however to guard myself against being supposed to maintain that those efforts of physiological and experimental psychology which are prominent to-day are not fully justified within the limits imposed by the nature of the subject, and appropriate to the furtherance of our knowledge. It merely seemed to me necessary—and here I hope to find support in other quarters—to emphasize all the more decidedly certain considerations which are sometimes allowed to fall into the background.

THE AUTHOR.

TÜBINGEN, *September*, 1893.

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PART III. (Technical)

LOGICAL METHODS

LOGIC

PART III

INTRODUCTION.

§ 60.

THE general problem of METHODOLOGY is to show how we may apply our natural mental activities in such a way that, starting from a given state of thought and knowledge, we may attain the object of human thought by an ideally perfect process ; a process, that is, in which none but fully determined concepts and adequately grounded judgments are employed.

1. The task which we have set before us in logic is to show how propositions which are certain and valid for all might be attained. With this object we were occupied in Part I. in investigating that function of judgment which finds expression in propositions claiming to be certain and universally valid, in bringing to light the presuppositions naturally involved in it, and in stating the natural laws by which it is always governed. In Part II. we endeavoured to ascertain the general conditions to which our judgments must conform if they are to attain to universal validity, and we developed these conditions so far as they are involved in the nature of judgment itself. The two essential characteristics of the ideal state to which our thought aspires proved to be the complete determination of concepts, and the consciousness of the grounds of our judgments.

The question how to attain to this ideal state, with the means at our disposal, forms the subject of the third and last part of our investigation, *i.e.*, of Methodology. Its problems in their most general features are already implied in our preceding discussion.

2. With reference to the first point in our question, we showed in our Introduction (§ 2) that no really practical Methodology can aim at a complete reconstruction of our thoughts in accordance with technical rules ;

it must, on the contrary, accept as given a mental activity already at work, which can merely be amplified or corrected. Unless there were a supply of ideas already in existence, as well as a natural impulse to employ them in forming judgments, there would be neither pupils to learn the art, nor material upon which to exercise it; the very possibility of mutual understanding presupposes a store of mental contents which must in the first instance be taken as it is found. The art of gymnastics is practicable only for those who already have control over their limbs, and its object is merely, on the one hand, to develop and multiply movements already familiar, on the other to eliminate any which are superfluous; it cannot suddenly annihilate all previous habits, or create new joints and muscles. In the same way a logical gymnastic must always bear in mind that the mental activities at which it aims must depend for their execution upon rudiments already possessed by the individual learner, and can succeed only in so far as they can be developed and elaborated from activities which are already naturally initiated and exercised without reflection.

3. With reference to the second point, our first object in Part II. was to develop the condition of complete determination of our concepts. It proved to be necessary to discover the simple conceptual elements and to determine the forms in which they might be synthesized; and this had to be done in such a way as to ensure on the one hand that these simple elements should be thought by every one in the same way, thus making it possible that there should be a conceptual system the same for all; while on the other hand the conceptual elements should be extensively complete, and sufficient for every object of thought to be conceptually determined and reduced to generally established elements. Only when this is done shall we always be provided with predicates for our judgments which will enable us to make our statements precise and unambiguous; and under these conditions alone is it possible for judgments to be strictly necessary and universal.

From our investigation of the demand that there should be a ground for every judgment, there next arose the problem of ascertaining those judgments which contain their own ground, *i.e.*, are necessary by virtue of the contents of the ideas connected in them, or possess the evident certainty of immediate self-consciousness; with this is also connected the necessity of finding the axioms indispensable to all proof. Here again the general object which we set before us includes the need for extensive completeness; we can only ensure that our judgments shall be thoroughly grounded

when we are fully conscious of the ultimate presuppositions of judgment, and recognise them as self-evident and immediate principles.

4. Thus we get the following questions for our Methodology : How is it possible to determine all the simple elements of the whole contents of mind, in such a way that we may be certain of their being the same for all thinking beings, and the forms of their synthesis in such a way that every one may proceed alike in combining them into composite concepts? How is it possible to become conscious of those ultimate presuppositions of all judgment, which are the ground for judgments which are not immediately evident, and how may we show each particular judgment to be grounded in a way which may be convincing for every one?

§ 61.

The more exact form which Methodology must take depends, on the one hand upon the nature of the actual conditions of the thought which it is to regulate ; upon the other on the content of the aim we set before us in our deliberate thought.

This aim is to know the universe which is accessible to perception, and to determine the final objects of the will. The ideal of knowledge contains, in the first place, a representation of the universe as complete in space and time, next a classification of all that is given in a perfect conceptual system, and finally discernment of the necessity of what is given in the form of perfect causal connection. Consideration of the objects of the will results in the selection of a supreme end which comprehends all particular actions, and in the recognition that this end ought to be unconditionally willed.

1. By formulating in § 60 the questions to be answered by Methodology we obtained only the most general and formal definitions. To make them really practical we must first realize more exactly what is that store of ideas and generally accepted judgments from which we start, taking into account not only the conditions under which we gather and increase the material of thought, but also those natural tendencies to reflection by which we are impelled to form ideas and judgments and to believe in their validity ; and secondly, some more complete idea of the aims of all serious thought is indispensable. We must assume that we think intentionally, that our thought does not wander vaguely and at random, with no definite object in view ; an object will be supplied, at any rate in its general

outlines, by the permanent instincts of human nature, and by the current standard of intellectual development.

2. With regard to the natural conditions of human thought we find that a great part of our conceptual elements comes to us through the activity of the senses and their connected functions, which give rise to the idea of the spatial universe, which persists and changes in time ; and in the same way our composite ideas are for the most part occasioned by perception. We are stimulated to the synthesis of certain conceptual elements by the manifold composite external objects which present themselves to perception, and which we try to compare and to arrange according to similarities and differences in our first beginnings of thought. Here our thought depends for its contents upon external conditions ; what will be presented to it does not depend upon its own activity ; and even when the will directs itself to perception of the external world its chance of success is limited by many conditions. The perception is of inexhaustible variety, hence we can never be certain of completeness in this direction, either of the particular elements or of those infinite combinations which perception may offer to us.

And this is true, not only of the objects which Nature affords to observation, without the intervention of man, but also, and in a still higher degree, of those objects of apprehension which are produced by human activity, of all the forms and products of mental life, of all inventions and arts, of all social relations and contrivances. Here again we are constantly stimulated to new conceptual combinations, and we generally confine ourselves to forming those concepts which are to serve as the expression of what is given in perception, or which arise out of occasions which stimulate the will in a definite direction.

3. In judgment, on the other hand, we are largely determined by tradition ; the individual learns most of what he believes, and his judgment depends upon the prevailing convictions which rule those around him, and which are far from being all of them grounded, at any rate for the consciousness of the individual. Corresponding to this receptivity, we find the universal impulse to communication, the expansiveness by which we endeavour to reproduce our own thought in the minds of others, to fortify and strengthen our own belief by the acknowledgment of others. Any treatment of thought which abstracts from the characteristic of community, from the social nature which man shows here as everywhere, must be one-sided and untrue. Hence it is especially important to take account of all

those ideas and presuppositions of thought which every one receives in his appropriation of language.

4. If now we ask what is the aim towards which all human thought is working under these conditions, and what is the final use of the methods by which logic seeks to direct it towards this aim, we must start from an actual tendency, which we accept for the present as it is given, without asking whether it is to be regarded merely as an irresistible craving of human nature, or whether we can or do recognise in it a higher ethical necessity.

5. Here we find that the significance of human effort in the region of thought is, in the first place, determined by the fact that before we attain to any logical reflection we have learned to refer our manifold sensations to a real world, which is not only the object of perception and curiosity, but is also the means of enjoyment and the source of pain, and therefore the object also of practical activity. When, rising above immediate necessities, we yield to the desire for knowledge and by generalizing make it our conscious object, there next arises the ideal of an exhaustive knowledge of the particular, as it presents itself in the form of perception, the ideal of a complete world-image, in which the whole manifold which exists in space and time shall be displayed before us. Just as at any given moment our surroundings present themselves to the eye, each particular object accessible to observation and intuitable by the senses, so the whole universe is to become an object of perception. As with the traveller landscape succeeds to landscape, and new scenes open out beyond the last, so we are to be able to send our searching glance throughout the fulness of the perceptible, and the universe of existence is to be spread out in one vista for us to survey in detail and gather up in memory.

6. The order in which we thus arrange the particular is, in the first instance, that of space. In any given field of vision everything is localized in a definite place, and we endeavour in the same way to assign to every object of perception its place in one space, and to comprehend as it were in one survey the spatial relations of the universe. The world must lie before us in its ground plan; we endeavour to determine its position in space for every star in the heavens, to gauge every height and depth of the earth, to learn how stones and plants and animals are distributed upon its surface. It is as if we could not feel at home in the world until we knew it in every nook and corner.

7. This effort to attain a complete spatial image of the world is

accompanied by the attempt to get a complete picture of its history, to arrange in a time series every possible object of human perception, and to extend into the infinite past that immediately intuitable flux of changes which we are able to follow with uninterrupted glance. We must be present in the spirit to watch the heavenly bodies describe their orbits myriads of years ago, to see the earth take shape, and its surface break up into land and water, to notice races of plants and animals appear and disappear ; we can no more tolerate a gap in the annals of the universe than we can in the chart which maps out its spatial relations.

8. These aspirations, which it has remained for the present age to grasp clearly and definitely in their full extent, arise out of our longing to break through the limits of the individual range of vision and the individual memory, and to widen our narrow consciousness until it comprehends everything in one intuitive knowledge, thus satisfying both our natural curiosity, and our desire for communication. Whether or not we are justified in assuming that in this way we shall arrive at complete knowledge of a real world as it actually is, is not a question which arises for subjective effort or affects the means of its satisfaction ; it is enough that our perceptions are independent of our will, that they are given to us as a number of intuitions, that they present themselves in inexhaustible abundance and thus yield a constantly new content for consciousness.

9. But such a world-picture could never be the object of one comprehensive consciousness unless the activities by which we apprehend particular phenomena and reduce them to spatial and temporal order were accompanied by others which make their manifold contents comprehensible to us under the form of unity. We should have a bewildering chaos of details, of forms, of things and events, which no memory could grasp and recall unless thought could master the plurality of content by means of comparison and distinction ; and could on the one hand recognise sameness and similarity, on the other, estimate degrees of difference. Not until we have discovered the one which is common to the many, have discriminated the like and unlike elements in phenomena separated by space and time, have ranged differences according to degree and so brought logical order into our intuitions, can perception become real knowledge, and each particular fall into place in a system of ideas which, as predicates of judgments of perception, enable us to transform the particular phenomena into a permanent idea. Such a process, if completely carried out, would lead to a comprehensive system of concepts, in

which the whole contents of the perceptible—of things as well as of events—would be displayed in order of similarity and difference : to a classification extending over the whole sphere of perception, and finding its expression in a well established system of notation in a scientific terminology.

10. Let us suppose that we have succeeded in reducing all objects of perception to this logical order of similarity and difference, and have drawn up a table of more or less general concepts into which every single form of a given thing, every event, every relation can be fitted, so that each part of our world-picture has not merely its position in space and time, but also its logical position in the hierarchy of concepts. We should then have satisfied the impulse to bring everything into relation by comparison and distinction, but this logical classification would be only like an external framework over the manifold of the given reality, and the plurality of species under one and the same genus, the number of differences into which a common element develops itself, would remain pure matter-of-fact. It might, indeed, give rise to wonder at the inexhaustible fertility of nature in producing different combinations of qualities ; it could never be the object of real understanding.

Unless we can find necessity in the differentiation of the general, and in the comprehension of different characteristics in the unity of a thing, we cannot even be certain of the completeness of our classification ; we must be able to see not merely that a genus has such and such species, but that it must have just these and no others ; we must understand what is the bond which holds together the characteristics constituting a composite concept, and by what necessity differences are developed from a common element. The perfection of thought lies in seeing that co-existents are necessarily connected, and we could not be certain that our classification was complete unless we knew that those combinations of attributes which were absent were also impossible. All attempts to deduce special concepts from general by logical development, above all that latest and most ambitious attempt, the Hegelian method, are expressions of this effort, repeating in an intensive form the characteristic of the effort after totality of extensive comprehension—*i.e.* penetration of the given manifold by the unity of one consciousness. In what way we are to endeavour to find this necessity, and whether it can be found by means of our thought, we must consider later on ; at present we are concerned only to point out the aims towards which thought strives.

11. But our desire for knowledge needs for its complete satisfaction something more than this insight into the necessity of conceptual articulation in the manifold of the world ; if the given reality is to be completely penetrated by the necessity of thought, then neither the spatial and temporal arrangement, nor the number of the objects in which concepts are realized, can be regarded as something indifferent and accepted merely as given. The theory of the universe, as conceived by Plato and Aristotle, was one-sided in thinking the problem solved by an enumeration of the general concepts which determine the manifold forms and differences of the sense-given, and in neglecting in comparison with the purely logical system of concepts—if not entirely excluding from scientific interest—the plurality of phenomena as definite in number and arrangement. Why some forms are more frequent than others, why these are to be found in one place and those in another, is a question which will make itself heard sooner or later. Bacon complained¹ that as yet no one had attempted to explain why some things are and can be so numerous and abundant in nature, while others appear so seldom and in such small quantity (for it is certainly impossible that there should be as much gold as iron); and in this he gave emphatic utterance to our effort to understand the whole position of matter-of-fact, even in that aspect which, as compared to the concept, is merely fortuitous.

12. The question just raised refers in the first place to the permanent parts of our world-picture, to the things which in their diversity form at any moment the constituent parts of the whole. But similar observations arise when we turn to the course of events in which these parts are changed in themselves and in their relations to each other. Here again our desire for knowledge is not satisfied by a conceptual comprehension and classification of all kinds and forms of change, by judgments which tell us that one thing undergoes a particular kind of change in this place, another in that, and that in this way the form of the whole passes through constant change from moment to moment. Even if we succeeded in a complete survey of the course of all changes and could reduce each one to its appropriate concept, there would still remain the pressing question as to the *necessity* of all this happening, and we should want not merely an external connection of events in our consciousness, but a knowledge of them as connected in themselves. We need to think of them proceeding from each other, as our thoughts depend upon each other, with logical

¹ *De dign. et augm.*, sc. iii. i.

necessity, and so to find in the external world a counterpart of the unity which binds our thoughts together.

13. It is in the general thought of causality in its many forms that this tendency seeks to satisfy itself. When defined with scientific exactness, this concept is the means by which we comprehend the necessity of all happening, and thus find again in Nature that relation which exists for thought between the particular and the general, the conclusion and its premises. Hence the thought of causation has its full effect only in connection with the conception of a general law from which particular events follow of necessity and are comprehensible as the results of a general principle. Thus there now appears with reference to happening a logical subordination of the particular under more and more general laws, like the subordination of the particular under species and genera in the world of things and their forms. For our thought a judgment which states a particular event is well grounded when we can recognise it as the consequence of universally valid laws.

14. The more exact form in which this thought is to be carried out and applied is left undetermined by the general tendency to find a ground for the particular. This tendency would be just as well satisfied by the Leibnitzian theory that all changes of particular things are only developments of their own essence, and follow as a consequence of their nature, independently of external influence; but the course of the world as we perceive it will not allow us to find regularity in the series of changes which each particular thing undergoes, so long as we refer those changes merely to its own condition. On the contrary, we find sufficient instances of connection between the changes of a thing and its changing relations to other things to enable us to formulate general principles, and sufficient ground to think that we must look for knowledge of necessity in events, not merely to the laws of self-development of independent substances, but also to laws of action and reaction amongst substances which are mutually dependent.

15. Thus out of the attempt to comprehend the given as necessary in the forms of thought there arises the ideal of a universal knowledge, which regards the perceptible world as the realization of a system of concepts, and the course of events as the expression of necessary consequences following from ultimate principles. By a complete arrangement of all particulars in their manifest logical connection our thought would be satisfied, the functions of intuition and thought would be harmoniously combined.

Both outer and inner life, material and spiritual events, would be alike removed from the sphere of isolation and chance, and that ideal unity of consciousness which we set before us in our pursuit of knowledge would be at once complete in itself and the expression of the totality of being.

16. But it is not only the perceptions in which we obtain without effort of our own an intuitable content of ideas to be elaborated into the world-picture, which afford us objects of thought ; nor does our mental activity exhaust itself in seeking the necessity of that which is or happens. In addition to these we find within ourselves a constant activity of will.

We are not mere passive spectators of what goes on without and within, letting ourselves be surprised by successive events as in a dream, and seeing our own action intervene only as necessity may happen to give rise to it. We are constantly putting forth action as conscious self-determination, and realizing aims which we have designed and affirmed as our aims. It is true that it is our wants, or some impulse which we cannot explain, by which we are led to devise an imaginary state of things for the future, and it is true that in the choice of means to realize this state we are limited by our knowledge of the existing world and our own powers of acting upon it. But that we can take such an imagined aim for our own, and control our limbs in such a way as to lead to its realization, this is for us something final, due to ourselves alone ; and the true pulse of life beats, not in the activity of receptivity, but in the continual devising and realization of aims which through our conscious acceptation alone attain validity and rise to something higher than idle play of fancy. The most comprehensive knowledge could neither alter nor kill this activity of effort. However clearly we might see that even mental life, even the particular acts of the will, obey an inevitable necessity, we should never succeed in calmly watching our own action and efforts of will as the infallible consequences of the course of Nature. We may indeed attempt in the interests of knowledge to regard the self as object only, as a part of the universe ; to take up our position, as it were, outside the self, and, regarding the Ego as a sort of double, to explain the causation of its will by an analysis of its motives. But this very process is itself an act of will, and the true self remains the living Ego which effects these duplications of self in the struggle for knowledge. However exhaustive reflection may have been, the object of observation is the mere shadow of the self, an *εἶδωλον*, of which the life-blood has retreated from the observed to the observer ; the actual will, through which alone an activity and the aim towards which it

is directed exist, must still be an irresolvable remainder. Of course a necessarily imperfect knowledge of the manner in which acts of will occur at any moment in the subject will always be sufficient to secure for us the restless excitement of willing and resolving ; but even if we had the fullest insight into a psychical mechanism for bringing about any given resolution, our knowledge would still not suffice to make us accept the act of will thus produced as a matter of course, to affirm the aim contained in it, and to be content to will something because we were constrained to will it. Because something is and happens it does not necessarily follow that it *ought* to happen, and that we are therefore constrained to acknowledge it as something which we ought to will and which is worthy of being made our aim. This is the more to be borne in mind because knowledge itself can only be finally realized through the will, it grows only through the setting of definite aims before thought. The relinquishment of a will which is independent of what we actually do would be self-destructive, for it would annihilate that distinction between true and false which arises from a comparison of actual thought with an ideal, and it would be open to error as well as to true knowledge to appeal to the justification of actuality.

17. In proportion as self-consciousness develops it strives for unity of the will, for subordination of all particular aims under one which is higher than all and embraces all, for the determination of every practical question by one supreme law. Until a man has made his purposes coherent and connected, and set himself to govern his changing and complicated activities by one principle, he drifts at random, and, like a child, is a prey to every chance stimulus from the external world and to his own manifold impulses. The reasonableness by which he is distinguished from the rest of Nature consists in just this, that by means of a clearly conceived system he brings order and harmony into his own activities and keeps one and the same object in view in every single exertion of his power over himself and the external world. Here again his thought ends in a consciousness of the necessity and unconditional validity of the end which he accepts, in the consciousness of "ought," and from it springs the idea of a law which is valid for all reasonable beings alike, the idea of something which is right and good, not for him alone, but for all.

From the nature of purposive will it further follows that here, at any rate, the validity of the particular ends is dependent upon the validity of the general and highest end ; the particular is only to be willed because it

is a constituent part of that object of our will which is supreme and includes all others. In the purely theoretical sphere it is conceivable that investigation might lead us to a plurality of independent principles which are merely not contradictory, but cannot be derived from each other. A practical principle demands unity, for my will is only *mine* when it is one with itself in all its activities, when it is ruled in all particular actions by one aim. Nor can sceptical prudence reserve to itself any loophole of escape in these practical questions; we may defer the theoretical question, and hope that later on the progress of knowledge will afford an answer, but in practice we must choose between yes and no, for inaction is itself an answer here.

Thus reflection upon what man ought to do reveals itself as the highest and most urgent problem of thought,—a problem moreover which is not only incapable of complete solution by the mental elaboration of what is given, but which proves upon enquiry to include this purely theoretical activity as dependent upon it.

18. We need only point out briefly that perception can never suffice to give any final teaching as to the ends at which we must aim, and that history would become totally incomprehensible if it should deny the influence of ideals which have transcended all that is given in perception. Or, again, if we argue upon psychological grounds that we can will nothing seriously which does not promise us some satisfaction, the question at once arises what does satisfy us? and no analysis can ever show that only the repetition of an experienced pleasure or pain can exercise this stimulus upon the will. On the contrary, the unprejudiced observer must acknowledge that the most intense satisfaction of all, that which is found in the consciousness of harmony with self, presupposes a will directed towards that harmony, which, as universal, can be determined by no experience, and which must therefore be regarded as ultimate and original.

19. But we rest upon the same ground even in our scientific investigations of the perceptible world. That comprehensive knowledge for which we seek is no gratuitous gift of a self-developing nature, nor is its acquisition merely incidental to the necessary process of satisfying our wants. It is a freely-willed *end*, for which we work with conscious and voluntary activity, and our justification in pursuing this end is ultimately derived from the validity of the moral ideal as including the conception of the most comprehensive knowledge. We must assume that the voluntary thought presupposed in logic, when in its concrete form it is directed to-

wards a definite end valid for all, is contained in the universal duty of mankind, and is a necessary aim of human activity.

§ 62.

In directing our thought towards these ends we assume on the one hand that our perceptions submit to the claims of thought so far as to allow of their being arranged in a conceptual system and in orderly connection; on the other hand, that all our actions can be subordinated to a single end.

These assumptions are POSTULATES, and our acceptance of them rests in the last instance upon our will.

1. We shall not attempt here to show how far the ideal of theoretical knowledge sketched above can be derived from general ethical principles; for the present we must assume that it is a part of man's duty to realize it, and that the very nature of his thought in its effort to find necessity compels him to seek just this form of knowledge if he will have knowledge at all. But we are concerned to point out the consequences involved in those presuppositions, and thus throw light upon the true character of the starting-point of our Methodology.

2. Though our thought aims at ranging the whole perceptible world in one system of concepts, and at showing its complete submission to law and necessity, we cannot assume *a priori*, nor actually demonstrate from our always incomplete perception, that the content and course of our perceptions will submit to the claims made by thought as it arranges its material and endeavours to find in it unity and necessary connection.

Kant has pointed out¹ that our perceptions might be so constituted as to make it impossible for us to divide them into genera and species, or to make them the object of a comprehensive knowledge, and impossible to derive their manifold laws from a few ultimate principles. It cannot be taken for granted as an axiom that all perceptions are necessarily capable of being ranged in a conceptual system in which it would be possible to group together similars and distinguish differents by accurately fixed gradations. On the contrary, the difficulties by which we are met in any classification teach us that the logical arrangement of what is given is no easy matter, that similarities do not always force themselves upon our view nor differences fall into clearly marked divisions. Those departments in

¹ *Kritik der Urtheilskraft.*

which conceptual classification can be successfully carried out are surrounded by and mingled with others in which the varieties of form are so capricious, or transitions are so gradual, as to seem to mock the frame-work of our concepts; we have therefore no invariable experience to which to appeal as a proof that a system of genera and species is realized in the phenomena of perception. If we nevertheless hold by our assumption that we must be able to arrange all given phenomena under definite concepts we are justified only because it is on this supposition alone that our aim can be attained, and the infinite number of our intuitions mastered by the unity of consciousness; in assuming as possible that which, if unattainable, would render our will futile, and therefore unreasonable, we lay down a postulate of knowledge.

3. The same is true of the conception of a thorough-going causal connection. It must be allowed that no strict proof of the absolute impossibility of a purposeless and lawless happening has ever been brought forward. Experience can show no invariable system according to which events subordinate themselves to laws which may be discovered by the mere comparison of similar instances; nor can constructive metaphysics succeed in convincing us that the existent cannot be thought otherwise than as determined according to rule by some other existent, or that the logical law of identity contains the principle that all change in the existent, whether from internal or external grounds, is necessarily determined by universal laws. The attempt to prove upon subjective grounds that perceptions cannot be combined in one consciousness unless they obey the law of causation has been no more successful, and has failed to show that experience is impossible unless every change is preceded by another upon which it follows according to law. True, a science of experience which can be expressed in universal propositions is possible only when we assume the law of causation; only then can we say of two events, *a* and *b*, that *b* *always* follows *a*, and that if it ever does actually follow, it is necessary according to a universal law. But Kant has not shown why events should not be combined in one consciousness which as a matter of fact only follow each other in one instance, without our being conscious of any necessity. On the contrary, unless it were possible in the first place to establish the fact that here or there *b* follows *a*, it would be absolutely impossible to discover the law according to which *a* is the cause of *b*. The science of experience of which Kant sought the conditions, in the sense of mechanical physics, is preceded by another science which merely observes and records

the temporal sequence of events and maintains the same attitude towards occurrences which seem to us causal as towards those in which we can recognise conformity to law. Unless it had been possible to determine the successive positions of Mars by observation before its movement was recognised as the expression of a causal law, Kepler could never have discovered his laws, nor Newton have explained them by the law of Causation. Thus what Kant has shown is not that irregular fleeting changes can never be the object of consciousness, but only that the ideal consciousness of complete science would be impossible without the knowledge of the necessity of all events; his principle of causality is no principle of the pure understanding in the sense of a synthetical judgment *a priori*, but a postulate of the effort for complete knowledge.

4. Thus we see that the universal presuppositions which form the outline of our ideal of knowledge are not so much laws which the understanding prescribes to Nature, or rather to our sense-perceptions, as laws which the understanding lays down for its own regulation in its investigation and consideration of Nature. They are *a priori* because no experience is sufficient to reveal or confirm them in unconditional universality; but they are *a priori* not in the sense of self-evident truths, but only in the sense of presuppositions without which we should work with no hope of success and merely at random, and which therefore we must believe if we are in earnest in our endeavour after knowledge. They are postulates, and are akin to the ethical principles by which we are wont to determine and guide our free conscious activities.¹

It is for this reason alone that however we may fail in our attempts to subordinate the world of perception to a complete conceptual system and to derive all events from universally valid laws, we never doubt the truth of our principles. We still maintain that even the worst confusion is capable of being resolved into comprehensible formulæ; again and again we start our work anew, and believe—not that Nature opposes an inexorable refusal to our endeavours—but only that as yet we have failed to find the right way; but this perseverance is due to the conviction that we ought not to despair of the accomplishment of our task, and the courage of the explorer is sustained by the obligatory force of a moral idea.

5. It is the same in the sphere of *practical* thought. Here again an ideal presents itself which has arisen from our endeavour to find a com-

¹ Cf. Lotze, *Logik*, p. 567 sq. Laas, *Kant's Analogieen der Erfahrung*, p. 175 sq. A. Riehl, *Vierteljahrsschr. f. wiss. Phil.*, 1, 365 sq.

plete unity and harmony of will such as no experience can yield, for it is never realized. Here again it is no matter of course that thought will succeed in finding one comprehensive end, from which all particular ends are necessarily derived, and through which we may determine unequivocally at every moment of life what is to be done, so that all our actions may be knit together by a complete logical necessity ; nor is it certain that all the actions proceeding from the nature of the subject, from his inextinguishable impulses, will submit to such a logical order and unity. The plurality of our wants and of external stimuli corresponds to the manifoldness of perception which theoretical knowledge attempts to master by its logical forms ; the fact that our life can contain one temporal series alone and that many impulses present themselves at every moment is to the unity of the end as the chaos of external impressions is to the forms of regulative thought. And in the same way we may have the general idea of connected unity without the particular form in which it should and can be realized ; just as the idea of complete knowledge is the motive power of theoretical effort, so the idea of the supreme good is in the first instance merely the motive power of practical thought.

6. Thus every attempt to construct a methodology refers us back to definite aims towards which thought strives, and which depend for their validity as ends upon the will ; and herein is involved a further subjective presupposition without which the instruction to be given to thought would remain barren and unfruitful—the presupposition that the end is attainable by actual thought. This involves the assumption that the psychological conditions under which actual thought takes place oppose no insuperable obstacles to the attainment of the end or to the carrying out of the rules demanded by it ; that consciousness of the end, and of the laws prescribed by it, really can determine our particular acts of thought and direct them towards the desired end ; that it does not follow from the psychological conditions of mental activity that we must passively and inevitably fall victims to error and contradictions. But this is none other than the postulate of freedom as referred to our thought as a voluntary activity ; the postulate that the concentrated will, when directed towards the idea of an end, has the power to rule all particular acts of thought into correspondence with it, and thus to establish the ideal unity of self-consciousness throughout its manifold thoughts. Descartes was right in saying that our judgment is free, and that we have the power to withhold our assent from any judgment ; not indeed as he put it, as if we could at any moment,

without further conditions being introduced, believe or not believe as we like by a mere act of will, but right in the sense that one of the conditions of true knowledge is that consciousness of the laws demanded by the unity of self-consciousness should be able in the course of time to bring about this unity. Even in the sphere of practice the idea of freedom is important not as enabling an arbitrary caprice to choose at any moment contradictory actions, but by making it possible for the will to be one and harmonious by virtue of the end which, when thought and willed with a consciousness of duty, determines the particular temporal acts of volition.

7. The postulate of freedom, then, is none other than that of the unity of self-consciousness, if by self-consciousness we mean, not merely the one form of unity in which every content is comprehended, but also the one source of volitional activities. When we recognise that the thought which Logic investigates depends upon volition, then we have recognised the supremacy of the will in the sphere of theory as well as of practice, and the final presupposition is not merely that the "I think" must accompany all my ideas, but also that the "I will" must govern all my acts of thought; in other words, that the natural laws of thought by which it is guided in its particular acts must oppose no obstacle to their complete interconnection, nor the natural laws of the will to the subordination of all ends to one supreme end.

8. Thus we have on the one hand the general principles which are involved in the ends towards which thought is directed, and which in their ultimate dependence on a will share in the characteristics of the *a priori*, and on the other hand that which as object of an immediate self-consciousness referring to a particular moment is itself immediately certain; and between the two thought moves to and fro weaving the links of necessity. But if we wish to bring to light and make sure of that necessity, we can do so only by ascertaining the invariable natural laws of the activity of thought itself.

§ 63.

In pointing out the paths which thought must take to the ends it has in view methodology must call to its aid the History of Science, it must investigate the methods through which those ends have actually been reached, or through which at least an approximation has been successfully made, and it must show the grounds upon which these methods have been based, and examine their justification. Its procedure therefore must be both historical and critical.

1. Our task now leads us to endeavour to point out the ways by which the ideal before us may be reached—or at least approached—under the universal conditions of human thought, and starting from such ideas and judgments as have been formed in the natural course of psychological development. Another element now appears in our investigation, the history of science, in which we find recorded progressive attempts to attain to concepts and judgments which are logically perfect and appropriate to the highest ends of thought, and which shows us, on the one hand under what conditions and through what means problems have been solved, and on the other hand what circumstances have until now hindered a complete solution and what expedients have at least facilitated an approximation to the end.

2. Generally speaking, a technical science appears after the art with which it is concerned has been for some time practised, and it reduces to rules that which has already been successfully carried out by proficient in the art. Where the art is one so generally practised as that of thought, it is not for Logic to attempt to offer new and original methods and artifices; we shall rather content ourselves with promoting the security of the procedures already discovered and in use by a consideration of the general grounds and presuppositions of methods employed, in order to find out how far and under what conditions they are fitted to attain the perfection aimed at and to secure freedom from error. Thus we have not to invent methods, but by considering the presuppositions involved in them to establish the extent of their validity, the limits of their application, and the significance of their results. The work is therefore essentially historical and critical; we take from actual practice the manifold of particular processes, and we estimate their logical justification by the conditions of normal thought, the determination of the concepts, and universal validity of the judgments, and their value by their importance for the attainment of the highest aims of our thought.

3. In all this we must of course confine ourselves to the proper sphere of Logic. Those methods within the departments of each particular science which depend upon the special nature of the objects and contain directions for treating these so as to bring about the most favourable conditions for knowing them (*e.g.* for making certain phenomena artificially perceptible, or for avoiding certain illusions) belong to the special technology of the sciences. No one will expect to find in a general methodology instructions how to use an astronomical instrument or to institute an

accurate census, although methodology involves the measurement of magnitude and the enumeration of the individuals in given groups.

Our subject-matter is limited to those mental processes which, starting from given ideas, lead to the formation of definite concepts and to knowledge of the necessity of our judgments.

§ 64.

The course of our investigation is determined for us partly by the logical ideal, partly by the fact that this logical ideal is not equally easy of attainment in every direction in which we seek to attain the ends of thought.

The first and fundamental task is the attainment of COMPLETELY DETERMINED CONCEPTS. It necessitates in the first place the analysis of all our ideas into their simplest elements, and next a synthesis guided by fixed rules. The second task is the formation of PERFECT JUDGMENTS which are absolutely certain and well grounded. It necessitates the discovery of judgments which are self-evident, and of the way in which all other judgments may be grounded upon these.

But in this logically perfect form the end at which thought aims can be only partially attained in a direct manner. Where fixed rules of conceptual synthesis and absolutely self-evident judgments are wanting, and the need nevertheless remains for the conceptual arrangement of a given material and for assigning the ground of suggested judgments, then we have to make use of *indirect*, in other words, hypothetical or tentative methods.

Our first task, that of carrying out strict methods, will show of itself when taken in connection with the whole problem of thought, the limits outside of which other methods must be applied according to the nature of the problems in view and of the conditions under which they must be solved.

1. The first condition to be realized for the perfect fulfilment of the problems of thought is the attainment of completely determined concepts.

According to § 41, 1, p. 253, the possibility of the perfect formation of concepts depends upon the analysis of our ideas into their simplest elements, and upon the regulated synthesis of these elements into compound ideas. Hence it follows that in methodology we must begin on the one hand by indicating the way in which all simple conceptual elements may be discovered and a fixed meaning assigned to them; on the other

hand by pointing out the forms of synthesis, and developing the rules according to which composite concepts are formed from their elements. Perfection would be attained if this synthesis were carried out in such a way that the genesis of each concept could be seen to be necessary, and the interconnection of its characteristics to be demanded by general laws, while, further, the process should itself guarantee the completeness of the conceptual system, inasmuch as the elements of thought, when completely reviewed, would naturally develop into their manifold combinations according to obvious principles. Two opposite poles present themselves towards which this ideal construction tends: the one is the greatest possible specialization of concepts which arises out of the actual enumeration of all determinations, and serves to develop all possible differences, so that every subject may be denoted by an exhaustive predicate; the other is the greatest possible generalization, which serves to bring to light on all sides the unity of differentials, and to make possible the most comprehensive universal judgments.

2. A glance at the results actually attained in the construction of concepts shows that these ideal logical conditions are fulfilled in very different degrees. In some departments, such as mathematics, we find both that analysis leads to completely determined mental elements which are the same for every one, and that the subsequent synthesis is perfectly clear and governed by self-evident axioms; while in others, such as psychology, we are met by the greatest difficulties even in the analysis of mental elements. Whenever we are dealing with external perception, even if we could imagine the analysis of elements complete, we fall short of finding necessity in the synthesis, and certainty as to when there is a reason for combining characteristics and when there is none, so that we lack a safe and general clue for the construction of all concepts. Hence it becomes necessary, in those departments where strict rules for conceptual synthesis fail, to find some other means of satisfying the needs of science so far as possible, and these, in the absence of rules, can exist only in provisional assumptions. Thus we get the distinction between methods which are certain and strict, and those which are only hypothetical or tentative.

3. A similar distinction appears with respect to the construction of judgments. The universal and final aim of methodology is to attain to absolutely certain judgments; *i.e.* to judgments which are either self-evident or deducible as necessary from others. This is possible with purely analytical judgments, which merely explain the contents of concepts

which are already formed ; it is possible, further, whenever we are able to draw conclusions by valid inferences from established axioms. But in Part II., §§ 45-48 we have already shown that all basis for such a process fails us as soon as judgment begins to deal with the reference of our perceptions to an objective being ; particular perceptions, if we attempt to make them express more than a subjective fact, cannot lay claim to immediate certainty, nor are there any universal propositions given to us with axiomatic validity which would enable us to establish the particular judgment of perception as necessary consequence of some necessary and self-evident presupposition. Here again then, in addition to the methods which bear the character of strict deduction, we must seek for others which will satisfy the need we are under of referring the given to an objective being and of arranging it in complete interconnection ; and the attempt to find these will teach us that we can only apply hypothetical or tentative methods.

Meanwhile we must bear in mind that the construction of concepts and of judgments are not processes which can be carried out independently of each other. At that point in our construction of concepts where we satisfy logical conditions every conceptual synthesis contains the judgment that certain characteristics belong together, and this judgment must have its ground in some law. It is only while we are analysing the process that these two sides of thought can be separated ; as they actually occur they are always passing over into each other.

4. Our exposition, then, will take the following course. We start with the fundamental problem of finding the simple elements of concepts ; in doing this we shall naturally be led to distinguish those departments in which it is possible to have a synthesis corresponding to the logical ideal and determined by fixed rules, and we shall discover the rules of this synthesis. The synthesis itself will lead us to the consideration of a logically perfect construction of judgments which are certain and obviously necessary—this is the sphere of DEDUCTION. Then we shall turn to problems where strict methods fail us ; for the most part this is when we are dealing with the reference of the empirically given to general principles. The theory of INDUCTION teaches us under what conditions and presuppositions judgments concerning the objects of our perception may be formed and what degree of certainty pertains to them ; the theory of CLASSIFICATION treats of the construction of concepts in this department. The fact that we here place the construction of judgments before that of concepts must be justified by the course of our discussion.

CHAPTER I.

CONCEPTUAL ELEMENTS, AND THE FORMS IN WHICH THEY ARE SYNTHESIZED.

§ 65.

THE first precept enjoined by the art of thought is that we should make ourselves clearly conscious of what we do when we think of any object, and that we should make this consciousness invariable. From such a consciousness arise the ideas of IDENTITY, of NON-IDENTITY, and of UNITY as the interconnected results of reflection upon the form of those activities through which our self-consciousness realizes itself in its unity.

According to their contents objects thus distinguished are further determined by their RESEMBLANCE or DIFFERENCE.

1. Our investigation of the simple elements of our concepts would have attained its purpose (according to § 41) if we had full knowledge of the laws of the genesis of all our ideas, and could show the simple acts through which their elements arise and by which they enter into more and more complicated connection ; and if, moreover, we could control the means of evoking these functions and their products in ourselves and others at will, as, *e.g.*, we can be sure of producing the sensation of a particular yellow for the normal eye by burning natrium. But we have no such science and art in our possession ; our logical theory must start from an infinite multitude of the most manifold ideas, which we may take to be represented by the whole wealth of language,—ideas of which we rarely know the origin, since they have taken shape so unintentionally and without reflection that they only come into clear consciousness when complete, and concerning which a closer consideration shows us that we can be certain neither that they are constant in themselves nor that they are the same for all.

2. The first step in working up this material must be to get a clear consciousness of all our ideas such as they ordinarily present themselves—a consciousness such that we are able to grasp the content of a given idea,

and to be sure in repeating it that it remains identical. At the moment, of course, we are always conscious of what we are thinking, but this consciousness is fleeting, and often passes but lightly over its objects ; it does not guarantee that stability and constancy in our ideas which is demanded by logical requirements. We must accept here as granted by psychology that with increasing practice the conscious will, the inner attention, does succeed more and more perfectly in grasping the contents of ideas with a certainty and accuracy equal to that with which the sounds of words are remembered and reproduced without particular effort. The first step in learning to think consists in nothing else than in this careful attention to mental activity, with the object of gaining complete mastery over it.

3. Language is a powerful aid to the reproduction and remembrance of different ideas, but as it stands it is not altogether favourable to this logical endeavour. Words have many meanings, and according to the context in which they appear express very different shades of thought which are often remote in significance, and for our present consciousness altogether disconnected. Hence they often hinder the attempt to keep distinct and reproducible the ideas with which we work, and present difficulties when we try to excite in others ideas exactly like our own, or to form our own from theirs. It was with the sure touch of the master hand that Aristotle was wont in his analysis of concepts to begin by distinguishing the different meanings of words and to guard against the dangers of the confusion incident to speech by an investigation of the *πολλαχῶς λεγόμενα*. He carried out to its completion the Socratic method of always directing the attention from the word to the thought which accompanies it.

4. If we assume now that we have fulfilled this condition, and made ourselves clearly conscious of the ideas which we connect with words, our next task will be to analyse these ideas into their elements and to reduce them to simple acts,—to acts which every one performs in the same way. Where, when we think of the infinite manifold of the content denoted by language, shall we begin our operations? How shall we find any clue to making even a start in the process?

5. What we are always told is that we must compare our ideas with one another, so as to show wherein they are similar and wherein they differ, and in this way to break them up into parts. An object, it is said, is like others in various respects, and thus it breaks up of itself into different constituent ideas ; by virtue of each of these it is like some other objects, different from others ; by continuing this process in all directions we must at last

come to the simple parts which cannot be further analyzed, and at the same time we shall attain to abstraction, the power of thinking them by themselves, apart from their connections. Thus in the visible world form and colour fall apart inasmuch as an object is like a number of others in form, but unlike in colour ; while, again, it is like others in colour but different in form. This process is facilitated where an object undergoes changes in which some aspects change while others remain the same ; the leaf in fading forces me to distinguish the unchanging form from the changing colour.

6. This process is of course begun, and to a large extent carried through, under the psychological laws which govern the formation of language ; and it would almost seem as if our purpose could be achieved by merely completing what is already given naturally.

But even if it were an easy task, one which might be said to proceed of itself and without training or practice on our part, to recognise the like in the different and disengage it from its actual connections, yet this method would still fail to tell us where we must begin. It would fail again in its process of elimination to distinguish those characteristics which appertain to all objects as such, just because they are our objects ; and, finally, it would fail to give us any guarantee that the ideal elements to which it attains are always the same for all and possessed by every one alike. It is these alone of which logic can make use, and what we must aim at is first of all to find those elements which are the same for every one, and we shall then have in them a standard by which to estimate individual differences.

7. The content of thought is of infinite variety ; and if we begin our process of comparison and abstraction at random, we have no guarantee that we are starting from data which are the same for all. One thing, however, must be assumed to be the same in every one ; that is, the activity of thought itself, and the manner in which, together with its content, it appears in consciousness ; unless we assume this, Logic itself becomes meaningless. We may, therefore, begin with the problem stated above (2), for this activity itself involves certain original ideal elements. The fundamental condition is that we shall attain such a knowledge of our mental activity that we may be able to repeat it as consciously the same, and the very form of this activity involves elements which we must accept as absolutely the same for all. Such a consciousness of our own mental activity would be impossible without the ideas of unity, of identity, of non-identity, of similarity and dissimilarity ; we are conscious of our thought when we

observe how we distinguish one from others, and identify it with itself as opposed to something else. The same assumption is involved in the method of comparison and abstraction; it presupposes that thought always begins by distinguishing differences and recognising similarities, and by maintaining the distinction between one and others; and it thus assumes these elements as the natural properties of thought. But we really possess them only when we observe our mental activity, when we become aware that we are distinguishing and unifying; we must recognise them as simple, interconnected acts, through which alone the many and the different can enter into consciousness, and which are therefore involved in every content of thought. In so far as we are conscious of performing these acts always in the same way and of necessity, we have in them an original possession common to every one alike.

There is no moment of our waking life in which we do not find a number of objects presented to consciousness. Reflection upon what takes place when we think of these objects shows first a process of distinguishing in the sense that in passing from one object to another we are conscious of a new and distinct act, without ever losing sight of the first object. In this way we distinguish the acts through which we apprehend some object, and then another and another, and by comprehending together a number of such discriminated acts we obtain the consciousness of a plurality of ideas. Each of them is one in so far as we are conscious of grasping it in one act, and see no distinction in it itself, but merely contrast it as different from some other; each stroke of a bell, each distinct figure which we seize with one glance, is in this sense one.

The simplest example may be found in the contents of our field of vision. When we are not staring absent-mindedly in front of us, but are thinking about what we see, we discriminate objects by their different colours. As we glance from one to the other, the consciousness arises of successive apprehensions; and at the same time, inasmuch as the objects first perceived are still present, either towards the extremes of the field of vision or in memory, these apprehensions are connected together, and there arises the consciousness of a plurality of visible objects side by side. The same thing takes place with successive sensations; when I hear the strokes of a bell one after another I am conscious at each fresh stroke of a new sensation, while the one just past still lingers in memory; in this way I distinguish the new perception from the one before, and combine them in one consciousness into a temporal series. Finally, when I revert after some interruption

to a visible object which I remember seeing before, I am conscious of a new perception which I both distinguish from the original one and connect with it. And, in the same way, when I survey anew the present field of vision and come again upon an object which I noticed before, I am conscious of perceiving the same object over again, and discriminate it from others as the same.

In the distinction between successive mental acts we have the basis for the opposite concepts of identity and non-identity. What I represent to myself in a second act is either a second object distinct from the first, or it is the same presented over again, after the intervention of some other, and with a consciousness that the repeated presentation implies no difference of object. In a motionless field of vision non-identity and identity become intuitable through difference or sameness of position in space.

The mere perception that mental acts which are separated in time, and particular objects which are separated in time or space, are distinct, must not be confused with the recognition of likeness or difference of content in objects which are in the first place merely distinguished from each other. Before I can speak of likeness I must have distinguished at least two objects, and be able to maintain the distinction between them ; but it does not follow that objects thus distinguished are *different*. We are aware of the likeness or difference of given objects (visible figures, sounds, etc.) by a sort of immediate impression which may so far be included under the concept of perception in the wider sense. But the abstract concepts of these relations, through which alone they can become predicates in judgments, have their origin in reflection upon psychical processes, upon what we do and feel when we compare one thing with another ; and they presuppose the development of conscious comparison, of investigation of different relations.

The same relations extend to everything which is the object of perception and thought ; we can have a definite and clear consciousness of nothing except by a continual process of distinguishing, identifying, comparing, and finding likeness or difference. Opinion may differ as to whether those most elementary functions by which we merely distinguish amongst different impressions and get the idea of plurality are to be called judgments or not ; if we start from the formula "*A* is distinct from *B*," we assume the abstract concept of distinctness which can be got only from reflection upon our original activities, and is therefore based upon these ; hence there is some ground for saying that we merely notice distinctness,

thus distinguishing between those simplest operations and the statements which are contained in complete judgments. But we cannot have a clear consciousness of even that simplest action except in the form of the judgment "*A* is other than *B*." Clear consciousness consists in just this statement of what the objects of consciousness and their relations are, which have their origin in immediate psychical apprehension ; it is the expression of them in judgments.

8. Thus from the source of all methodical procedure, observation of what we do when we have the idea of an object, there arises the consciousness of certain original ideal elements. From this point of view the idealistic philosophy is right in teaching that the essence of intelligence is in that continuous reflection by which events at first unconscious are brought into consciousness, and that the self-intuition of the ego in its activity is the source of all knowledge. In looking for the conceptual elements which are the same for all we must begin, not with the fortuitous plurality of the content manipulated by thought, but with the activity of thought itself. That these fundamental determinations are simple and constant is guaranteed by the unity of self-consciousness, which would disappear if the functions in which it realizes itself were not always fulfilled in the same way.

9. Here enter the so-called principles of identity and contradiction, in the form in which they are generally presented by logic ($A=A$, not non- A). Generally, it is true, they are meant to apply only to the content of concepts, and not to the merely numerical, spatial or temporal distinction between objects ; but the formulæ may be extended to every object of thought, even to the particular which is distinguished only in time and space. On the one hand, they are imperatives, commanding us to maintain every object in memory as the same, and guarding against all confusions, all unnoticed blending amongst our ideas ; on the other hand, when presented as fundamental laws of thought, they state that the concepts of identity and non-identity are given with thought itself, and are the first immediate results of a reflection which is directed upon our mental activities and apprehends them in their fundamental forms (cf. 1, § 14. 6, p. 88, § 22. 6, p. 129).

10. In the fact that this reflection apprehends constant activities which repeat themselves in every mental act, lies the difference between the consciousness of these universal elements of thought and the mere apprehension of some particular mental event, such as the seeing of a particular

colour or the feeling of a pain of a certain intensity. They are the factors which constitute the unity of self-consciousness itself, not factors which refer to the plurality of its objective content; and in deriving the main concepts of the understanding from the transcendental apperception Kant was right, in so far as the self-consciousness which is one and the same in all particular acts realizes itself in distinguishable activities, amongst which the most original are just these processes of distinguishing and identifying.

11. But this is so only in connection with the function of which the result is the concept of unity. For when we recognise something as identical and as different from another, by that very fact we recognise each as one; and when we become conscious of these combined functions in their relation to one another, there arises with the concept of one that of *two* also, and here we have the foundation of all numerical concepts.

The justice of this view of the manner in which we arrive at the most primitive conceptual elements must be confirmed by the derivation of number and numerical concepts from these first beginnings. For if anything is perfectly determined and universally valid, thought and understood by every one in the same way, it is numbers; moreover numbers can be shown to be amongst the oldest and earliest constituents of language.¹

§ 66.

From consciousness of the activities which take place whenever we think of objects, there develops COUNTING and the concept of NUMBER, in which we are conscious not only of distinguishing and comprehending the particular acts by which we proceed from one to another, but also of the law of this procedure. Hence there develop also the concepts of particular numbers and the general concept of number as well as of relations of MORE and LESS and EQUALITY.

The possibility of continuing the series of numbers indefinitely at will depends upon our consciousness of the law of counting; the possibility of reversing the numerical series is due to the presence of the successive steps from unity to unity in the consciousness which combines them; and from the application of counting to numbers themselves arise those methods of calculation which again lead to an extension of the concept of number by the introduction of negative, fractional and irrational numbers, though they can never overcome the originally discrete nature of number.

¹ H. Hankel, *Zur Geschichte der Mathematik*, p. 7 sq.

Thus all numerical concepts are nothing but developments, in higher forms of synthesis, of the formal functions which we practise in every mental act by identifying and distinguishing.

1. It might be held that the idea of number arises at the lowest stages of human mental life from the sense-impression of things which can be counted, and that the ideas of numbers 2, 3, 4, etc., are got by simple abstraction from different groups containing the same number of objects. Not only does Mill hold this view, when he says that all numbers must be numbers of something, that there is no such thing as number in the abstract, and that the properties of number are really properties of things,¹ the algebraical symbols *a*, *b*, *c*, etc., symbols of things in general; even Bain² refers to sense impression, at any rate for the simpler numerical concepts. Number is for him a series of discrete sense-impressions, coloured spots, notes, etc.; unity is the abstraction from numerous concrete things, *i.e.*, from many *particular* impressions. Hegel himself says³ that the operation which gives rise to number is the counting of fingers, points, and so on; we can only *show* what we mean by four, five, etc.

2. There can of course be no doubt that it is in this way that we learn to count, that is, the examples which are shown to us make us aware of the meaning of the numerical words we hear, and that we make use of similar sensuous helps to facilitate our understanding of addition, subtraction, multiplication, etc. This process presupposes that it is immediately evident to us that each of the many things thus grouped together in counting is *one*. But it is not clear that this "one" can rest simply upon the sense-impression; Mill himself tells us that we can think of everything as divided into four equal parts, so that we may predicate of it every attribute of the number four. If so, what determines whether we shall think of it as one or as four? Evidently not the sense-impression, which is the same in both cases, but the manner in which we apprehend and interpret this sense-impression. But if this is so, then mere abstraction from concrete things cannot give us unity and the simplest numerical concepts, in the sense that nothing more is necessary than that a series of particular discrete impressions should be presented. The idea of one finger is certainly different from the idea of two fingers; but this does not mean that in the one case the idea of one, in the other the idea of two, is given with the objects themselves, and that just as we may abstract the idea of red colour from a number of red

¹ J. S. Mill, *System of Deductive and Inductive Logic*, I, bk. 2, ch. 6, § 2.

² A Bain, *Logic*, ii, 200 sq.

³ *Logic*, i. p. 238.

things, so we can without more ado abstract the idea one from a number of concrete things, or the idea two from a number of pairs of things. When we ask what is the point of resemblance between all things which we naturally think of as one—sun, moon, and stars, animals, trees and sounds—we find that in respect to their perceptible contents they are so absolutely different as to show no common element; and Bain shows the confusion of his presuppositions in attaching special importance to interrupted sensations, to transitions of consciousness. If the common element is to be the transition of consciousness, the interruption, then in order to perceive this we evidently have need of another function than that which makes us aware of the particular sense-impression itself; we hear the single toll of the bell, but we do not hear the pauses between the tolls, and in the same way we do not hear the number of tolls, although we hear the tolls themselves. In hearing a sound three times nothing is given but a succession of three sensations, not the idea of this succession nor the idea of the number three; it is only when we group them together in memory and become aware of the “transition of consciousness” itself, that the idea of plurality can arise. In the same way the idea of unity presupposes that we are conscious of the self-contained and definite act of perception of one object, in distinction from the repeated perception of other objects. All that we think of as one in the domain of sense is first disengaged from the continuum by such an act of defining and comprehending; the particular sound from the continuum of time, the particular form from the continuum of space. Change of sensation affords the *occasion* for this function, but purely passive change is not the function itself. It would be simply inexplicable how we could regard the same thing as one and as many if it were not due to the different way in which we *draw our limits*, and become aware of this limitation and progress to something else. We may regard one of the words before us as one, inasmuch as we combine a plurality of letters by one act of limitation into one image, and separate it from other images which are divided from it by intervals; we may also regard it as many when we turn our attention to the transition from one letter to another, and notice each step. Thus it appears to be a matter of choice what combinations we shall think of as one. Wherever it is possible to combine even differences in one consciousness, so that memory can survey them and connect the beginning with the end, and discrimination include the whole within one boundary, there we have the possibility of the one; wherever it is possible to think of differences within

a whole, or to combine a number of different objects while still maintaining the consciousness of their difference, there we have the possibility of the many.

3. This is the foundation of number. Each number is not mere plurality, but a plurality which is bound together and limited, and so far thought as a unity, in one act of thought which combines together the repeated acts by which we have distinguished the units. The possibility of number lies in our consciousness of the steps we take, and in the fact that the one act by which we combine two of our consciously performed unifications is distinct from that by which we combine three,¹ and can be clearly retained in memory.

We need hardly show in more detail that the formation of numerical concepts means the *determination* for consciousness of spontaneous activities, which are occasioned, but not necessarily produced, by sense-impressions; and that these activities are of a purely formal nature, in so far as they can not only be repeated with respect to any given content whatever, but may even guide the creation of a content to correspond to them, as when in rhythmical movements our production of a numerical content is guided by the thought of number. In this sense we may speak of number as abstract, but not as meaning that it is abstracted in the ordinary way from a given content. We should rather think of it as a spontaneous creation of thought in becoming conscious of its own activities, and as therefore, when it has once arisen, absolutely independent in its further developments of all that is sense-given.

Thus number shows itself to be the simple consequence of the fundamental functions of thought itself, the development grounded upon these functions of the discrimination of unities. In it we merely bring into definite consciousness what we always do when, in thinking of and discriminating unities, we pass from one to another. Hence it is of the most universal application, and in developing its laws we do no more than develop in one particular direction the laws of activities which are the basis of all thought.²

4. Not only is this view of number instructive for our methodological studies; it is a further confirmation of what was shown in § 41, 6. 7 (I. pp.

¹ In this sense Bain is right in saying (p. 200 and elsewhere): "By memory we can easily retain a small succession of interrupted sensations or transitions of consciousness and identify it with another small succession."

² See Appendix A.

258 sq.) viz., that the assumption (generally involved in the schematic representation of concepts as a sum of attributes), that we can carry the analysis of our *ideal elements* back to isolated, independent attributes, is false. Unity and difference, one, two and several, are inseparably connected for thought; the acts of unification and discrimination are mutually conditioned in such a way that neither can come into consciousness without the other. Counting itself, as the general form of conscious progress from one unity to another proves to be a function which, though it presupposes discrimination, cannot be regarded as simply a combination of other acts. Thus instead of isolated elements we find interrelated functions, and get already a glimpse of the end to which the analysis of our conceptual elements leads; we see that the theory which hopes to derive everything from particular elements of sensation, originally separate and given in successive moments, overlooks the essential determinations of a thought which has its root in self-consciousness.

5. The consideration of number shows also that just because related functions are so inseparably connected certain general forms of synthesis are involved in the simplest and most primitive mental functions. In our consciousness of one together with another which is different from it, we have the most fundamental synthesis which is consciously realized in the number two. And since this synthesis can be applied to any series whatever, counting appears as a general form which brings into consciousness its own laws as a process from one unity to another, accompanied at each step by the combination of previous acts into a new unity. As we become conscious of this form and this law we avail ourselves freely of them, and create an infinite series of particular numbers in accordance with the law; we need no other help from sense than the numerical word or symbol which enables us by its characteristic form to remember and distinguish the particular numbers, in a way which the uniform law of their production could not do.¹

6. When we actually carry out this process in the numerical series $1 + 1$ or 2 , $2 + 1$ or 3 , etc., we obtain a series of definitions in which both the particular elements and the form of their synthesis are fully determined. They have the character of nominal definitions in so far as they convey the meaning of the particular numerical terms, but from the first they are also synthetical definitions (§ 44, 4, I. pp. 290 sq.), inasmuch as the

¹ On Helmholtz's proposal to base arithmetic on a fixed series of symbols see Appendix B.

corresponding ideas must always be constructed. Each of these concepts is completely determined and has therefore no extension (§ 42, 5, 1 pp. 270 sq. cf. § 26, 4, note pp. 159 sq.) and each is logically perfect because we are conscious of its origin in definite conscious acts.¹

¹ J. S. Mill, in the passage cited above, contends that propositions such as $2+1$ is 3 may be regarded as definitions. Certainly, if we regard them as propositions referring to *things*, they have all the appearance of merely identical propositions. The assertion "Two pebbles and one pebble are equal to three pebbles" does not affirm equality between two collections of pebbles, but absolute identity. It affirms that if we put one pebble to two pebbles those very pebbles are three. The objects, therefore, being the same, and the mere assertion that "objects are themselves" being insignificant, it seems but natural to consider the proposition "two and one is equal to three," as asserting mere identity of signification between the two names. This, however, continues Mill, though it looks so plausible, will not bear examination. "The expression 'two pebbles and one pebble,' and the expression 'three pebbles,' stand indeed for the same aggregation of objects, but they by no means stand for the same physical fact. They are names of the same objects, but of those objects in two different states: though they denote the same things, their connotation is different. Three pebbles in two separate parcels, and three pebbles in one parcel, do not make the same impression on our senses; and the assertion that the very same pebbles may by an alteration of place and arrangement be made to produce either the one set of sensations or the other, though a very familiar proposition, is not an identical one. It is a truth known to us by early and constant experience, an inductive truth; and such truths are the foundation of the science of numbers. The fundamental truths of that science all rest on the evidence of sense; they are proved by showing to our eyes and our fingers that any given number of objects, ten balls, for example, may by separation and rearrangement exhibit to our senses all the different sets of numbers the sum of which is equal to ten."

The whole of this explanation shows in a most characteristic manner to what expedients we must have recourse if we try to base arithmetic upon the crude empiricism of sense-impressions. It seems almost as if thought such as this concerning the nature of number had stopped short on the level of the elementary school; as if the empiricist did not know how to count three unless the objects counted are all together, and could not recognise them as the sum of one and two unless he laid one of them apart. He forgets that in order to be certain that after such an operation we have still the same stones, we must ultimately rely upon counting, and would have to trust the evidence of his senses just as much if a juggler should manipulate the three pebbles in such a way as to show them producing two parcels of two, and again appearing as three when put together. Merely for the sake of showing that man is no more than an animal, merely in order to eliminate all thought from logic, the harmless proposition $2+1=3$ must be taken to signify the physical law that three pebbles which are arranged thus \blacktriangle can always be arranged thus, $\bullet \bullet$, and our repeated experience of this is to be our only ground for believing the proposition! Here again Bain follows his predecessor, though his language is not so clear, when he says (p. 201, and elsewhere): "Addition is the next fundamental notion; also obtained, in the last resort, from the senses. When we bring two detached groups, or successions from different places to the same place, or into one continuous group or succession, we are said to add; the implicated contrary is to subtract." Here the important distinction between bringing together and adding is obliterated; the external means by which we stimulate counting and facilitate consciousness of it is taken as the cause producing it.

7. Here again we may see a relation of the general to the particular comprehended under it, which seems contrary to all accepted ideas concerning general and special, super- and subordinated concepts. Looking at numbers as they stand, it might certainly appear as if the general concept of number were a simple and easy abstraction from particular numbers, these being the first known, and the concept of numbers in general being attained by disregarding the differences in particular numbers. But we forget here that counting as the universal form of the process is necessary to the formation of particular numbers, and that by it the series of successive numbers is produced; hence it is rather the nature of the universal which gives us the plurality of the particular, and the generality in question ultimately resolves itself into the consciousness of the spontaneity of thought, in which lies both the power of continuing the same process indefinitely and the consciousness that the law of the process is always the same.

8. This peculiarity of the general concept of number shows itself clearly in the endlessness of the numerical series. When a general concept is obtained by abstraction from a number of ideas we may indeed be uncertain whether the particular ideas we know exhaust the extension of the general concept, and the indefinite possibility remains that the same concept may be found in other particulars; but here it is part of the nature of the general concept to admit of an indefinite progress, so that it cannot be exhausted by any number of particulars. Inasmuch as every number to which we come can be increased by one it seems to be a part of the concept of number itself to actually comprehend an infinite series of numbers; but inasmuch as it is necessary to think of every series of ones as a unity, the complete concept of number in general can only be applied to finite number, and an infinite number is a *contradictio in adjecto*, since the term implies that the limitation by which number becomes what it is, is now absent. Thus the expression indicates rather an impossible task to which we cannot even approximate, since, count as long as we may, we shall always be as far removed as ever from the infinite. From this point of view, therefore, number is a concept of which one element is limited by the other, and is thinkable only when so limited. The concept of infinity means logically only that in the concept of number itself no definite limit is set to the process of counting; but to speak of an infinite number as if it could be used for calculations as finite numbers are, leads necessarily to contradictions.

9. From that nature of numerical concepts as above interpreted, purely logical as it is as first, still further developments proceed. The comprehension of successive steps in one consciousness, together with the consequent continuation of the consciousness of the former steps in that of the later, gives rise to the process—at first in memory—of running through the series backwards; this takes place in the simplest cases when we break up the number two into its units in order to combine them afresh, and it is only in this backward and forward play that the permanent consciousness realizes itself. As with the sight, which does not fix itself, but travels through the field of vision and returns to the same point, so with any given number, the series of steps through which it has been produced may be traversed backwards, and the numbers be thus produced in reversed order. Here another point arises. In counting upwards 1, strictly speaking, cannot be regarded as a number; counting does not begin until we pass to the second one, and hence 2 is the first number. But when we count backwards, and find that 1 proceeds from 2 just as 2 does from 3, then 1 takes its place as a uniform member of the series, and according to the same principle brings with it zero also; the original distinction between unity and number disappears, and only appears again in the special significance of unity in the more advanced methods of calculation.

But this apparent extension of the original concept of number is not real. A unity can never be grasped by itself; it never appears in consciousness but as member of a series, for we can be conscious of it only as we distinguish it from others, and only discriminating reflection can isolate the one act from the process with which it is connected. We may arrive at the same conclusion in another way. We have treated counting as a progress; but this progress presupposes the continuing consciousness of whatever objects were distinguished, even though these are represented merely by external or internal acts which have remained in memory. Each one of these co-existing objects forms a member of the series, and when we think of them altogether it is indifferent with which we begin; the first is as much a part of the series as the others. Moreover, because the consciousness of a something presents itself as distinct from negation, zero also presents itself as the natural conclusion, or, on the other hand, as the natural beginning of counting; so that the comprehensive consideration of the complete functions which lead to number justifies us in ranking one and zero as numbers.

10. We must distinguish between the general concept of number, which derives its meaning from the law of counting alone, and the use of letters in arithmetic as general numerical symbols. In the formula $a + b = b + a$, a and b do not express the general concept of all number, applying indifferently to the infinite series of particular numbers, and the propositions referring to a and b state nothing about this general concept; the letters are symbols of any particular number whatever, but always imply that it is a definite one. We cannot make calculations with number in general, but propositions may easily be stated which are true of any number in the whole series, and they are expressed by means of these symbols for which all values can be successively substituted. Here then a and b are not symbols of concepts, but only common names of all the particular objects contained under the concept; only in so far as the series of these is incapable of completion do they participate in its consequent indeterminateness.

11. The relational concepts of More and Less (which we can distinguish as greater and smaller numbers only indirectly) are included in counting itself. That which we reach by counting further is more, and when we look backwards every previous stage is less; the proposition that if $a > b$ and $b > c$ so much the more is $a > c$, expresses merely a consciousness of the law of progress, and is therefore involved in the law of the construction of numbers; it is the most elementary comparison without which numbers themselves cannot arise.

The concept of equality, when applied to numbers, depends in the same way, upon those processes by which numerical concepts are formed. Its ground lies in our consciousness of the sameness of the process by which in different instances or at different times we obtain the number two or three; in this sense we should speak of the *identity* of numbers. The concept A , though thought at different times, does not appear as a plurality of similar A 's; and in counting 2 or 3 over again we should not speak of counting equal numbers, but the same.¹ Equal can be used only of numbers of different things; the number of fingers on the left hand is equal to that on the right hand; the distinction presupposed in equality is distinction in the objects to which counting refers.

The concept of equality as applied to abstract numbers is justified only when these numbers are attained by different methods, when the same processes are not repeated, and when therefore the difference lies in the

¹ Cf. Helmholtz, *Phil. Aufs. zu Zeller's Jubil.*, p. 24.

method of counting itself, not merely in its external occasion. But the symbol of equality has a different meaning in arithmetic and in geometry ; in the latter it applies to objects (lines, surfaces, etc.) which are separate in intuition, in the former to concepts which are ultimately identical.

12. In addition we have the first example immediately connected with counting of a method of attaining a number which differs from elementary counting itself. In simple counting we proceed from 1 as our starting point, but in addition a twofold counting takes place, inasmuch as the unities which are added to the first number must be counted themselves ; thus addition rests upon the possibility of referring the same increment of unities to two points at once, of regarding the same unit as part of two numbers—hence upon a consciousness of the absolutely identical repetition which is contained in counting and which enables us to take any point in a numerical series as the starting point of a new process of counting. To this is due also the possibility of breaking up any number as we like, of representing it as a sum of smaller numbers, and from this is derived subtraction, which presupposes such an analysis. The proposition $a + b = b + a$ tells us nothing more than that the particular groupings by which we reach the same number through different combinations of smaller numbers is a matter of indifference.

13. It is unnecessary to repeat for every method of combining numbers the same proof that all are originally concepts of functions in which the fundamental process of counting is repeated, syntheses which are applied to syntheses, and that these concepts can arise only from the consciousness of a spontaneous activity. Even the arithmetical signs $+$, $-$, etc., are always on the one hand symbols of a problem, imperatives which command some operation, on the other hand symbols of the simple numbers obtained by this operation ; and all calculation consists finally in reducing manifold combinations of numbers to simple counting, the value of a sum or a product to its expression in the natural numerical series. Progress beyond simple counting depends upon the power of voluntary combination, from which every form of numerical combination could arise, just as they actually do arise when excited by the relations of enumerable things.

14. These combinations lead us naturally beyond the original sphere of the simple numerical series. The reversed methods of calculation, such as counting backwards, subtracting, dividing, finding roots and logarithms, are of course first obtained by simply retracing the steps taken in counting, adding, multiplying, and involution, but as soon as the concepts have

been found in this way they can be applied—at any rate in the form of a question or problem—to any number whatever. In the expressions $a - b, \frac{a}{b}, \sqrt[3]{a}, \log_2 a$, we can give any value from the actual numerical series to a and b , and thus we get the problems $2 - 5, \frac{5}{2}, \sqrt[3]{5}, \log_2 5$ —problems which are insoluble by the original concept of number, and cannot therefore be equated to any number. Nevertheless, we speak of negative, fractional, irrational numbers, on the ground of an extension of the concept of number, which we begin to apply to all combinations arising out of the methods of calculation; and the extension is justified by the fact that those combinations resemble actual numbers, at least in the one respect that they are all comparable as more or less, and can therefore be brought into a series, and that therefore the different methods of calculation—similarly extended in meaning—can be applied to them with all their formal laws of combination and transmutation.

Imaginary numbers take us a step further in the extension of the concept of number, for in them appears not merely the impossibility of finding in the original series of whole numbers the number which is yielded by the formula, but also a contradiction of the formal laws of combination which must hold good for negative numbers. If all square numbers must be positive, the problem contained in $\sqrt{-a}$ contains a contradiction in the combination of signs, while the formula \sqrt{a} merely fails to represent any number for certain values of a ; the former problem does not simply arise from the possibility of applying reversed processes of calculation to all possible actual numbers, because $-a$ could not be obtained by a process of squaring.

From a purely arithmetical point of view the distinction between fractions as rational numbers and roots which cannot be expressed in numbers as irrational, cannot be justified; $\frac{1}{3}$ has as little claim to be called a number as $\sqrt{2}$, and it is only because we are more familiar with the process of division as applied to given continuous magnitudes that we imagine ourselves to be still dealing with something tangible. Only because we are always able to divide any area taken as a unity into three equal parts, each of which we regard again as a unity, does it seem to us that $\frac{1}{3}$ or $\frac{2}{3}$ is a comprehensible number with which we can reckon just as we do with 1 and 2, and find no difficulty when called upon to multiply it by 3. In the case of $\sqrt{2}$ this easy analogy fails us, and we are confronted by the twofold impossibility of conceiving of $\sqrt{2}$ as a unity of parts in any

way attainable by counting and of actually multiplying it by $\sqrt{2}$. Supporters of the view that all arithmetical concepts are based upon the evidence of the senses might well be called upon to show upon what sense-impression the concept of a root or a logarithm is based.

15. The possibility of inserting any number of fractions and irrational numbers between any two consecutive numbers, thus diminishing to any extent the difference between successive members, seems to reduce number—which originally arose out of discrete steps—to a continuum, and to transform it into a magnitude in the same sense in which space and time are magnitudes; hence we often speak of numerical and spatial magnitudes as if they were co-ordinate. But a strict interpretation of the concepts forbids us thus to bridge over the cleft between them by an inaccurate analogy; number is always discrete, for however far we may push our process of interpolation, we can never succeed in reaching more than a finite number of intervening members, and the law according to which the interpolation takes place, though it fixes no definite limit, must always, from the nature of the numerical expression, call upon us to break off somewhere. It is with infinite division as with infinite number, it contradicts the logical character of number and numerical expressions, and the infinitely small is of as little avail for purposes of calculation as the infinitely large. The continuous progress which intuition gives us in space and time can never be really expressed in the logical forms of number, which depends upon unity and difference in determinate acts. In the differential calculus, we attempt to break down the limits which arise from the nature of number, but we succeed in forcing upon it the expression of continuous change only by means of artifices in which we avoid assigning any numerical value to the differentials themselves, and confine ourselves to expressing their relations to each other in numbers and combinations of numbers.

16. By tracing these conceptual elements and their developments we come to see that a number of interconnected and mutually dependent functions arise from the conditions under which all consciousness takes place; we find that our ability to reflect upon these functions and their connection is the basis of the arithmetical concepts, and that the fundamental function of counting is re-directed upon particular acts of counting, as, for instance, in multiplication, when we count how often we complete a given number in counting straight on, thus performing three series of countings applied to each other, the whole activity consisting merely in

becoming conscious of the positing of one unity and progress from it to another.

It is evident, therefore, that in arithmetic we are in the same sphere as in logic, and have to deal merely with an increasing complication of those activities upon which all thought depends, facilitated by a method of notation which owes its efficacy to the perfect determination and yet complete generality of its symbols, qualities which are possible because they apply not to the manifold of an ideal content, but to formal activities of consciousness which are the same whenever they recur.

17. These concepts are *a priori* in nature, inasmuch as the functions must be regarded as ultimate and of immediate certainty, and inasmuch as we can find no explanation for them in the nature of external impressions, but only in the nature of consciousness itself. In every one whose consciousness resembles ours we must assume the same functions and the same power of reflecting upon them, and are therefore certain of their identity in all thinking subjects; and as any given content can come into consciousness only through these activities, we have here conceptual elements which enter into all thought, and from which nothing can escape which can be distinguished and compared in definite acts of thought—nothing, therefore, which can be thought of at all.

But these elements are purely formal; they leave it quite undetermined what is to be comprehended and distinguished as one and as many. If we turn now to the content, of which we gain a clear consciousness by means of these forms, then our investigation comes in the first place upon ideas of the external world, and in these the first elements which present themselves, and which, because of their peculiar nature, can be easily and certainly distinguished as part of the objects even by unskilled analysis, are the ideas of space and of time, and of spatial and temporal determinations and relations.

§ 67.

The attainment of elementary and perfectly definite concepts of the Spatial presupposes that the idea of SPACE must have already arisen without any conscious activity on our part; no conscious process of the creation of the idea of space out of particular elements takes place as in reference to number.

On the other hand, the continuity of space prevents us from establishing by means of mere abstraction from perceived forms and positions any

perfectly definite and invariable elements of the idea such as are actually given in the straight line and the concepts dependent upon it—the plane, the right angle, etc.

The possibility of forming the geometrical concept of the straight line within the general idea of space must in the last instance depend upon the fact that the straight line is a determining element in the genesis of the idea of space itself; it is the direction in which we project objects into space; but the mutual relations of different straight lines by means of which all the particulars contained in our idea of space are originally determined as to position are themselves determined by an unchanging and invariable law of our space intuition. The necessity of this law can only be recognised; it cannot be deduced from any other law, and so far it is empirically known.

The means by which we determine different spatial relations according to the established laws of spatial intuition is MEASUREMENT, the reduction of these relations to comparisons of lines and angles and to numerical relations.

1. It would seem that in order to obtain simple fundamental concepts of the Spatial which are the same for all we must adopt other methods than those which lead to the numerical concepts. We create number consciously, we *learn* to count, but we do not learn the intuition of space; we find it always present in the idea of the external world. Hence we cannot speak of the creation of the idea of space by particular conscious acts; when we learn to count the idea of spatial things is there already, and we cannot think of a conscious state preceding the idea of space as we can think of one in which the concept of numbers is not yet contained.

Hence the construction of geometrical concepts must always start from an idea of space already present, and the question is, how do we pass from this idea to concepts which are logically perfect, and what are the elements of these concepts?

2. The most obvious way seems to be by a simple process of comparison, by means of which we pass by abstraction from the sensuous qualities of colour and so on first to the concepts of different forms, and then by abstraction from the difference of the forms to that which is alike in all perceptible objects—the concept of Extension. Here it is assumed as a matter of course, or regarded as proved by experience, that this attribute of extension is the same in all objects, however different, indepen-

dent of their position as of their other attributes, and of the same nature in the most different magnitudes.

Descartes, for example, employed this concept of *extensio in longum, latum et profundum* as needing no further analysis ; he regards extension as an attribute of things like any other attribute, and for this reason cannot conceive of it, wherever it may be perceived or thought, except as attribute of a thing, of a *res extensa*. By attending to the most general differences of the extended we may get the concept of Magnitude from the intuition of one body containing another, or by the comparison of a part with the whole ; the Divisibility of extended objects, whether actual or ideal, is also given in immediate intuition.

3. But this manner of explaining the genesis of the concept of space by abstraction from the extended objects of intuition overlooks one essential element—the fact that particular objects are presented to us not only as each extended in itself, but also as spatially related to each other in different directions and at definite distances apart. It overlooks moreover that our immediately given space-idea includes extension to which there is no corresponding sensation (none, that is, which is ordinarily perceptible). The air in which we move and through which we see makes no direct impression upon our eyes ; for original sense-perception space filled with air is empty space, not an extended thing amongst other things, one body amongst other bodies ; indeed it would be impossible either by sight or by touch to get the idea of external, visible and tangible bodies if space were entirely filled by visible and impenetrable objects. Hence it is not necessary in order to get the idea of empty space to get rid of the bodies which fill it by an effort of thought ; our ordinary idea always contains both ; extended things are in empty space, which surrounds and separates them, and the expression that perceptible things fill space is made perfectly comprehensible and familiar by our original way of regarding it. It will not do therefore to attempt to construct the idea of space only by abstraction from sense-perceptions, such as might be regarded as the primary consequences of affections of sense, and which, together with colour, contain extension ; we cannot in this way explain the position and arrangement of particular images in an all-embracing space. To say that the extension of perceptible bodies is similar to the extension of empty space, and that our experience of extended bodies moving through empty space justifies the abstraction of an extension common to that which is empty and to the body which fills it, is true from one point of view ; but in order

to carry out these comparisons we must first imagine the whole of space, and we might just as well say that not until we think of particular extensions as parts of a total extension are we conscious of their homogeneity, and if therefore we speak of space as an enlargement of that which we abstract from particular objects, we must speak also of a limitation of the totality of space which is presented together with particular objects.

4. If we turn to psychology for information as to the genesis of the space-idea and for an indication as to where we must seek the real beginnings of the idea and the law of its construction, we find the difficulty rather confirmed than solved. The problem before psychology is to explain how our sensations give rise to our idea of space ; how, from the superficial ideas which alone, from the nature of the organs of sight and touch, can be the immediate consequences of particular sense-affections, a three-dimensional space can be constructed, and whether even the superficial arrangement of colours in the field of vision and of touches upon the surface of the body is not something composite, and needing to be explained into psychological processes.

5. With reference to the psychological genesis of the idea of the spatial world the question as to whether the space-idea is *a priori* is relevant, in the sense that it is possible to maintain the thesis that the mere fact of a number of adjacent nerve-endings being touched is not sufficient to ensure the production of a spatial idea of a superficially extended field of vision, much less to explain the third dimension ; but that there must also be something in the nature of the thinking subject which necessitates his arranging the plurality of sensation in just this spatial order—a necessity which is merely directed and guided by the sensation as immediate consequence of the stimulation of particular nerves. In this sense a careful psycho-physical analysis could subscribe even to Kant's statement that "that wherein alone sensations can range themselves and be placed in a certain form cannot itself be sensation" ; but not to what follows, that for this reason we must be able to consider the form apart from all sensation. For in psychology it remains certain that the idea of space is never given to us apart from actual sensations ; that in the first instance we think of space only in that extension to which actual sensations impel us, and that even when we pass beyond in fancy we are always accompanied by dim images of the visible ; we help ourselves to increase the distance by imagining certain marks in space which we are forced to think of as traversed by something which is presentable to sense. Kant's proposition that space

is thought of as an infinite given magnitude is certainly false as the expression of a general psychological fact ; we can never think of infinite space in the same way as of a space which is finite, even though it may be indefinitely extended. It is also false that all the spaces of which we think are thought of as parts of one and the same continuous space. The space in which my imagination constructs geometrical figures is not part of the space in which I place my real world ; it is of the latter only, and not of any conceivable space, that we can say that it is one and only one, and that all particular spaces are only its parts. But it is true of all alike that we can never actually think of it except in connection with some content.

6. For this reason the fact that space is psychologically *a priori* cannot be simply utilized for our conceptual determination in the sense that in the space-idea we are quite independent of external affections and guided only by a subjective law of thought. It might be true that from the nature of our representative activity we were forced to range our sensations in this form of continuous proximity, and yet that the particular manner in which we construct our space-idea depended upon the nature of our sensations, and ultimately upon the nature of an objective space containing the things which give rise to our sensations, and was therefore merely empirical. For example, it might be that the perfect equivalence of the space-idea in all its parts, the possibility of conceiving any figure as transferred to any part of space without change, without being either increased or diminished, was only due to the fact that there are actual solid bodies, unchangeable in form, which, according to natural laws, do admit of these changes of position in every direction. Then the impossibility which we feel in imagining any other than the space to which we are accustomed would not be absolute, but would have arisen merely from an unbroken experience ; we could only say that we actually do represent space in this way without being conscious of any necessity in so doing. On the other hand, when we say that twice two is four, we see not merely that it actually is so, but that it is necessarily so ; and even if the world were so ordered that by a constant trick of magic a fifth thing came into existence whenever we put two and two together and vanished again when we separated them, this could only justify the proposition that two things and two things make five things, never that twice two is five.

7. The issue of this dispute has no immediate and decisive importance for our investigation. Whether the psychological processes which give rise to the space intuition have their origin exclusively in the subject, or

whether they are partly conditioned by the nature of the object of sensation, there is no doubt that so far as concerns the logical analysis of the space idea, the intuition must be taken as given. We can show neither processes by which we could consciously change or vary this space idea, nor processes by which we could consciously produce it in such a way as to refer the particular steps to simple acts, as in counting. The chief obstacle to such an analysis is that continuity of our space intuition which Aristotle recognised as the fundamental difference between space and number. We might attempt to discover creative acts which could be consciously repeated after being unconsciously practised, such as the traversing of space in different directions with the movement of sight or of imagination; but then this activity itself is not simple, since a plurality is distinguishable in what it is to create; nor is it determinate, containing its measure in itself. The simple method of drawing a straight line from some point and then moving it in such a way as to describe a plane, and the plane so as to construct a solid, shows that the idea of space cannot be really constructed, but that these operations can only take place in a space which is already there and from which they derive their meaning and results. Moreover the concept of movement conceals just that element of continuity which cannot be treated as something simple, and the directions of movement always presuppose space. The distinction between intuition and thought, between intuitive and discursive ideas, expresses just this fact—that we cannot produce space in this sense by means of simple conscious acts.

This being so, our logical discussion must start from the general idea of space such as we actually find it. All we can do is to note distinctions within it, and thus to obtain ideal elements which are as simple and definite as possible, but which are all derived from the general idea, and presuppose it.

8. Such distinctions we find appearing in language with reference both to the spatial extension of things themselves, and the manner in which they are spatially related to each other.

With respect to the first, language distinguishes not only the different forms and magnitudes which present themselves to perception, but it also draws a distinction between the superficies and the solid, between the line and the plane, the point and the line; and in doing so it is guided by the nature of the intuition itself, which could never take place without these distinct elements, and in its concrete determinations always involves them.

For sight as for touch surfaces alone are the direct objects of sensation ; they are an element which can be grasped independently, and to them is added the idea of the solid of which they are the superficial boundaries. Surfaces themselves however appear in the field of vision as variously coloured, each one bounded by others of different colour, and it is due to the primarily superficial arrangement of the parts of the field of vision that these boundaries also appear in consciousness as lines. They do not, indeed, appear in direct sense-perception ; the boundary line which divides two surfaces is not seen in the sense in which the surfaces are seen, for it has no colour ; but unless we were conscious of the limits by which two different coloured surfaces are divided, their arrangement side by side in a continuous field of vision would be impossible. In the same way the edge in which different planes intersect reveals itself to touch as the boundary separating dissimilar sensations. It is true again of the point as the end of a line which was followed so far continuously ; the point is invisible, but we are conscious of it as the apex of a solid, or as the corner of a superficial figure, because of the sudden and contrasting changes of direction.¹ It is only through this power of perceiving and representing boundaries that we can perceive forms, it is attention to the course of the boundary which makes us conscious of them and able to remember them.

With respect to the second point, the spatial arrangement of objects, ordinary perception distinguishes right and left, above and below, before and behind ; and in so doing we take as starting point our own body with the positions of equilibrium and possibilities of movement determined by its organization, directions of movements being referred to and distinguished by its principal axes. We further distinguish distances in these different directions, and measure them by the paths which have to be traversed by movements of the eye, the hand, or the whole body.

¹ Mill shows little reflection upon the nature of seeing as it ordinarily takes place, when he says (*Logic*, vol. i., bk. 2, ch. v. § 1), "The points, lines, circles and squares which any one has in his mind, are (I apprehend) simply copies of the points, lines, circles and squares which he has known in his experience. Our idea of a point I apprehend to be simply our idea of the *minimum visibile*, the smallest portion of surface which we can see . . . we cannot conceive a line without breadth ; . . . all the lines which we have in our minds are lines possessing breadth ; but we can attend only to the length of these lines, and abstract from the breadth." No doubt the marks by which we indicate lines upon paper are planes ; but where is the breadth in the line where the sea ceases and the sky begins, or in the line made by the ridge of a house against the background ? And is not this line perceived and distinguished as a line from the plane surfaces, notwithstanding that it can never be seen without these ?

9. It might be thought that by continuing our process of abstraction and analysis of the forms and positions presented in perception we must at last come upon the simplest and most completely determined elementary concepts of space ; concepts of lines, surfaces and solids, as well as concepts of directions, distances and positions. But even if the point might perhaps be reached in this way as a perfectly simple and definite idea, yet as soon as we pass beyond the point simplicity ceases, inasmuch as no space-idea which is in the least extended can really be brought into consciousness by one indivisible act ; there is always presupposed the comprehension of a continuous manifoldness, distinguishable into parts, a comprehension moreover which cannot take place merely as a repetition of the simple, of the point, since points can never give rise to a line. Determinateness also disappears, for the forms actually presented are so incalculably manifold, and pass into each other by such imperceptible differences, that the method of mere abstraction would be confronted by a chaos the confusion of which no art could regulate ; the bare evidence of the perceiving sense could afford no inducement for preferring one line or one form rather than another, nor any guarantee that in the ideas presented and reproducible in intuition we have elements which are unchangeable and always repeated in the same way. Geometry would thus be inexplicable.

10. More especially would it be quite incomprehensible from this purely empirical point of view why the straight line and the right angle play such a predominant part amongst geometrical concepts, and are looked upon as perfectly determined standards for all forms and relations of position. Apart from the creations of human art nature is very sparing with anything approaching to a straight line such as might give rise to the construction of the concept, while she is wastefully lavish with curves and curved surfaces. In the wilderness of the uninhabited earth there are but very few objects which could leave as their copy the idea of a straight line, and what should draw attention to such exceptional cases, especially as they merge through many gradual transitions into forms of increasing irregularity ? Do we not regard every perfectly straight line which we discover in a landscape as a sure sign of human work, whether it be a straight road, or a watercourse, or the outline of a pyramid ? And it is the same with the right angle.

If the importance of the straight line cannot be explained in this way, then the fact that man theoretically regulates space by means of the

straight line and the right angle, and practically takes the straight line as the basis of his own creations as persistently as nature rejects it in hers, must have its origin elsewhere than in our perception of external forms. It must be due to something else than mere reception of what is presented, that the straight line is accepted as a perfectly determined idea, thought by every one in the same way, and therefore a concept which can be utilized in science.

11. The problem finds its solution when we remember that our sensuous representation of the spatial world first arises when—as it is generally expressed—we project our sensations, when more especially we transfer our visual images into space and so localize them. The manner in which this is done may be very obscure, the psychological functions called into play quite unexplained, but the fact that our intuition of objects which are separated from us by space has its origin in a mental activity which supervenes upon particular sensations is unquestioned and indisputable. It is as indisputable that the straight line is the direction in which this activity presses forward, and as it were repels objects outwards from some point in the body in order to range them at different distances; to speak in more concrete terms, as our space-perception is mainly developed through sight, if not entirely due to it, the straight line is originally the line of sight,¹ the direction in which we look upon coloured images, the line moreover in which our actual movements tend, although the movement of our limbs in a straight line is hindered by their construction. We are too much accustomed to assume first that objects exist external to us and then become visible by means of their rectilinear rays of light, to grasp easily the reverse idea that first of all we have projected them in straight lines issuing from ourselves, and that our localisation of them always continues to be determined by the original idea of direction which is accurately expressed by straight lines alone. Thus the straight line is really the vehicle of all actual spatial intuition in which the position of each visible object is referred to the self as the intersecting point of all directions. It is independent of direct sensation and in so far it is non-sensuous in nature. Inasmuch as it is presupposed in every space intuition which we have, it

¹ I know of no thorough and complete investigation of the development of spatial intuitions by means of touch alone, such as must take place in the case of those born blind. The fact that their space-idea coincides with that of people who see proves that similar conditions must be present, although it is difficult to conceive how this may be since the movements of the organs of touch naturally take place in curves; this is only an additional proof of the non-sensuous nature of the fundamental idea of the straight line.

is *a priori*; and this reveals itself in the fact that no sensation can place a limit to the mental activity which presses forward into space, and that we find it possible to think of every straight line as indefinitely lengthened.¹

When visible and movable objects at different distances intercept each other, we become conscious of rectilineal direction, and this consciousness is confirmed by movement towards fixed points. This ideal of the straight line must always be present if we are to recognise as straight any perceptible line, the limit of some visible plane or an edge; we test it by trying whether all its points cover when brought into the line of sight. It is not until afterwards that we find that the rays of light are straight because we really come upon the objects when we move towards them in the direction of the line of sight, the correctness of our localisation depends upon this fact, but not the process of localisation itself.²

If our concept of the straight line arises in this way, then it is closely connected with those elements in our space-idea which cannot find an explanation in sensation—with the third dimension and empty space. We find, moreover, that in constructing the concept of the straight line we merely bring into consciousness what we do when unconsciously and involuntarily we first project our visual images; we find here the fundamental form of the procedure in which our space-idea has its unconscious origin.

12. Together with the directions in which we project objects, we get,

¹ Zindler in his *Beiträge zur Theorie der mathematischen Erkenntnis* in the *Sitzungsberichten der Wiener Academie*, vol. 118, ix. p. 32, gives the name of "axiomatic concepts," or "axioms of definition," to those ultimate conceptual elements which cannot be further defined but are fully determined. The name is, indeed, open to objections in so far as axioms are always understood to be propositions; but I am in material agreement with him in counting the straight line amongst these "axiomatic concepts" (p. 3).

² It might no doubt be urged that we also distinguish directions from right to left, up and down, within our field of vision; directions which we traverse by moving the focus of sight by means of the muscles of the eye, and of which therefore the idea is associated with the idea of this movement. But this consciousness of opposite directions, left and right, up and down, and the reverse, does not reach the full determination of the concept of the straight line; just as we can speak of East and West, North and South without referring to accurately determined points upon the horizon, just as we can say the sun moves in the sky from East to West, although it moves in a curved line, so the completely determined straight line may be absent from that idea of direction according to which we distinguish the different directions within our field of vision; it is indeed much more difficult to say with certainty of a line drawn across the field of vision than of a line receding from us, whether or not it is straight. We always refer mentally to the line of vision, and we learn to judge which of the lines given in the field of vision would prove to be straight if placed in the line of vision.

by help of our consciousness of the movement of eye, head and body, the difference of these directions, and the concept of the angle made by the different directions which have the self for their centre. The magnitude of the revolution to which corresponds that area of the field of vision which the sight traverses gives us a measure for the various angles, and the fact that we can repeat this process in the same way in all directions is a guarantee that all angles are comparable as to their magnitude, and makes it possible to make of the right angle as determinate an idea as the straight line itself.

13. But though we can bring into consciousness these definite forms of our space intuition in the concept of the straight line and the angle, we must note that in so doing we can merely single out elements which, in the total space-idea as it is always present, are given all at once in a way which we can never understand; that is, can never resolve into discrete, numerable acts. It is obvious that we can never construct space out of any number of straight lines radiating from us at different angles, or visible surfaces out of the terminating points of these straight lines, we come upon the antagonism between the continuous and the discrete, an inexhaustible field is presented to our discrimination of directions, which can be repeated from every point of space at which we may place ourselves actually or in thought.

When therefore we realize the position of visible objects in space, we draw an invisible network of straight lines from every point which we discriminate to every other. These lines all *pass through* space, but they do not *fill* it; still less can they create it. They merely single out particulars and thus make us realize that something is represented in space which can never be exhausted by this activity which distinguishes discrete elements.

14. By the introduction of movement, which cannot be resolved into discrete elements, we do indeed succeed in producing the plane surface from the line, and space from the plane surface. But this movement, as we have already noticed, always takes place in an already existing space, and all we really see is that the space arising from the movement of the surface completely fills the space already present. We are aware moreover of the connection which exists between the continuity of space and that of time, but we cannot overcome the difficulties which are presented by the intuition of the continuum to conceptional analysis. The famous demonstrations of Zeno merely expose the im-

possibility of the attempt to comprehend the continuum by means of a number of conceptually determinable steps.

15. But though space cannot be created in this way, the existing space-idea may be rendered perfectly determinate by connecting it with a framework constructed from the most simple elements and capable of being always thought of in the same perfectly definite manner. We are aware that, wherever we may represent space, we always localize in the same way by straight lines which radiate from one point, that when we turn a straight line in a plane in space round this point the same four right angles must always arise, and that space can always and everywhere be divided by three planes perpendicular to each other, and that it is indifferent in which direction these planes are laid. We know that the space which we regard in this way is not merely that which results from sense impressions, but that wherever we like to imagine anything spatial it contains exactly the same elements combined in the same way.

16. From a purely logical point of view it is of course matter of fact that our space-idea is of such a nature as to be always alike throughout, that we can move at will from one place to another and turn in all directions without involving any change in our space ; we are made aware of this invariability by the fact that we can always construct the same forms from any point by drawing lines, and that whatever point we start from we must always refer space in the same way to some system of axes. But although we recognise this as matter of fact, we do not therefore mean that our space-idea is empirical in the sense that it might chance to be altered by some change in the nature of our external perceptions. External impressions do not create a continuum of this kind by themselves, for no space is ever presented to us which is completely filled by sensations ; nor could they elicit this particular kind of projection upon which all localization depends. What is empirical is only our recognition of the fact that all men agree in their manner of representing space just as they agree in the form of their self-consciousness and the fundamental laws of their thought ; and from the actual invariability of the space-idea in each individual and its sameness for all we infer a necessity which we can neither understand nor prove, but can only recognise as actually given.

Again, the belief that on the presupposition of an objective space filled with real objects our localization is correct, is empirical ; but this belief means, in the last instance, merely that the different ways in which we can ascertain the position of a body leads to consistent results, or that dis-

crepancies which present themselves when we localize objects by projection in straight lines, as in cases of refraction, can be explained according to general laws by just this assumption of the objective validity of our space-idea. Finally, the belief that the forms of bodies, of which the solidity corresponds to the constancy of our images, and which we have therefore learned to regard as invariable, maintain their invariability even when in motion in our space, is again empirical. Should the forms of bodies vary when in motion, this would be no reason for modifying our view of space, but only our idea as to the solidity of the bodies.

17. This attempt to discover the simple elements of the space-idea has taught us that isolated elements are not to be found, that the straight line and the angle which we find within all space as strictly determined ideas only obtain real significance when they are accompanied in consciousness by those relations between different lines and angles which are prescribed to them by the idea of space as a whole. These relations are therefore themselves essential elements of the space-idea, and the same logical character must reveal itself in them.

The means of obtaining a clear consciousness of the relations between the particular elements and parts of our space-idea is measurement; and measurement presupposes that it is possible to think of everything spatial as consisting of equal parts or as constructed by the addition of equal parts.

18. The more indefinite relations of larger and smaller are given directly by ordinary sense-intuition; a body which encloses another, a plane which includes another as its part, a line from which a portion is separated, give us the difference between larger and smaller upon simple comparison. The relation of part to whole, upon which this comparison ultimately rests (§ 6, 3, 1, p. 35 sq.), is always given in the continuum of spatial intuition and the homogeneity of extension. We are also familiar with the process of comparing objects which are exterior to each other; we lay them together or think of them as laid together, and either perceive how the one extends beyond the other, or judge it to do so with immediate certainty on the ground of their juxtaposition in our imagination. As to the assumption involved in this operation, that things do not alter by being moved to another place, we have no doubt, for we certainly do move our spatial images here and there without difficulty, and a hundred experiences teach us that the spatial image of a moving body also remains the same.¹

¹ I cannot persuade myself that if objects were to shrink upon being moved in a certain

For the same reason we never have any apprehension lest the dimensions of a body present to sense should vary because it turns about in space.

19. But these expedients, although they lead to the process of measurement, do not suffice to explain either the geometrical concept of equality nor the development of geometrical concepts of measurement. So long as we have to do with real objects, we cannot attain absolute coincidence either of lines or of points. Our sense-perception of equality and inequality is bewildered by our knowledge of the continuity of space, by the continuity of increase and decrease in spatial magnitude which it involves, and by the gradual disappearance of perceptible differences. Not until we mentally construct our non-sensuous straight lines and lay them upon each other do we get the idea of their absolute coincidence when they have two points in common; not until we exclude even the smallest difference which escapes perception have we the ideal concept of geometric equality, to which in intuition we can approximate only negatively by defining the equal as that which is neither greater nor smaller—an ideal concept which we can never realize in intuition.¹ Only in the thought of a perfect straight line have we any foundation for an absolutely invariable definite measure; and only by thinking of the parts of a straight line as strictly continuous do we get completely geometrical division and addition. The same is true of angles, only the angle which is enclosed by straight lines has an absolute definite magnitude. Moreover, we have here, in the invariability of the angular revolution of a line in a plane back to its original position, an invariable unit of measurement, which the straight line lacks because it can be indefinitely produced.

Thus these relational concepts also ultimately depend for all their determination upon a mental construction and movement of lines, which

direction, we should not notice it, provided that we ourselves and the standards by which we measure were to shrink in the same proportion. It would have to be assumed that not only the percipient surface of the bodies and the standard of measurement, but also our memory-images, were subject to the same law of shrinking. But then we should have another mental life altogether, for all our present apprehension is based upon our ability to reproduce sense-impressions, at any rate for a short time, without perceptible change; even our confidence in the constancy of our standards of measurement depends ultimately upon the fact that as at present seen they do not differ from the image reproduced from the previous moment; in this way alone can we get the idea of constant solid bodies.

¹ Cf. the pertinent remark of Zindel, p. 11: "When we say 'two lines are equal, if, being placed the one upon the other, they coincide,' we tacitly assume that the lines have not changed while being moved. But this assumption, that is, the concept of unaltered length, already involves the concept of equality," etc.

again has its basis in the consciousness of an activity which never changes in its representation of space.

20. The idea of difference in magnitude acquires definiteness only through number, which states what part the smaller is of the larger, or how many equal parts the one or the other of the compared magnitudes contains, and thus reduces the indeterminate larger and smaller to determinate relations of magnitude. But in the application of number to spatial measurements we must never overlook the conditions under which the counting takes place and which are contained in the continuity of the unities counted; that which is added is not merely a plurality of unities, but a plurality of unities which lie in continuous contact and thus form a connected whole. Only spatial intuition can show what is meant by adding one line or angle to another in such a way as to express that continuous contact of parts without interval which is not to be found in the concept of number by itself. Thus the expression of the relations of spatial magnitudes by numbers and their relations is always connected with certain fundamental presuppositions as to the nature of the parts to be counted and the meaning of the process of addition, etc.; and has no significance unless it is based upon intuition.¹

21. But here we come again upon the antagonism between the continuous and the discrete. The spatial continuum can never be exhausted by numbers; it always resists exact numerical measurement. From this point of view also intuition maintains its original supremacy against every attempt to resolve it into mere relations of measurable magnitude. It is only by passing beyond the sphere of number in the strict sense by means of irrational numbers and the expedients of the differential calculus that we succeed in overcoming this resistance. At the same time when we have learned that there are incommensurable magnitudes in space, we have the most striking proof against all empirical theories of space; no actual measurement could convince us that it is impossible to express the side of a square and its diagonal by numbers of the same unit.

22. On the other hand, intuition affords a perfect counterpart to the peculiarity in the relation between the general and the particular which we found in the meaning of the general numerical symbols. It is part of the nature of the continuum that we can think of any magnitude as changing and diminishing throughout all values; the power of increasing at will every element of our space-idea, according to the same law of construction, in-

¹ On the conception of a non-Euclidean Geometry see Appendix C.

volves the certainty of being able to give to every numerical value its exact counterpart in spatial magnitude. From this point of view, then, the applicability of numbers, even in their general expressions, is unlimited ; indeed, it is this applicability alone which enables us to apply the idea of a continuum—although only in the form of an approximation—to the series of numbers which progresses by constantly diminishing differences. The actual numerical values must always from their nature continue to show finite differences, even though we can diminish them at will.

23. But still another and different form of generality belongs to the particular spatial ideas. Every straight line of which we think is a particular straight line ; but since we can repeat it in exactly the same way in every part of space and in every direction, it represents an endless number of counterparts exactly like itself, which are not identical, as the number 3 is always the same identical number, but can be thought of as an actual intuitable plurality.

24. In the sphere of numbers we have, then, developing itself from a few interconnected elements which are subject to a simple law of synthesis, a series of which we are first conscious when we construct it ; in space we have a presentation in its totality within which we have to distinguish and to determine the simplest definite elements in order to reconstruct by their spatial combination according to fixed laws the idea already given, thus attaining for the first time a consciousness of its actual necessity.

§ 68.

The investigation and determination of the elements contained in the idea of TIME also depends in the last instance upon our power of reflecting upon the functions which act in our consciousness of the time series as the synthesis of memory and upon our consciousness of the mutual relations of these functions as necessary, and as always the same whatever the temporal content may be.

The parallelism of the processes by which we determine the idea of time and that of space is grounded upon the intuitable continuum of time. In antithesis to both, number stands in a like relation both to space and to time.

1. The idea of time is inseparable from consciousness, and is contained in everything which we experience immediately as our own states or our own actions. We find ourselves existing in time and acting in time, and the manifold which fills consciousness contains time as an element and

is arranged for us in the form of a time-series. In this sense a time-series filled with a definite content is given before reflection. If we should attempt by abstraction to disengage time from the content which is in time, we should first obtain—corresponding to extension in space—duration as the common property of our different states, and of objects in so far as they are present to consciousness; and from the continuation of one state after another had ceased (*e.g.* the continuation of sight after cessation of hearing), we should also get the distinctions of longer and shorter periods of duration. But duration does not exhaust the idea of time any more than extension, as the common property of all objects, exhausts that of space; the ranging of distinguishables in before and after, temporal succession, is also involved in the content of consciousness, and even the idea of empty time is not wanting as the interval between successive and intermittent impressions, acts of will, etc., corresponding to the intervening space between external objects. An absolutely empty time cannot, indeed, be immediately given in the way in which an empty space is intuitable in the distance of objects. Every moment is filled by our own existence and the unobtrusive mental changes which accompany our consciousness of it, and it is only in connection with this background that the intermittent activities appear to be separated by pauses; the same time must be related to distinguishable series of events, of which the one is continuous, the other intermittent, and thus there arises the second temporal relation, contemporaneity.

Where every conscious content disappears the idea of time also disappears, as in sleep. When we imagine our experience of time to be extended, as in forecasting the future, we cannot lengthen time except as accompanied by the thought of some content, however indefinite.

2. It is more easily obvious than in the space-idea that this fact of the time-idea being given does not prevent its being *a priori* in the psychological sense, for it is evident that the succession of particular perceptions does not amount to a perception of their succession. On the contrary, in order to explain the actual state of our consciousness, we must refer back to some original connection between the particular movements of consciousness which is inseparable from the process by which we become aware of the particular, and appears first in the simple form of memory leading from one particular to another and drawing one moment after another. By virtue of this connection what is presented is never an instantaneous Now alone, but accompanying it a longer or shorter series of

preceding moments. Time is *a priori* in the sense that in the laws which regulate consciousness this function is involved as necessarily taking place; it may be called a form in so far as the mode of connection is independent of any particular content, but we can no more attain to the idea of time without a content which is experienced and retained in memory than we can attain to an idea of space which is not occasioned by stimulation of sense.

3. What really remains when we attempt to gain the pure concept of time by abstraction from this content is nothing but a consciousness of this connection itself and the fundamental relations contained in it of before and after, which are based upon the way in which we pass from the present moment backwards and forwards through the series contained in consciousness. Our power of passing beyond our immediate experience and representing in the same way whatever content we like, of imagining time as extended either in the past or in the future at will, depends only upon the spontaneous manipulation of that activity which when raised into consciousness gives us the concept of a time of unlimited extension. But all the ideas of times which we get in this way combine to form one all-embracing time of which the particular times are parts only on the presupposition that we expressly accept the content as existing, and place it in actual relation to our own actual existence, determining its distance from our Now. But the concept of the one time in which everything exists is not the concept of time in general. The antithesis between the one present moment which is and the past which is no longer, as well as the future which is not yet, is also grounded upon this reference of the concept of being to that which is represented in time; apart from this reference time is for us a uniform continuum, without any prominent point. When for mathematical purposes we imagine a body rotating round its axis, this emphatic opposition between the Now and the Before and After disappears altogether; we are not obliged to attribute to the image with which we are concerned an independent being or to assign to it a place in real time, a reference to our Now, and we therefore are able to regard an indefinite period of time as if it were a uniform whole like a spatial area. The particular parts of this time are mutually exclusive in no other sense than are the particular parts of a line; for in contemplating a line also we direct our attention successively to different parts.

4. As the universal concept of time in general arises only through our consciousness of what we do when we represent a temporal series, so the

more exact determination of the particular elements contained in time and supported by it depend upon the same activity of reflection. Here, again, it is true that all we can do is to determine differences and particulars within a total idea. A point of time as an indivisible moment is never given to us in the sense that we can think of it as filled by any definite content; it is only by discrimination within a flowing continuum that we can grasp it as a *limit*, at first occasioned by the change of content as it ceases and begins, and then applied to uniform duration within which the same process of limitation can be repeated.

5. By division we get periods of time, which are comparable as to their length, are greater, smaller or equal. The overlapping of one duration by another, or the equal duration of two objects which begin and cease together, was given by immediate perception of co-existing objects, so that here our comparison rested upon a definite given content; but the power of comparing successive periods of time depends ultimately upon the manner in which time enters into consciousness. When we break up the combination of successive moments into a repetition of connected acts and compare them together, we are able to distinguish the act which embraces more from that which embraces less and to think of an equal time as a limit between the two. In the last instance it is always an unanalyzable impression through which two periods of time appear to us as equal, and we are really certain of it only with very small intervals, such as occur between the strokes of the metronome. It is however from this subjective measurement of time that all our knowledge of equal durations starts, just as measurement by the eye alone makes it possible to find fixed standards of measurement.

6. The resemblance between the processes by which we compare and combine successive intervals of time, and those in which number as the unifying combination of a plurality has its origin, is obvious; and it is easy to understand how they should come to be thought identical. But in number we have to do with a series of indivisible acts, which need not necessarily be observed as passing through consciousness in a temporal series: so that when we have completed our idea of number we lose sight of the time which we used for counting. On the other hand, the basis of the connection between successive periods of time is the continuum of moments which pass into each other without break; the acts which correspond to counting are limitations of a duration, and thus contain something which always remains divisible and cannot be pro-

duced from a combination of unities. To this extent time stands in the same relation to number as space does ; counting is applied to both in exactly the same way.

7. But it follows further that it is only by calling in the assistance of number that periods of time can appear as definite magnitudes and be reduced to conceptual expression. On the other hand, immediate subjective measurement is extremely limited ; it suffices only to yield a principle and general mode of procedure of measurement which we can extend and apply at will in our idea of time, so as to construct the general scheme of temporal magnitude, but which needs artificial aids for its application to the time of actual experience.

8. The product which results from the combination of all these elements is the thought of a time which progresses uniformly and without limit, and is divisible into periods as short as we like ; it does not signify how small we make the unit of time if we adapt our actually limited power of discrimination to existing conditions by imagining every period to be so far magnified as to admit of still further division in intuition. This artifice, without which we could never perceive intuitively the progressive division of any given period into smaller and smaller periods, is made possible by the uniform flow and the simple proportionality of all temporal relations to which it gives rise.

9. Thus it appears that definite, conceptually fixed ideas of the temporal—as of the spatial—are attainable only by means of a reconstruction of that which is immediately given, and that this reconstruction depends upon our attention to activities of continual and uniform occurrence, of which the invariability appears to consciousness as the sign of their necessity, and which thus disengage themselves from any definite content. We find also that in the idea of time, as in that of space, there are given a number of inter-connected elementary ideas, the mutual relation of which is determined by the general idea of time, which regulates it and gives it meaning ; and that we have not succeeded in making our idea of time conceptually clear and fixed until we have attained to a consciousness of this inner connection and its necessity.

§ 69.

The concept of MOTION presupposes for its complete determination first, reflection upon the co-operating functions of the ideas of space and time as connected in the idea of movement, and then the reduction

of this connection to its mathematically determined elements. The concept cannot be perfected by the method of abstraction, but only by the reconstruction of what is given.

1. Space and time unite in the idea of motion. Kant himself contrasts motion as something empirical, presupposing a perception of something moved, to the *a priori* forms of space and time, and it certainly does seem as if here we could get no conceptual analysis, no determination of simple elements, except by abstraction from movements given in perception. But here again closer observation will show that the matter stands just as with space and time, that while the *original idea* of motion is given empirically, the *definite concept* of motion can in no way be obtained from simple sensuous apprehension.

2. The simple motion of a body in the field of vision, or the motion of our own limbs, seems at first to be an immediate perception and to need no further analysis. From various moving solids we easily abstract, by disregarding the particulars in which they differ, an abstract general concept of motion, and we as easily find the contents of the idea to be change of place in time, motion being thus distinguished from *rest*, which is continuance in the same place.

3. But if we proceed to ask what is involved in the idea of change of place, and why it is so easy to abstract the general idea of motion from the perceived changes of place of the most various objects, then it appears that there is certainly something in the activity of perceiving a movement which is independent of the form and magnitude of the moving object, and which is always the same whenever we perceive motion. It is not one indivisible act with which we have to do when we say that we perceive change; there is always involved the combination of different temporal moments in one total idea, and this involves that activity of memory which connects with the present that which is distinguishable from it as preceding, and thus constructs a whole from the successive stages retained in consciousness. But the intuitable element in the idea of motion, that which cannot be further resolved into discrete elements, consists in the fact that we do not think of and distinguish an enumerated number of separated places, which we afterwards combine as a whole, but that in the process of combination we apprehend the continuum between the particular and momentarily fixed points of the path of the moving solid.¹

¹ Phenomena are known, such as those of the stroboscope, which prove that this

Now the form of the process, through which motion is thought, is evidently always the same, inasmuch as the same functions of distinguishing and combining must always be involved; difference arises only in its reference to the different material of moving objects, and the different magnitudes of successively distinguishable intervals of space. This is the ground of the ease with which we abstract the idea of motion, and raise it to generality.¹

4. But not even the general idea of motion can be raised in this way to complete conceptual determination. Inasmuch as sense-intuition gives us only moving solids (or surfaces) their motion breaks up at once into the motion of their parts, and we find no end to this division; the smallest perceptible element which moves is still extended, and cannot be resolved into ultimate perceptible elements. And if we further proceed to determine the differences which appear in motion, sensuous intuition leaves us completely at a loss, and we find that only spontaneous construction in merely imaginary space and time, that is only the mathematical consideration of motion, can help us to reduce to fixed and unchangeable ideas that wavering representation of motion which we get from the senses.

5. From the nature of our spatial concepts it follows that a completely determined place is only a point in space, and hence that the only way to get an unambiguous idea of change of place is to follow the motion of a point. In this way alone is it possible to determine the beginning and end of a motion, and therefore the distance traversed, and in this way alone the path which is always involved in the thought of motion becomes fully determined. Even in ordinary thought we reduce the path which a body describes to a line, of which alone the direction can be easily distinguished and determined. But this reduction is uncertain so long as we retain the whole which is given in perception; only when in mathematical abstraction we move a point in a straight line, a circle,

continuum is present in thought, even where the perceptions were really intermittent; but they do not prove that the whole idea of motion is due only to the intermittent perceptions of separate successive positions, and that what takes place in the intervals is not perceived but only added by imagination. The intuitable idea of motion is composed of both elements; it is not possible without consciousness of the difference between the successive positions which are traced by intermittent movements of the eye, and form to this extent discrete points within the one comprehension; but it also contains continuous passage through the areas traversed, as something entering into immediate apprehension itself.

¹ Cf. i. § 41, 12, p. 254.

an ellipse, do we get that mathematical ideal of rectilinear or circular motion with which we compare motion as actually perceived, and this is only possible when we fix upon definite points in the moving solids. We often see the motion of a passing cart-wheel, we easily distinguish between the progress of the whole and the rotation round the axis; but do we see in the ordinary sense the path described by a point in the circumference, or could we get an image of the cycloid from the simple apprehension of sense?

It would seem then that from perceived motion we can never arrive at clear and unambiguous concepts of motion, such as we could retain and reproduce with a certainty of their identity. Only the purely ideal thought by which we move an invisible point in space can give both the general concept of motion and in it all the different elements contained in motion.

6. This is especially true of velocity. It is true that when we perceive motion we are at once struck by differences of greater or less velocity. One movement declares itself swifter than another by a peculiar stimulus which is closely connected with motor sensations of the limbs and the following eye; when one body overtakes another in the same path we get a still more definite clue. But we get no further than the mere comparatives of "quicker" and "slower" until we introduce measurement of the relation between space and time in motion; and this measurement, if it is to be accurate, again involves the mathematical ideals, the point as the beginning of motion, the line as its path, and a strict equality of spaces and times which no perception can absolutely guarantee.

7. When we reduce the idea of velocity to the concept of the relation between spaces and times, we find that the particular form of the synthesis of space- and time-magnitude gives us at once all the specializations which result from the nature of magnitude and of numbers as they pass through all values. It is possible to extend the concept to cases in which sense-perception fails us owing to the slowness or swiftness of the motion; it is also possible to separate uniform motion from different degrees of retardation or acceleration by indefinitely small intervals. Thus nothing more than the concepts of numbers and numerical relations are needed in order to develop the general concept into its differences. No apprehension by the senses, however favourably endowed, would ever be in a position to abstract the idea of uniform

acceleration from one or more observations of falling bodies. The discriminating capacity of our senses and the subjective measurement of time are far from being competent to a problem which requires that the relation between space and time should be measured into the smallest intervals ; a Galileo was needed to construct mathematically the law and the course of such a motion, and then to find it in reality.

8. Here again, then, we find that the only means of making our fundamental ideas fixed and unambiguous is conscious reconstruction of what is given from simple definite elements ; here again, we find involved in the nature of these ideas their fundamental relations to one another, and the form and law of their synthesis. The ideas of space and time rule that of motion, while on the other hand definite concepts of spatial images are formed by help of the idea of motion, and time is most easily represented in motion. They form a trio of interconnected spheres which cannot be resolved into simple isolated elements ; rather is it characteristic of them that they act as a compulsory bond, holding together in mutual dependence all which is distinguishable and different within them ; they are a universe of ideas which, once brought into consciousness, bring with them the consciousness of a necessary activity by which they are produced. This constitutes the characteristic which has been denoted as *a priori* in the logical and epistemological sense, a word so often misinterpreted and therefore unjustly attacked. Neither space, nor time, nor motion is *a priori* in the sense that it is originally due—as number is—to a conscious, spontaneous activity. They precede all thought ; we find them as mental products and can alter nothing in them, but the idea in its totality prescribes invariable mutual relations to all the particular parts which can be distinguished within it, and is thus perfectly determinate and exclusive of all ambiguity. A definite principle of synthesis governs all particulars which we can represent in time and space, and thus governs also the sensuous content of which the varied change can in no way affect the general form of spatial and temporal apprehension and combination.

§ 70.

For the conceptual determination of ELEMENTARY SENSATIONS the first requisite is a resolution of what is given into homogeneous parts.

These parts fall naturally into definite groups, which are clearly distinguished from each other—the spheres of the particular senses ; but

within these groups mere comparison of subjective sensations can give us neither fixed points nor a universally valid measure for differences.

In a logical determination of these elements we start from the assumption of a constant relation between differences in the cause of sensation and differences in the sensation itself as it would occur in an assumed normal subject, and we endeavour to represent the causes of the sensation directly by spatial and numerical relations, or indirectly by substituting for the effect of the cause upon the feeling subject an analogous effect upon an external constant body.

Thus in this sphere all methods depend upon the assumption of fixed causal relations, and the only subjective element involved is the estimation of likeness of intensities and of qualities in sensation.

1. In number, space, and time we have found conceptual elements which can always be presented in perfect clearness and determination because they have their origin in functions which are always being exercised in a uniform manner so long as our minds are active at all, which are to be found in all alike, and in the representation of which we are conscious of an inherent necessity. The ideas of the sensuous qualities, of colours, notes, temperatures, etc., are given to us under essentially different conditions, which call for other methods in their conceptual determination.

2. The process of abstraction by which they are disengaged from their complication with other elements is indeed easy, and has therefore been everywhere carried out in language; nor can any doubt arise as to their simplicity when once we have analysed spatial objects so far that the particular parts present homogeneous sensations, and have broken up into simple constituent parts that which is given at one time so far as it still contains a distinguishable plurality.¹ For example: the impression of a colour uniformly spread over a surface cannot be further analysed into different elements; it is simply there, and we are conscious of it all at once in its completeness. But then it is there only amongst external conditions, which vary in many ways, and without these external conditions it can be only incompletely reproduced. It is out of the range of that voluntary manipulation of which mathematical concepts allow, and thus it happens that the memory images become more and more uncertain as the differ-

¹ The fact that artificial expedients may be necessary here, as in the resolution of a musical sound into its constituent notes (by means of Helmholtz's instruments), refers primarily only to the conditions under which we are able to carry out the analysis accurately; it does not affect the proposition that we do ultimately arrive at simple and unanalysable elements of sensation.

ences given by sensation become more numerous and manifold. Difference between individuals also presents itself here; both as regards the extent of sensations and the clearness of their discrimination we have no right to assume that all people have the same ideas, quite apart from the impossibility of ever proving perfect agreement in this sphere between different individuals. Finally, the main difficulty by which we are confronted in a logical treatment is the gradual nature of the transition between distinguishable colours, sounds, tastes and sensations of temperature (although it is not, as with spatial and temporal differences, strictly continuous, nor contained within a continuum of perfectly homogeneous progression), and the absence of fixed elements, which can easily be established for all, such as are given in the straight line. We are, moreover, hampered by uncertainty as to the completeness with which we can collect and compare all possible sensations. Who can be certain of having really seen all visible colours, of having really heard all possible sounds, so that he can arrange them in a fixed conceptual system including all? Nor can imagination, as with space or motion, supply the deficiencies of perception; the limits of perception and the distinctions contained in it cannot be extended at will, or new colours invented as we can invent all possible curves.

3. If we now consider what it is which we have to start from in reducing ideas of this kind to fixed elements, we find it in the fact that natural abstraction and discrimination have already begun by sorting out different groups, incomparable with each other and disconnected, but containing comparable elements, which belong to different senses, or, as with sensations of pressure and temperature, to different functions of the same sense. Colours which are seen fall naturally apart from sounds which are heard; they form two closed wholes of similar and comparable impressions, although we can abstract no further element which is common to all. When they are classed together as sensations of light or sensations of sound, this means no more than that they are all either seen or heard.¹ The division between sensations of smell and of taste is indeed less certain, for these, owing to their constant association, are sometimes confused, and sensations of taste and of touch are connected in the same way; but for the present we can neglect these limiting cases. Language itself has further initiated a series of distinctions within this sphere, and similar impressions are grouped together under a common name and separated from others which are clearly different; thus amongst colours

¹ Cf. i., § 41, ii., p. 256.

reds, blues, and yellows are classed together; amongst sounds whistlings and rustlings.

4. In our logical treatment we must simply recognise the first division; but the second gives rise to difficulties. The similarity of one colour to another, of one sound to another, is logically indefinable; it rests upon an impression which cannot be further analysed, which indeed always guides our classification, but which can be utilized only after we have obtained a measure for the difference which still remains—a difference which distinguishes the merely similar from the absolutely same, and the magnitude of which determines greater or less similarity and thus institutes a series of gradually increasing differences up to a limit beyond which difference preponderates over similarity.

5. It seems to be a matter of course that in comparing our sensations we should distinguish two kinds of difference—that of intensity and that of quality; in the latter we are clearly conscious of difference in the felt content; in the former, whether the content be the same or different, we are conscious only of difference in the strength of the peculiar subjective excitement, though we are accustomed to regard this at once as a difference in the strength of the stimulus. But it is open to question whether, if we were able to apprehend sensations purely as subjective phenomena, without thinking of their significance as representing objective things, these two aspects would be as distinct as are two dimensions of space; whether sensations which we consider only intensively different really show no qualitative difference at all, or whether we do not merely overlook the qualitative differences of two intensively different sensations because we know that they proceed from the same object, and therefore refer the difference merely to a different strength of stimulation under different conditions (as of illumination). In the sphere of colour every difference in brightness is really a qualitative difference. We are apt to think that one and the same object must present the same colour, only in different intensity, according as it is more or less strongly illuminated; but the painter knows that the illuminated and shaded parts of a surface of uniform colour present two colours which may be as different as the colours of two objects under the same illumination. It is easier to maintain that the same note may be heard as stronger or weaker, although here again it is a question whether the weaker A of the same violin-string does not differ from the stronger by more than mere intensity.¹ But if we are to have any firm

¹ Cf. the remarks on this point in Stumpf's *Tonpsychologie*, i. 240, 349. I can accept

foundations at all, we cannot cut ourselves loose from the objective sources of sensation, and it is desirable to lay down the distinction between intensity and quality at any rate for the beginning of our investigation, the more so that even in our immediate apprehension we find sometimes the more or less of subjective excitement preponderating, sometimes differences in the objectified content of sensation.

6. The next question is where to find fixed standards for intensities, and a measure for their differences.

If we take as an example heat, in which we think that we know merely intensive differences most clearly, we see how difficult it is to determine these differences conceptually and for all alike, and to fix upon any starting point. The power of discrimination is very far-reaching, but not only does the problem as to how much greater the intensity of one sensation of warmth is than another remain insoluble; even the dividing point between the contrary intensities of cold and warm remains indefinite, and is not merely variable and subjectively different as compared with external temperature, but cannot even be determined subjectively with any certainty. When we try to present immediately the series of distinguishable intensities and to state their differences, we are confronted by none but fluctuating determinations; we seem nowhere able to get beyond the indefinite more or less so long as we work only from immediately given sensation.

7. Fechner's brilliant adaptation of Weber's observations of just perceptible differences as a universal measure of intensity of sensation, seems at first sight to claim also an eminently logical significance. The just perceptible increase of sensation is taken as unit, the point from which every just perceptible sensation begins as zero-point; and thus it seems as if every intensity could be expressed as a sum of such units in a definite number. If, for example, we fix upon 20° as the zero-point at which heat and cold as felt by the hand divide, and agree¹ that a rise in temperature of $\frac{1}{4}^{\circ}$ yields a just perceptible increase of sensation, then we should be able to express the intensity of the sensation given by 22° as 10, of that given by 28° as 40. In this way the Kantian principle—"in all phe-

the explanation on p. 240 of the qualitative difference which I seem to hear, and certainly do not dispute that in comparison with the consciousness of differences of intensity, the qualitative changes which may be present are extremely small.

¹ Assuming that Weber's law is not true for sensations of temperature within certain limits, but that the just perceptible differences of sensation correspond to an *equal* increase of objective stimulus.

nomena the real, which is the object of sensation, has intensive magnitude, *i.e.* a degree"—could be definitely formulated; the synthesis producing the idea of the magnitude of a sensation would be represented as the sum of demonstrably equal units.

This just perceptible increase of sensation is however not presented, as any measurable spatial or temporal unity may be presented; nor even in the same sense as a particular sensation of definite intensity is presented. The whole calculation is based merely upon a judgment that of two successive sensations—successive at least for our attention—the one is stronger than the other, and the value which we attach to the just perceptible increase only measures our power of distinguishing two sensations according to their intensity by means of this judgment of comparison. This process of discrimination, moreover, presupposes not merely the sensations themselves, but also the power of remembering them accurately, and is dependent upon the time which intervenes between the two. This alone makes it impossible to assume that the just perceptible differences of sensation are all equal and to treat them as fixed units of measurement. The theory implies, moreover, that that which we recognise as difference of sensation is itself sensation and forms a part of the stronger sensation; but the difference between two degrees of brightness does not present itself to consciousness as a third sensation, any more than the difference between two colours is itself a colour. But even if we could regard the just perceptible difference between two degrees of sensation as itself a sensation of given intensity—if the difference between dark and light grey were itself grey, as the difference between two lines is a line—we should be still more entitled to raise the question whether we are justified in using this just perceptible difference as a unit of measurement. It is obvious that the degree in which a loud note must be increased in order to yield a still stronger sensation cannot be equivalent to the corresponding increase of a note which is only just audible, and that the perceptible increase of a brilliant white, if itself regarded as sensation, must be quite different from the increase which distinguishes a dark grey from still darker grey. The differences in different regions of a scale of intensities cannot be taken as equivalent, and for this reason any given intensity cannot be represented as a sum of equal units; it follows that it is not measurable in the ordinary sense of the word. Weber's observations merely go to show that our power of recognising sensation *a* as more intense than *b* depends upon the degree of intensity of *b*; what this itself depends upon we cannot investi-

gate here, but we cannot in this way obtain a direct measure of intensity of sensation. Fechner's method of true and false instances indicates clearly that he is not dealing with something immediately given; his very statement of it assumes that an increase of sensation corresponds to every minimum increase of stimulus, but that our judgment of comparison is at fault.¹

In this way the method suggests the necessity of finding other standards of measurement if we are to attain to logical definition, and these are the objective standards upon which it is itself grounded. If we look more closely, it is evident that the above method of determining the subjective zero-points of sensation, and the differentials of increment, is made of no avail for logical purposes by differences of individuality; the problem is, how to eliminate these subjective differences. Psychologically it is of the greatest value, for in psychology these subjective differences have also an essential significance; but logic aims at utilizing these elements of thought for a universally valid, objective knowledge, and the nature of our immediate judgments concerning intensity makes them unavailable for this purpose.

8. If we consider the methods actually used by science to make these questions clear and definite—the simplest example is the thermometer—we find that they generally start from the assumption that differences in intensity of sensation correspond to differences in the objective constitution of things; in this way alone could we perceive that heat expands quicksilver, and cold contracts it. But they rely more upon the constancy and uniformity of the effect which the source of heat has on an inanimate body than upon the uniformity of its effect on the feeling subject; and,

¹ The fact that practice produces a marked increase in the certainty with which we discriminate small differences, implies that complicated psychical processes condition the result, and that we are not dealing with a fixed magnitude; we have still clearer evidence of this in the interesting observation made by Volkmann that when, in consequence of often repeated experiments, the ability of one side of the body to estimate small differences correctly has been increased, a similar acuteness appeared in the corresponding part of the other side without being preceded by experiment. This proves clearly that we are dealing with an increasing accuracy in apprehending stimulated sensations, with a process like that which always occurs where what is at first held to be the same reveals differences when more closely attended to. It is only gradually that the painter can learn to apprehend the finer differences in colour, the musician in the pitch and intensity of notes, and can practise their power of remembering them.

With reference to the whole of this question cf. v. Kries, *Ueber die Messung intensiver Grössen*, etc. *Vierteljahrsschr. für wiss. Phil.*, vi. 273 sq., and the careful explanations in Stumpf's *Tonpsychologie*, i. § 1-4; there is also much which is relevant in A. Grotenfeld's *Das Weber'sche Gesetz und die psychische Relativität*.

most important of all, they endeavour to find that increase of effect upon external objects which corresponds to increase of sensation in instances where it appears as simple expansion in space, and is thus immediately measurable down to the smallest portion. We have now accustomed ourselves to determine degrees of heat by the thermometer, but the process is really based upon the fiction of a normal sensation, which always follows the same stimulus and increases according to the same law. In speaking of heat we speak of a subjective quality of sensation; but for scientific purposes we do not make use of it as it is felt at the moment, but as it would appear, assuming that our sensation were always constant under the same external stimulus. We do not measure the more or less by the unit of just perceptible difference, but for this subjective standard we substitute an objective, which we can rely upon to express in always the same way any increase in that which gives rise to the sensation of heat.

It is much the same with sensations of weight. Light and heavy, lighter and heavier, are primarily statements of immediate sensations, and of differences in their intensity; but in this state of indefiniteness and relativity, without any fixed limits, the terms are useless for logical purposes. We have recourse to the balance which enables us to determine the differences numerically, by means of spatial movements which are measurable, and to establish a system of predicates which are objectively valid.

The methods of photometry also make use of spatial relations, and presuppose only the simplest of immediate perceptions, the power of judging that two degrees of brightness are equal; but from immediate perception they obtain neither a fixed starting point nor a standard of measurement.

9. As the general result of these considerations, it appears that in conceptually determining the intensities of sense-impressions we are obliged to have recourse to spatial magnitudes and numbers, by which they can be expressed according to one standard of measurement; this of course involves assumptions as to the uniformity of the effect of objective causes of sensation, of which the validity and origin cannot be investigated until later. Where we fail to make such a reduction, as in the case of intensities of smells, there no logical determination is possible.

If, moreover, this reduction is to have any value, it must hold good—first, that our perception of spatial magnitudes in the process of measurement is itself free from subjective variations, that there is universal agreement; secondly, that our power of discriminating the spatial magnitudes

which correspond to differences of sensation is at least as great, and if possible greater, than our power of discriminating the differences themselves. We must be able to read off from the thermometer an increase of heat which would escape a comparison of sensations.

10. The methods of conceptually determining different qualities are still more difficult and involved. Discrimination between different colours or notes is one of the facts which does not admit of further analysis; what we have to do here is to find fixed points, and a measure of differences.

Turning first to colours we find the course taken by language very instructive. A few unmistakably distinct colours—black, white, grey, red, brown, yellow, green, blue¹—are selected as fixed points from among the incalculable number which present themselves to the eye, and there is no doubt that here we have a primary psychological fact. Certain colours make a predominant impression, which is moreover characterized by their value for feeling, and are thus especially attractive to attention and favoured in reproduction; in this way we obtain a number of ideal colours, which are distinguished for their æsthetic value, and by this standard we measure all which actually present themselves, in order to see how far they contain the ideal element for which we are looking. The extent of the limits within which we are able to trace a similarity to one of these main colours is, again, a purely empirical quality of our minds, and follows partly from habits determined by language; no universal objective principle can instruct us as to where in the spectrum it becomes impossible to discover any similarity to red, any reddish tinge in orange, or where green begins to be bluish. The analysis which (led by the names in use) we make here is of a peculiar kind; the one undivided impression made by the colour orange is broken up by us into a similarity with red on the one hand and yellow on the other, and our tendency to speak of mixture and mixed colours is not sufficiently explained by our experience of mixing colouring matters. No doubt experience shows us that when we mix colouring matters we frequently get mixed colours; but green is not regarded as a mixture of yellow and blue, because blue and yellow colouring matters make green when mixed; for it is subjectively impossible to

¹ We may consider that the attempts to infer from the history of the names of colours in language that the colour sense has been developed within historical times, have broken down before the results of the discussions from different points of view. The history of such names is psychologically interesting, as showing how gradually attention to the differences has grown; just as some children are still very slow to use the existing names with absolute correctness; beyond this it proves nothing.

find any similarity to blue in green. We speak of a mixed colour, as distinct from the colour of a mixture, when the immediate impression—say of violet—allows us to trace a resemblance to blue on the one hand and red on the other.

11. In gradual transitions we find almost insuperable hindrances to complete conceptual determination ; and here, again, by making use of objective and orderly processes, we have recourse to spatial and numerical relations in carrying out a methodical limitation of the various colours. The spectrum has long been recognised as the fixed basis of all conceptual determination, but it is only lately that it has been turned to full account ; it is by division of the spectrum into parts, and by mixing the spectral colours together, and with white, according to the spatial relations of the sectors of the colour-circle, that we are able to represent the continuum of gradual changing differences in all its variations. As soon as this is done we revert to the method employed from the first by language ; instead of marking off a space to each colour-name, within which it is to hold good, we fix upon certain values (*e.g.* particular Fraunhofer lines), and represent intervening colours as intervals of definite magnitudes and combinations in definite proportions.

12. We determine qualities appertaining to the sense of hearing in a similar manner. In language there is recognised first a number of noises, distinguished above others by the liveliness of the impression which they make, but not admitting clear lines of demarcation. It is only where sounds are musical that we can apply the artificial means of exact conceptual determination. Here we are guided by musical intervals ; starting from a key-note, we give names to a series of notes, but refer to the indefinitely many which intervene according to their distance from those which are named. For reference to the key-note itself and for exact determination of the intervals, we must again turn to spatial and numerical relations, to the length of organ pipes and the number of oscillations. Starting in this way, we can follow up the combinations of simple notes to definite consonances, which, like the vowel sounds, appear as simple to the ordinary apprehension, and then proceed to represent the different harmonies ; but our art fails us if we endeavour to represent the many kinds of *noises* as a definite combination of simple elements. The application of this method again calls for nothing from sensation and the judgment of comparison but a statement of the equality of two sensations, which are given together or in immediate succession.

13. From what has been said it is evident how little that which is immediately given in sensation, the purely empirical in its rude form, is adapted to yield ideas which can be used for logical purposes. Those ideas which have arisen from a simple repetition of the similar all share in the fault of indefiniteness and variation from one individual to another ; any conceptual limitation and arrangement of these simplest elements is only possible by means of various expedients which presuppose a knowledge of the causes of sensation, and succeeds only in so far as we assume a normal typical relation between objective causes and subjective sensation, and reduce differences of sensation to measurable and enumerable differences of objective causes, in which we think we have a sphere of ideas free from subjective differences and capable of perfect conceptual determination.

14. When, finally, we attempt to obtain a complete survey of all qualities of sensation, we find it only possible when we are able to represent the manifold of what is given in spatial dimensions or in numerical series. The manifold of tastes and smells have so far defied all attempts at comprehensive arrangement, because no measurable objective relations can be found corresponding to the increase of subjective variations. The series, on the other hand, formed from the number of oscillations represent a scale from the lowest note audible to the highest, which is complete at any rate from this point of view, and in which we are certain of finding a place for every audible simple note. With colours the task is more difficult ; the fact that a mixture of colours produces the impression of a new simple colour favours the operations by means of which these mixtures are exhibited ; but then the variations even of the so-called pure spectral colours are so numerous, and we are so little certain of their completeness (witness those colours which appear only under certain conditions), that we can only aim at an approximation, and can attempt this only by having recourse to a representation of colour variation in a spatial diagram. These defects are the less important because the chief and most direct service rendered by colours is not the knowledge as to differences in the material of objects, which they give by means of their innumerable variations, but the knowledge we obtain from their spatial limits as to dimensions ; and the essential importance of the services rendered by the spectrum is that it connects colour qualities with the sections of a linear extension and ultimately reduces the judgment about colour qualities to a judgment about spatial coincidence. Even the

meaning which colours have as indicating the nature of objects has only become fruitful since spectral analysis has overcome the imperfections of subjective sensation and reduced the problem to that of determining upon which line of the spectrum a bright or dark band appears.

§ 71.

The analysis of the idea of QUALITATIVE CHANGE brings us to elements analogous to those which we found in analysing the idea of Motion ; differences in the quality and intensity of sensations, following each other continuously in time, are connected together just as differences of place are connected together in motion.

The conceptual determination of the elements contained in the idea of change presupposes measurement of differences in intensities and qualities, and can only be completely carried out with the help of mathematical construction.

1. The idea of qualitative change (*ἀλλοίωσις*) in its sensuous meaning is connected with the simple predicates of sensation just as the idea of motion is connected with space ; and the function fulfilled by the idea of time is the same in both cases. We have already shown why the general idea of change can be so easily disengaged from the idea of that which changes ; it is the constantly recurring connection of a series of differences following upon each other in time.

In comparing change of quality with change of place we mean to include not merely change of quality taken in the narrower sense, but also of intensity of the same quality ; hence the differences which correspond to differences of place and constitute the basis of our discrimination and recombination, are differences of the quality and intensity of our sensations. Here again when we think we have an immediate perception of change it is based upon a continuum in so far as we do not find a sudden break between two immediately successive moments, but only become clearly conscious of the magnitude of difference after an interval of time which has been continuously filled by imperceptible transitions. Thus in the general form of the activities which lead to the idea of change there is a complete coincidence between motion and qualitative change.

But this element of continuity is again a hindrance to conceptual determination, and makes it difficult for us to obtain conceptual elements which might be perfectly determinate and absolutely identical. Owing to it the series of differences passed through grows continuously and without

limit from the minimum upwards, and the relation between the temporal magnitudes and the magnitudes of the differences passed through, the velocity of change, is capable of indefinitely many values. Because of our limited power of discrimination by the senses immediate sense-perception leaves the minimum of change indefinite, and is unable to follow all the stages of any change because a perceptible magnitude of difference between two successive sensations can only be presented in definite intervals of time, while the intervening differences, in proportion as they become smaller and follow upon one another more closely, withdraw themselves from the range of direct sense-perception. Moreover in the sphere of sight and touch every quality perceived is spatially extended. Hence change of these qualities consists in the sum of the changes of spatially distinguishable parts which may be indefinitely multiplied. Thus the limits of the concept tend to fall into confusion, and in the manifold comprehended in it we fail to grasp any definite point from which to work in our conceptual elaboration; the measurement of change according to magnitude and velocity is not included in the general concept of change, and cannot be obtained from mere observation if we demand exact accounts. We may indeed form a mental picture, which can be reproduced to a certain extent, of the increase of light when greater pressure increases a gas jet, or of the crescendo or diminuendo of a note; and we may remember the preliminary or final state if it is distinguished from the change by longer duration; but to distinguish the amount of increase for every moment is impossible.

2. Here again we turn for fixed concepts to an ideal construction, which presupposes measurement of the intensities and qualitative differences; it then fixes sharply the beginning and end of the change by a given standard, and represents the course of the change by a mathematically demonstrable relation between measured differences of time and measured differences of intensity and quality. Only so far as definite formulæ can be applied to this relation can we obtain fixed and invariable concepts within the chaos of particular mental pictures of change.

3. The simplest case, which we can apply as a standard, is uniform change, in which the variations are proportional to the times, and which corresponds to uniform motion. We have a perfectly definite concept of what uniform increase of temperature or light is, and what is meant by the uniform heightening of a note, and so on; in other cases, we can construct, according to the circumstances, the different formulæ which express varia-

tion in the course of the changes. If a light is moved with uniform velocity away from a surface and in a line perpendicular to it, and if the illumination at the end of the first second is i_1 , then we should have to express the decrease of the illumination (i) at the base of the perpendicular by the formula $i = \frac{i_1}{t^2}$, and the change of illumination in the other parts of the surface by a more complicated formula. The increase again, say of daylight when the sun rises above the horizon, would be expressed by still another formula.

4. We must be careful to distinguish between the concept of qualitative change, which expresses merely the continuous transition of a sensation through a series of variations of intensity or quality, from the concept of undergoing change and being altered, which has for its subject not the immediately given intuitable sensation passing from difference to difference, but a thing to which the change is referred, and which changes or is changed as its qualities alter. For the determination of this concept we must first of all fix that of a thing.

§ 72.

These last-mentioned determinations of sense-given qualities and their changes are always connected in ordinary thought with the idea of THINGS which have qualities and undergo change.

Our endeavour to make this idea of a Thing logically perfect, and to fix it in the concept of substance, must start from the analysis of that which is contained in the idea, and from the question why we form such an idea.

A. (a) If for the present we disregard change, then we find that the idea of a thing is based in the first place upon the comprehension into unity of a figure which is bounded in space and continues in time; that is, upon a spatial and temporal synthesis. Moreover, inasmuch as the sensations of different senses are referred to the same space, a synthesis takes place between different sensational contents to enable us to think as one that which occupies one position in space, and this gives rise to the distinction between the one thing and its different qualities. The series of elementary functions contained in the idea of the thing is completed by the thought of its existence.

The motive which gives rise to this thought can be none other than our

need to think of the subjective synthesis as well grounded, and of spatial and temporal co-existence as necessary.

(b) In our logical elaboration of the concept we must start from the question: what are the conditions under which these syntheses may be seen to be not arbitrary and fortuitous, but objectively necessary and therefore perfectly determinate? This leads us first to the antinomy between the unity of the thing and its spatial extension which always contains a plurality, which we may attempt to solve either through the concept of the unifying form or through the concept of the atom; in both the determination of the concept of substances must be carried back to mathematical elements, and in the latter we must call in the help of the concept of causality.

B. The idea of the *changes* of things contains still further problems.

(a) It has arisen from the necessity of comprehending into a unity that which happens continuously in one place; so far the synthesis which refers different successive sensations to one thing is exactly similar to those syntheses which give rise to the idea of the thing at rest and without change.

(b) In the logical determination of the thing which changes we are confronted by the difficulty of giving a definite expression to its unity, of saying what that is which persists through change. Thus there arises the attempt to reduce all change to variation in the relations of unchanging subjects, as the simplest way of grasping the unity of the thing throughout the course of its changes. But while on the one hand the apparent variation can only be a change of relation to the perceiving subject, there arises on the other hand the question whether the ground of simultaneous changes in different things lies in each thing independently or in their mutual relations, and thus the logical elaboration of the concept of substance leads here again to the concept of causality.

1. Our consideration of sense-given qualities and their changes leads us naturally to investigate those concepts through which alone the elements we have reviewed assume the form in which they are generally present to thought; the form of things to which belong attributes and activities which we can predicate of them in our judgments. If we are called upon to state unambiguously what we mean by that which we call a thing all the expedients of conceptual determination which we have so far recognised seem to desert us. In talking we always make use of propositions which presuppose this thought, but it is none the less difficult, as previous at-

tempts show, to make the idea which is involved conceptually clear and unambiguous. In proportion as it seems a matter of course that in the sphere of our ideas we are dealing with things, and to raise a question about them seems unnecessary and over-subtle, it behoves us to use greater caution; even in face of the fact that one of the most thorough investigations, that of Herbart, arrives at the conclusion that the familiar idea of a thing with many attributes is a contradictory concept, the contradictions of which have to be discovered by logic and swept away by metaphysics.

2. The method by which we reach conceptual determination even here can have no other starting point than the ordinary thought which is always present, and which it is the general problem of all methodology to interpret, to correct, and to extend. If, however, we attempt to apply the process of comparative abstraction, and to begin by finding the common element in all which we are accustomed to call Thing, we are met at once by the difficulty mentioned in § 40, 5 (i. p. 248); that is, the extent of the sphere within which we should begin our process is not fixed. That which we call a Thing, and which we treat as formally the same in logical operations, has not always the same meaning; the word is a *πολλαχῶς λεγόμενον*. In using the substantival form of speech we make no distinction between the particular thing which is thought of as actually existing, and the merely logical subject of which something is predicated; nor is it possible, until the concept has been to some extent determined, to see at once the limits where the one meaning passes into the other, and which divide the actual thing of daily experience from the fiction of imagination, and this from the geometrical figure, and this again from the abstract substantive.

3. In such cases there is no other way open than to relinquish the survey of the whole and to begin our operation with the most obvious examples, to analyse what is contained in the thought of these, and then to see what are the extent and limits which are self-imposed by the concept thus gained. This is, no doubt, an experimental process, but it is a process already familiar to us in the concept of number in which it is naturally involved.

Let us begin, then, with what is most obvious, which is unhesitatingly called "Thing" by every one, with the particular objects of the world around us, which even when unfamiliar give rise to the question, "what is that thing?" It is not difficult by analysing what we name in this way, and by disregarding differences in the examples before us, to determine

that we mean in the first place an object of thought which as first perceived and afterwards reproduced in memory, presents itself above all as a spatially bounded figure which persists in time. (A flash of light, a shot, a smell, are not "things" in the same sense as a stone or a piece of wood.) If we reflect as to what determines us to consider anything perceived as a thing, we find it to be the invariability of its figure; but the spatial limitation as well as the persistence of a figure is most easily obvious when it disengages itself in motion from others, and appears as the same in different positions in space. The attribution of real existence to the perceived figure is also involved in the sense in which we are accustomed to speak of things.

4. If we leave this last determination for the present, then that which shows itself as most important is unity. That which we regard as a thing must be one thing—a single object which we represent in one definite act, distinguishing it from other things and maintaining its distinction.¹ This determination of unity alone makes it capable of being the subject of a simple judgment.

But this determination of unity has itself many meanings. We mean by it, on the one hand, the singleness which is represented intuitively by spatial limitation, and the resulting distinction from everything else in other parts of space; on the other hand, that identity with self which is contained in the possibility of retaining the same elements as permanent for the idea throughout different moments of time. We are not here speaking of unity in the sense of simplicity; on the contrary, the unity of a thing does not exclude discrimination of its different parts or of different elements in our idea of it. A geometrical figure, such as a circle or triangle, may serve to illustrate this element of the idea; it is one, inasmuch as we think of it in one act of comprehension as spatially limited within surrounding space, and maintain it for any length of time as an unchanging object; it is not simple, for in the circle we can distinguish between centre and circumference, while in the triangle we can distinguish the sides from each other and from the angles; only the point would be absolutely simple. We may also regard the things of actual perception in the same way as geometrical figures in so far as they are mere images, which persist in intuition. A rainbow, or the shadow of a body at rest, have this bounded figure which *persists* for intuition; their unity depends upon spatial limitation, and upon the comprehension of a definite sensuous content which may be homo-

¹ Cf. § 66, 2, p. 31 sq.

geneous or not, but which is at any rate continuous, and that this unity is at first merely subjective is proved by the fact that it is often a matter of choice. When observing the stars it is open to us to construct figures by any possible combination of the bright points, and to regard the constellation of the Great Bear or of Orion as a connected group, as a figure ; all that is wanted is a frame-work in which we may combine a manifold. In the same way we can call a pile of wood, a pyramid of balls, or a heap of sand a unity or a thing, although it contains a plurality.

It follows from what we have said that in the first place we obtain ideas of things in this sense only from those senses of which the sensations are from the first spatial, and which are also distinguished by their uninterrupted excitement from the intermittent sensations of the other senses. That which can be limited as one and persistent is determined either by colour or by touch ; it is a seen or felt figure.

5. But it next appears as an essential point that the things which we know are not present for one sense alone, but generally manifest themselves to several senses at once, principally to sight and touch.

This is not the place to enquire how the reference of different sensations to the same object takes place psychologically ; all we have to do is to realize clearly what ideas are involved in our ordinary mode of apprehension.

The fundamental assumption here is that seen and felt space are one and the same, that the contents of visible space is none other than that of tangible space, hence that whatever is presented both to sight and touch in any given place must be one and the same, because the same position in space cannot be occupied in two ways at once. This unity of space, therefore, is not a product of our perceptions, but is the presupposition which enables us to construct ideas of things from our various sensations ; the synthesis of colour, hardness, coldness and so on, is governed by the law that whatever occupies the same position in space can only be one.

In this way we get the idea of the occupation of space on the one hand, and of spatial exclusion on the other. Qualities of colour and touch are given only in connection with extension, and in reference to extension different colours are mutually exclusive ; in the same way the thing of our thought is constituted by the fact that we find a part of space occupied by a particular colour and tangible quality.

6. It is upon the ground of this identity of space and of places that

different sensuous qualities appear to us as attributes of one and the same thing; the identity is in the first place that of locality, the unity is given by spatial limitations which are present in a consistent manner for both touch and sight. It is obvious that from this point of view the difficulties raised in Herbart's metaphysics against the concept of a thing with many attributes, disappear; that we merely employ it to enable us to think of space as the same for different senses, and to speak of something which is perceived in a given position in this same space. Our different senses would fall into hopeless conflict unless the spatial images of one were identical with the spatial images of another.

7. The ordinary popular idea with which we operate in the beginning of any science is completed when we connect with this geometrical concept of the unity and selfness of a thing the thought that it is, a thought which cannot be derived from anything else. Hence the principle that two different things cannot be in the same place is not one which is added to our idea of things after it is complete, but a principle which guides the formation of this idea itself, and as such might be included among the *a priori* presuppositions of our experience with far more right than many a Kantian principle; its origin in the unity of self-consciousness is moreover obvious.¹

8. Our consideration of what we really do when we think of a thing has shown us a threefold comprehension: it is temporal, in so far as we can only think of something as persisting through distinguishable moments by comprehending these altogether; it is spatial in so far as in marking the bounds of a spatial image we must comprehend its different parts; and finally, it is union of the space-ideas derived from sight with those derived from touch, by means of which the qualities of the two senses become attributes of one and the same object.

9. The originally spatial basis, however, upon which the idea of a thing as one thing rests brings its difficulties with it, and calls for further elaboration. From the fact that space is extended arises the question whether an extended thing can really be grasped as a unity, whether the limitation which is the first step by which it is marked off as a unity and distinguished from other unities is definitive; *i.e.*, whether it is necessary and such as could not have been carried out in a different way.

Even though complete uniformity in the general process by which sensations are worked up into the ideas of things indicates a natural law

¹ Cf. § 47, 9, I, p. 312.

which has its ultimate ground in the unity of self-consciousness as opposed to the manifoldness and incomparability of sense-affectations, yet this does not guarantee a uniform and objectively necessary application of the process to the whole content of sensation. We have still to ask how this application must be made if we are to make our concepts quite definite and free from all subjective variation.

10. We may disregard for the present the conclusion to which we have come that the reference of different sensations to one thing is conditioned by localization, which gives rise to the further question as to how it can itself be carried out in a universally valid manner; and we may disregard all the difficulties entailed by change and motion. Then, as we have already shown, we find that it is to some extent a matter of choice which of the contents of the spatial world we will comprehend within the limits of a unity, a matter of choice whether we regard a heap of wood or the single sticks contained in it as the unit. This is the case, moreover, with every extended unity, the mere possibility of thinking of parts within every extended thing, of regarding it as a whole consisting of different parts, of distinguishing between right and left, between the upper and the lower halves, reveals the contradictions between unity and extension which may be traced throughout the whole history of the concept of substance. From this has sprung the distinction between Form and Matter, and the attempt of Plato and Aristotle to place the unity of the thing in its form as something which is permanent for thought; and here we find the origin of all the attempts to reach by a division of extension a really simple and ultimate which is no longer spatial, or at least no longer actually divisible into parts, attempts to get a subject which would meet the logical demand for an absolutely single and simple statement. Thus ancient as well as modern atomism is to be explained as fundamentally due to the need of a logical determination of the concept of the thing.

11. But when we enter upon this question, the old antagonism reappears between the continuity of space and the demand for unity. So long as our division merely continues to give us smaller things we keep our original intuition, but never come to an end; if, on the other hand, the process is driven to a point where atoms lose all extension in gaining absolute unity, then there disappears at once all the content of the idea of the thing, and all possibility of applying those predicates which are primarily known as attributes of a spatially extended thing. Neither colour, nor hardness, nor any other of the sensational qualities which are insepar-

ably connected with the idea of extension can any longer be applied, and the concept of the thing is entirely removed from the sphere of sense-intuition; that which was a thing and a unity is only a subjective comprehension of a plurality of points which occupy definite spatial positions; the continuous occupation of space is an illusion, and the mutual limitation of things depends merely upon the way in which these non-sensuous things in their plurality affect our sensation. The concept of the sensuous attribute resolves itself into an effect upon our subjective sensation; and that which is really a thing ceases to be perceptible. If the ordinary expression that a number of atoms are comprehended in one thing can have any justification, it can only be found in the real relations which exist between a given plurality of atoms in their reaction upon one another; the concept of the composite thing is dependent upon the concept of causality, the perceptible form is the expression of the laws by which the co-existence of this given plurality is governed.

12. Whichever way we take, whether we fix the concept of the thing in that of the form or that of the atom, it is evident that here again we can reach conceptual determination only by means of mathematical concepts. We have no fixed concepts of forms except in so far as we can avail ourselves of geometrical construction according to established laws. The concept of the atom, moreover, presupposes the geometrical concept of the point, and in reducing an intuitable thing to atoms we need a geometrical construction of the relative positions of many points. So that here again the dependence of conceptual construction upon the development of our mathematical ideas becomes most evident.

13. We need not go further to see how completely unreliable the doctrine is, which endeavours to reduce the popular idea of the thing to the mere co-existence of sense-given qualities. Hume's attempt to eliminate altogether Locke's wavering and changing concept of substance (which had finally no intelligible content left but "a number of simple ideas which go constantly together"¹) was quite consistent from the empirical point of view, and Mill² formulated the necessary consequences of this attempt in explaining the import of every judgment concerning the attributes of a thing to be the statement of the co-existence of the attributes which constitute the meaning of the predicate with those which constitute the meaning of the subject. Here again the attributes can be finally resolved into

¹ *Essay Concerning Human Understanding*, ii. 23, § 1.

² *System of Deductive and Inductive Logic*, book i. ch. 3 and 4.

pure states of feeling, inasmuch as these are all which we really know of the assumed external objects; for sensations and the order of their appearance constitute all that we can know of matter. But is the popular idea of a thing—and it is only with this that these theories are concerned—really exhausted by the statement that a number of sensations, colour, hardness, etc., “co-exist?” In the first place, we cannot even say that this co-existence is in the strictest sense an object of immediate perception, for the sensations by which we learn to know a thing are generally successive. But a more important objection is that the concept of co-existence is much too wide and indefinite, it leaves the sensations independent of each other, and does not even include that local identity which is undoubtedly contained in the idea of the thing (Mill sometimes¹ says “*Order in Place*” instead of “co-existence,” but this is an inaccuracy; the different attributes of a thing are not arranged in space). Even if local identity is added to the concept, there is still lacking the point of union which Locke rightly recognised as forming part of the current idea, and which Hume was at pains to explain by subjective custom, but which finds expression in the principle that there cannot be two different things in the same place. To the empirical theory of a merely external association of different sensations, given in perception and strengthened by habit, we must therefore oppose the proposition that in the concept of a thing we have a synthesis, which cannot be explained by the sensuous factors of thought, but is ultimately referrible to an original function by means of which we refer the sensations of different senses to each other, in order to construct from them the idea of a spatial object. The genuine concept of the thing depends upon our bringing into consciousness the law of this synthesis, and finding its normal objectively necessary application. As to how this can be done, whether by the concept of the form or by the concept of the atom, can be decided only by the nature of the content to be combined and the development of the concept of causality.

If we enquire what is the ultimate motive of this synthesis, we shall find it on the one hand in the necessity for unifying the particular elements of consciousness; on the other hand, in the fact that this unification is only necessary and well grounded when the object itself is the ground why different sensations exist together in the same place. The unity of the thing makes that necessary which is given together in consciousness.

14. So far we have been concerned with the difficulties attaching to the

¹ See *op. cit.*, i. 5, § 6.—Ed.

determination of the concept of the thing considered in reference to its spatial extension and the plurality of its attributes ; a new series of questions arises when we attempt a conceptual elaboration of the ordinary view which ascribes to things not merely persistence through any length of time, but also change during their persistence. It was from this point of view that Kant undertook to find the import of the concept of substance by reducing it to that which persists through change ; and he endeavoured to make the necessity of the concept obvious by the fact that without something permanent change itself could not be thought as such, hence that all objective determination in time and with it all experience would be impossible.

15. If we examine the meaning in which, according to our ordinary apprehension, we speak of change with reference to things, then we find that the confidence with which in the simplest cases we speak of change of a sensuous attribute as change of the thing itself depends upon the immediately perceived continuity of the processes going on within the spatial boundary which is occupied by a given thing. When a blue paper turns red before our eyes, or a piece of wax melts in front of the fire, or a cold body grows warm in the hand and a hard one softens, we have in each instance a continuous transition, which takes place at the same position in space ; nothing occurs to make us suppose that what was previously there has been replaced by an entirely new substance. The unity of the thing which was due before to its limitation in space now rests in the temporal continuity of transitions of sensation within the limits, or in continuous change in these limits themselves. Because of this continuity we feel ourselves constrained to maintain the unity even when, as in the case of melting ice, every sensible quality changes, colour, form, temperature, hardness, etc. In this case we cannot speak of a permanent which is given to intuition, a complex of sensible attributes, of which a part remains the same while another part changes. Even if we choose to call this a transformation of one thing into another, still the transition between such an instance and another in which only a part of the attributes change is so gradual that no fixed limit can be drawn between change and transformation ; and if we had nothing to go upon but what appears to the senses, we should find ourselves puzzled where to draw the line between the concepts of change and of transformation, to distinguish when a thing remains the same and merely changes one or more of its attributes and when it entirely passes over into another thing.

We are met by similar difficulties in that aspect of change which refers

to increase and diminution in volume. Here again it is the continuity of the process, the continuous expansion or contraction of the local limits of the thing as first perceived, which forbids us to doubt that the growing organism, the rising stream, the contracting quicksilver, the shrivelled fruit remains the same thing, and here again we are led by gradual transitions to cases where the limits disappear, to cases of origination and annihilation. Immediate perception certainly justifies us in saying that clouds appear and disappear in the clear sky, that mists gather and the fire goes out.

16. We see, then, that change, growth and diminution, beginning and ceasing to be, as predicates of one and the same thing, are due to the fact that our continuous consciousness combines together a series of images which follow upon each other in unbroken transition, and combines them because there is no sudden breaking off, no gap in sense-perception to be a motive for drawing a line and separating the existence of one moment from that of the moment which follows. When we have realized this, it is obvious that the synthesis in which this combination of temporally successive phenomena takes place is of the same kind as the synthesis of a spatial continuum and the synthesis of a plurality of attributes belonging to the different senses, and we see the meaning of the unity which is thus conceived and which does not exclude, but presupposes, a plurality of successive differences. The concept of change involves no more contradiction than does the concept of the thing with many attributes, for to say that it is the same thing which is now hard and now soft is not to say that hard and soft are the same, nor yet to say that the thing is the same, in the sense that it is undistinguishably identical ; it is just this distinction between the thing and its attributes which makes it no contradiction to conceive of the same thing having many attributes. If the thing were merely the sum of its attributes, if the idea of it were due merely to the functions by which we perceive the attributes as such, we could never even think of a changing thing ; the smallest change would imply that the previous unity had disappeared and was replaced by a new unity, differently composed.

It cannot therefore be due to these supposed contradictions that science has from the first attempted to free itself from the thought of change, of beginning and of ceasing.

17. Even Kant's proof for the permanence of substance in which he admits of change, and merely gets rid of transformation and beginning, is

not convincing. His effort to show that a law of physics which he presupposes as the corner-stone of all natural science is *a priori* necessary from the conditions of the possibility of experience, obscures the course of the proof itself, and introduces an element—the invariable *quantum* of substance—which does not follow from the premises. If all that is wanted is that simultaneity and succession should be perceived, that something existing should correspond to absolute empty time, then this is sufficiently provided for if there is anything whatever permanent in the whole of my experience by which I can estimate change. But this permanent element is before all Myself, as the correlate of all the objects of my consciousness; and since time certainly depends in its origin upon this consciousness of mine, it involves the possibility of perceiving simultaneity and succession. Space, again, in which all external perceptible change takes place, is also permanent. Kant's further stipulation, which he connects with this, that we should find in objects of perception the substratum which represents time in general, is not really fulfilled in the general form in which he puts it; part of that which corresponds to sensation, of the real, is always changing, and the substratum which is to remain always the same is not the real, *i.e.* not that which corresponds to sensation; it is not an object of perception. It is not true that it is absolutely impossible for beginning and ceasing to be objects of perception because only the permanent renders possible the transition from one state into another, from not-being to being; it is sufficient to enable us to perceive beginning and ceasing if a part of the phenomena persist within which and by comparison with which the introduction of a new phenomenon can be perceived. The principle of the permanence of substance in the Kantian sense assumes a totality of phenomena which is never found in perception; the proof proceeds as if all were one unity.¹

18. According to our view, it is quite another motive which has led to the principle of the permanence of substance; the proof by which it is justified is only empirical, and the principle itself holds good only within the limits in which it is empirically proved.

The motive lies in the slippery way in which change in whatever form eludes the grasp of our thought. Just as the spatial extension of the thing impels us to overcome the difficulties of infinite divisibility by having recourse to the absolutely simple, so the temporal continuity of change,

¹ Cf. the searching critique of the Kantian principles by Laas, *Kant's Analogien der Erfahrung*, p. 63 sq.

its incomprehensibility for thought with its desire to analyse, impels us to get rid of it from the concept of the thing. According to the ordinary way of looking at it, although there may be change, yet because of the continuity of this change the idea of an individual thing is given as continuing the same in spite of the alteration which it undergoes; and thus we find ourselves called upon to say what this one is, to fix it in a definite concept. The first step is to separate that which persists from that which changes; in the autumn leaf the form persists, while the colour changes; hence thought can grasp the former for determination. Or sometimes it is the form which changes, as in fluids, which will take the form of any vessel, while their other qualities remain unaltered. Thus when we speak of things being changed it is generally the most familiar image which we have in our minds as the true thing, and in order to accustom ourselves gradually to the new image we survey the series of changes in imagination. But our need for definite concepts demands strict unity; that which we are to regard as one thing must not be this Proteus which eludes our grasp. And thus we are led to the attempt to find the permanent and persisting *behind* the perceptible phenomenon, to think of the attributes by which we distinguish it as something absolutely permanent, and of that which changes as mere relation which does not affect the thing itself in its constitution. In proportion as the change is more penetrating and extends to more attributes (as with melting ice or solidifying ore) we feel ourselves more urgently called upon to find some definite centre for the unity which was originally merely suggested by the comprehending process of perception, to show clearly how the subject which remains the same and persists throughout change may be thought. It is to this need of reducing the changeable to unchanging subjects that the old atomism is more especially due; from it is derived the *gigni de nihilo nihil, in nihilum nil posse reverti* of Lucretius, while the principle *ex nihilo nihil fit* is more often used as a causal principle than as referring to the permanence of substance.

The proof for the validity of the principle that the quantity of substance is invariable could not be established until a measure was found for the quantity of substance, until weight—as a relation, an effect—was recognised as this measure, and chemistry could prove that weight remained the same throughout all chemical combinations and analyses. In this, and in the possibility of reproducing the previous thing as externally recognisable, lies the empirical proof of an hypothesis which will meet our

logical needs ; but the principle is in no way necessary for a perception of simultaneity and succession, nor does it give us any indication (as it should do) what it is in the phenomenon which is to be regarded as the permanent, *i.e.* as substance. The mere proposition that the changing must be the different modes in which the permanent exists again assumes the permanent as a unity, as one general substratum, while from the first the question always before us is : What particular element is the subject which changes ?

In the proposition "in all phenomena the permanent is the object itself, *i.e.* the substance, while all that changes or can change belongs only to the way in which the substance or substances exists, hence to their modes of being," Kant has already indicated the gap in his doctrine by the use of the indefinite "substance or substances." But in addition to this, the difficulty still remains undiminished as to how the unity of the thing in the change of its determinations can be grasped ; the problem of change as emphasized by Herbart is not even touched. And yet it is just here that the chief interest centres in the scientific determination of the concept of substance. If we ask what, from a general point of view, are the considerations in favour of chemical atomism, we find that it is in this way that we get rid of this problem of the actual change of the attributes inherent in a thing. If we ask what the physiology of the sense-organs and the development of the mechanics of aggregate states have done for metaphysics, we find that it consists in the reduction of that change of attributes which ordinary perception seems to force upon us into change of relations only, into which invariable substances enter ; so that all change, with the exception of spatial motion, which alone remains objective, passes into the activities of the feeling and perceiving subject, which is affected in changing manner by invariable things.

In this way the *attributes*, by which we first attempt to determine the concept of the thing and its change, resolve themselves into *effects*, and here again it appears that the concept of the thing, when once it is removed from its popular vagueness, cannot be complete without that of cause.

19. We are brought to the same concept from still another point of view. According to p. 86 we are finally led to the synthesis performed in the thought of the thing by the attempt to think of the co-existence of qualities which our self-consciousness constrains us to combine in one place as necessary. We may regard the unification of changes in the same way ; here again we seek one ground for the successive co-existence

of different qualities. The concept of action as contained by intransitive verbs places the ground of this succession in the unity of the thing, and in itself it is no more contradictory than the thought of the many qualities of one thing. But we have also the succession of the actions of different things, and then the question arises whether there is a ground for this succession also, and whether instead of finding the ground of change in each thing by itself we must not look for it in their relation to one another, by virtue of which one thing necessitates change in another, *i.e.* in a causal relation.

§ 73.

A. Leaving for the present the question as to a universal principle of causality, the analysis of the idea of CAUSALITY must start from the idea of the operation of one thing upon another, which is always implied in ordinary language, especially in transitive verbs, and which is thought to be given beyond doubt in particular cases.

In the first place, an operation is always assumed where we perceive spatial and temporal continuity in the motions or other changes of different things; but mere succession of events does not exhaust the meaning which we connect with the term "efficient action"; it must be supplemented by the thought that the action of one thing (the cause) passes over into the other and produces in it a change which it would not have experienced by itself.

The motive which gives rise to this idea of efficient action lies in the need for having one ground for the perceived connection between changes, and it is thus akin to the motive which leads to the idea of the "thing."

From this motive the idea of influencing and being influenced (cause and effect) extends beyond the limits within which it was first applied to further cases, and guided by analogies it applies the same thought even to that which is at rest and that which persists, regarding it partly as the effect of a previous operation, partly as endowed with faculties and powers for an action which is to come.

B. (a) In logically determining these elements of the idea we have first to overcome the difficulties contained in the time-relations of cause and effect, then those which are grounded in the exclusive reference of the whole result to the efficient cause. We obtain, in consequence, in the first place the simultaneity of the action with the realization of the effect; in the second, the necessity of referring the effect to the object under-

going it as also its ground—of referring it therefore to a relation between two things. In this way the concept of force is determined as a relational concept.

(b) Logical determination aims further at deriving efficient action from invariable substances with invariable forces, and the change which occurs merely from their relations as the conditions of the efficiency of the forces ; and this invariability finds its expression in the unity of a law according to which every force takes effect. The term cause being then transferred to the changing relations which represent the conditions under which the forces take effect at any time, it can be equated to the totality of these conditions ; and in so far as this occurs the cause (as the entrance of the conditions which takes place at a given point of time) precedes the effect. When the change which appears is also defined as the measure of the action actual or possible under the given conditions, the conceptual determination of the connected elements is completed by the principle of the conservation of energy, and at the same time the concept of the thing is made complete as the concept of the one ground of its attributes and activities.

C. The so-called PRINCIPLE OF CAUSALITY has many meanings. From the nature of thought there arises the obligation of conceiving everything which we think of as existing as necessarily following from a real ground of its existence, and of its existence in a particular manner, an obligation which ultimately presupposes a simply and absolutely existent. From this obligation follows the assumption that whatever is given has a ground.

It is, however, not determined by the general principle what is the nature of this ground of the existence of a given thing. We may look for it in the relation to external causes, which act upon that which is given ; but we may also look for it in the nature of the thing itself, by which its activities and the order of their succession are necessarily determined according to a law of development.

The term "*causa*," which was originally used only to denote an external cause, being applied to this latter relation also, there results the distinction between the *causa immanens* and the *causa transiens*. "Cause" is again sometimes used of the subject as the one ground of its activities, sometimes of the particular activity in so far as another follows necessarily upon it.

In what sense we can show that which is actually given to be grounded, must be determined by our attempts to reduce it to its grounds.

1. The ordinary view which finds its expression in language has

answered the question raised at the end of the last § by developing the thought of an interaction amongst things.

In dealing with the idea of the action of a cause it is even more difficult than with the idea of a thing to find its ordinary content, as determined by our use of language, in order that we may define its limits and make it logically perfect ; nor is it more easy to show how such logical perfection is to be attained. The question as to the genesis of the idea of the action of a cause is more intimately confused with the question as to its content, and scientific language itself has failed to come to any conclusion, even more than with reference to the concept of substance.

2. It would be trouble thrown away if we were to make the origin of the causal idea the subject of our direct investigation, as a means of arriving at its meaning and content. Though this idea has doubtless had its origin in the course of the development of human ideas, and though it could not have arisen without the stimulus of the sense-impressions with which the development of consciousness begins, yet we are for ever shut off from any direct observation of its growth, and the variety of the theories as to the origin of the causal concept speaks clearly enough of the difficulties which beset the question as to the genesis of the idea. When we begin to reflect, the idea is already there, and is being made use of as something natural and familiar. How it has come into consciousness, no recollection can inform us, and all hypotheses as to its origin can be based only upon the content which we find to hand.

3. But this given content itself is not easily grasped nor always the same ; the results of a highly developed scientific elaboration of the concept of causality are easily confused with its earlier and more elementary form, and it becomes uncertain what we are to understand by cause or causality. Only too often the question as to whence we get our idea of causality in general has been entangled with, or quite mistaken for, the question as to the origin of the causal principle, the law that everything must have its cause ; and this general causal principle has then been further confused with particular aspects of it. More especially when we speak of the causal principle does the term itself suffer from ambiguity ; sometimes it signifies the real ground of necessity in general, sometimes the ground contained in an efficient cause.

4. We must begin by making a distinction. The conscious thought that everything has its cause is undoubtedly later than the thought that there is something which must be thought of as cause ; hence the *meaning*

of the causal relation must be present before it can be thought of and affirmed in its universality.¹

To get at the meaning of this causal idea, as it precedes scientific elaboration, the only way is again to turn to the most indubitable and simplest cases which are intelligible to every one, and which are already universally and confidently denoted as cases of efficient action by the use in popular language of words containing the thought of efficient action. For, here again, there can be no doubt that the idea of active operation, as presented in concrete particular cases, precedes the concept of cause, in which the idea of the relation of efficacy is mingled with that of the efficient thing; just as murderer is derived from the verb to murder, and not *vice versa*.

5. Here we may note three points :—

First, that which takes effect is originally always a thing, and, properly speaking, efficient action can only be predicated of concrete things with a particular existence; indeed, those verbs which express efficient action must always have such a concrete subject.

Secondly, efficient action, where we seem to find it most clearly and indisputably, is action which occurs at a definite time, is instantaneous or persists for a space of time and is directed towards some other thing.

Thirdly, that which is effected is a definite change of this second thing, and the action finds its fulfilment in just this production of change, in the realization of the effect.

When I overthrow an upright body by a blow, when a falling body wounds me or breaks a vase, then I act as a cause by moving my arm in the direction of the body, the stone by falling on to the vase or my skin; by the change that takes place in the object, the causation is exhausted, even though the effect, the state brought about by the change, persists. The position of the fallen body does not *continue* to be caused by my action, but *has been* caused by it; the pain of the wound continues, but the stone which wounded acts no longer; its causation was fulfilled at the moment in which it struck me; only the effect persists, not the causation. In speaking of after-effects, we distinguish between the events which occur after the real causation and the causation itself.

It is an obvious abbreviation of speech when we substitute for the

¹ This excludes neither that the causal idea is due in the individual to a kind of rational instinct, nor that the knowledge that *A* is the cause of *B*, depends for its certainty upon the known validity of a general principle.

efficient subject the action, through which it takes effect, as a unity, and denote this as productive of an event. In so far as the efficient agent can produce an effect only by virtue of and through its own action, this effect may be directly ascribed to the transitory action; we say, either, that my blow overthrows the body, or, that I overthrow it; that the stone, or that the falling of the stone, breaks a vase.¹ What significance is to be attached to this distinction will appear later on.

6. As in our ordinary practice we keep in view the beginning and end of the whole process as it occurs in time, and lay especial emphasis on comparing the conditions before and after the action, it naturally follows that the beginning of the action and the completion of the effect fall asunder in time; and inasmuch as we look at the action from the beginning and at the effect from the end, it follows that the effected state succeeds the action of the cause, or, more accurately, succeeds the moment when it begins to be efficient. First, I strike a blow, then the body falls; first the stone begins to fall, then it strikes my skin and wounds it.

But the idea which we connect with efficient action is not exhausted by this thought of temporal succession; it contains also the passing over of the activity of one thing into the sphere of another, the power which it can exert over it, the constraint which it can put upon it, the suffering which it can make it undergo. The body is not overturned "of itself," just when my hand touches it; the vase does not break "of itself," just when the stone strikes it; it is not merely this temporal connection which obtains between the one movement and the other, although it is only this temporal connection which is the object of immediate perception. The origin of the change effected lies in the efficient cause; the object of the action would not itself have produced it.

7. It will not be disputed that in these and similar cases we are immediately certain of the action of a cause, and in support of this we quote Locke,² when he says that mere observation teaches how change is brought about in a thing through the application or operation of another. If, however, we ask what it is in such instances which gives us this certainty of inner connection, we are first confronted by the answer of Hume and his followers, that it is perception of the regular recurrence of the sequence of *A* and *B* which finally brings us to regard *A* as the cause of *B*. According to this view, we mean by cause that invariable antecedent

¹ Cf. I, § 6, 3, p. 36-39, § 13, 3, p. 78.

² *Essay*, etc., ii. 7, § 8, 26, § 1; cf. *supra*, I; § 47, 12, p. 314.

of any phenomenon which we learn to know from experience, and by generalizing from numerous instances we arrive at the assumption that every change has its cause,—has, that is, an antecedent upon which it regularly follows. But though this observation of invariable sequence is of great value, both in giving rise to the thought that efficient causes act according to law, and—given this thought—in helping us to determine what is to be regarded as the cause of *B* and the effect of *A* and in correcting our mistakes, yet it is incapable of creating the thought of causation itself, for this contains more than mere sequence. It is inconceivable how mere repetition should introduce an entirely new element of which there is no trace in the particular instances which are repeated.

8. If we turn our attention first to the nature of the events in which efficient action is most immediately obvious, what we first find is the spatial and temporal continuity of the changes which take place in different things, and it must be this continuity which first impels us to regard them as one connected event. When the spade penetrates into the earth and pushes aside those parts with which it comes into contact, when the axe splits a piece of wood or the hand pushes forward a body, we are quite unable to think of the one movement without the other; the principle that two things cannot both occupy the same position in space, involves that whenever the one body moves the other should move out of its way. And since the two processes, that of penetrating or pressing forward, and that of receding, take place in unbroken connection, our original datum is the picture of the whole event. This we nevertheless, because of the duality of the things moved, distinguish at once into two events, into the movement of the body which presses forward and the movement of the body which recedes, only that we may again annul the division and look upon the receding movement as the immediate continuation of the forward movement. Thus we come naturally to think of the action of the first body as continued in the change of the second, to extend it spatially and temporally beyond its own movement, and not consider it as finished until the whole continuum of changes has come to an end. Thus in the thought of efficient action is contained the real ground of that unification which our self-consciousness in its temporal continuity and spatial comprehension performs between two spatially and temporally connected events. In this way this thought comes very near to the apprehension of the changing, and indeed of the unchanging thing; just as the continuity of change will not allow us to break off the existence of one thing and begin

that of another, so the same motive leads us to find the ground for the continuous progress of the change of one thing into that of another in the first, the action of which passes over into the second. From this point of view a new light is thrown upon persistence also; the ground of persistence is again contained in the unity of the thing, it is the thing itself which continues its existence in time. Although this persistence seems such a matter of course, it is none the less certain that it is based upon a similar comprehension of different moments in unity, only here the comprehension takes place most easily, because there are no differences but those of time itself to be overcome. That it does actually take place, becomes obvious when we are confronted by the violence which is done to the idea of the continuity of existence by the doctrine of Arabian philosophers, and adopted by Descartes—the doctrine that the existence of the universe at any one moment is absolutely independent of its existence in the preceding moment, and that it is only the recurrence of divine creation at every moment which produces an appearance of continuity.

9. This root of the idea of efficient action finds expression in the Aristotelian doctrine that bodies cannot act upon each other except when in contact; it has given rise to the obstinate and constantly recurrent opposition to any kind of action at a distance, and it appears also in the originally local significance of the cases and prepositions which denote in the active voice the object, in the passive the subject of the action. Nor is it wanting where spatial continuity falls into the background, and the temporal only seems at first sight to be present in the relation between internal and external events. The pain caused by a blow, the feeling of satisfaction which follows eating and drinking, the connection between volition and movement, all this is regarded as cause and effect as confidently as in the case of pushing, pulling, or pressure. The spatial relation indeed is not wholly wanting even here; the pain is localized just where the blow was struck and the will works from within the body, but the action and its result cannot be comprehended intuitively in a spatial whole. Nevertheless the temporal continuity is sufficient, at any rate where it is not contradicted by the spatial relations, to connect the different events as a unity, and to make us regard the second as the continuation of the action which constituted the first; the need of referring elements which accompany each other in perception to one ground of unity remains the same.

The theory according to which the whole idea of causality is originally due to the consciousness of my own voluntary action, forgets that after all

the movement of my limbs merely follows the volition which is directed towards them, and that here again it needs explanation how I come to regard this as the action of myself upon my limbs. To say that here the inner connection is brought about by an *aim* which governs my action, is to forget that before I can consciously set before me an aim I must have already experienced the consciousness of my *power*, hence the efficiency of my will. Moreover, the psychological development of gradual mastery over the limbs and of the consciousness of being able by means of my will to effect something outside myself, is only explicable if preceded by involuntary movements, and if the results of these have been noticed. Only this much is true in the theory: that these mutual relations between ourselves and the external world are the events which interest us most and are first to excite our attention, and that the thought of inner connection between successive events would probably not come into clear consciousness so easily or so soon if we were mere spectators of external events; our voluntary action and the pleasurable or painful experiences which we undergo from without make a livelier impression and form a sort of typical instance, so that we are inclined to interpret even external events anthropomorphically by ascribing endurance to the object of the action, and to its subject a kind of volition. In the same way it is our consciousness of exertion which is the original measure by which we estimate the magnitude of an action, and by this same feeling we determine how far we effect anything; when we place a ball upon an inclined plane, and let go, we know that it rolls down "of itself" because we have used no exertion to produce its movement.

10. Everywhere, then, where we speak of efficient action in the original sense, this synthesis of connected changes in the thought of one ground takes place, and this explains why, when we distinguish between acting and being acted upon, we first comprehend a continuum in a unity, and then call an event which occupies more or less of time, and which we regard as one, the action of the cause, and the change connected with it—again considered as one—the effect. When I throw a stone, the movement of my arm until the stone flies is one act, the flight of the stone until it alights the second act; the whole action which is apprehended as acting and being acted upon is concluded in these two acts. In this instance the efficient agent works from within outwards; it is impelled by itself to begin the action which is concluded when the stone ceases to move.

11. From these simple and comprehensible examples the idea of efficient action is extended in various directions. In the first place, we can trace chains of effects in which changes pass from one thing to a number of other things successively:—a body is struck by a blow, it falls into the water, drops of which splash up and wet the surrounding objects, etc., where we distinguish between near and remote, immediate and mediate, causes and effects. In this way the beginning and the end fall still further asunder in time, and we are led by analogy to connect as cause and effect events between which we perceive no intermediate links. Recovery does not follow immediately upon the swallowing of a remedy; we say “it does not take effect yet”; nor does illness follow immediately upon a chill; but our perception of many changes, which do not appear “of themselves,” causes us by means of simple association to look for the efficient cause and to find it even in remote events if these seem appropriate, from their similarity with known causes, to effect the change before us.

Analogy leads us further to regard even the quiescent in the same way, and its state may be looked upon as the effect of a previous action; its present position implies that it has been brought here, its present constitution that something has acted upon it, and it follows that the idea of being acted upon is extended to many instances where we have no immediate perception of change, but where we are convinced that the object owes neither its presence here nor its constitution to itself.

From another point of view efficient causes may also be looked at in this way. The action presents itself at a definite point of time, and so far the action of a cause is itself a change of the cause, and the question arises as to the origin of this change; the same need of unification works backwards, and seeks a ground for the action in a preceding state or event. This question marks the division between actions working from within outwards, and actions excited from without. Our own action is an example of the first; it proceeds from a volition which arises within ourselves and which we are wont to regard as something ultimate; we are conscious that we have only to will in order to be able to perform an action, that our limbs obey our command as soon as it is given. This is the origin of the concept of a faculty, of a power which is present as the permanent state of the capacity for action, and which is distinct from the momentary volitional impulse through which the power is set in action. By extending this aspect everything which passes from rest into action seems to be acting from a power which is always there, as we ourselves act in volition;

while on the other hand some stimulus to bring the power into action, some impulse to excite it, is needed as a substitute for the will. The idea of an effort which is checked by obstacles and becomes action when these are cleared away is based upon the same psychological ground; a state precedes the actual result which stands midway between rest and action, a state in which no visible result appears, but in which an effort is made to reach the result. It is unnecessary to point out how easily the state of a tense spring or a hanging weight may be compared with the exertion which a man feels when he strives to overcome an obstacle which surpasses his power.

We are taught by our experience that heavy bodies fall "of themselves" to the ground, or slide off a steep plane when they are not supported or held, that the string of the stretched bow springs back when we let go, to distinguish between the activity which merely lets go, and the action which hurls a body or strikes a blow; in the former case the ground of the motion must lie in the body itself, which is only freed from an obstacle. Aristotle's explanation was the most obvious when he derived the fall of a body from an inherent effort to reach its natural place; it needs no cause in the ordinary sense of the word, moving it from without, to make it enter upon the downward path. It is a much later reflection which attributes the fall to the attraction of the earth, that is, to an external cause to which is ascribed a power which is constantly acting upon bodies—a power, moreover, which the earth exerts while in rest without itself changing. No action, as when motion is imparted by moved bodies, now corresponds to the producing of an effect; and in the pressure again which a body exerts upon that which supports it, power is also manifested which elicits no movement, which does not therefore act in the ordinary sense.

Here again, then, the quiescent becomes involved, not this time as an effect, but as possessing a faculty or power of activity, or an effort to act; the idea of the quiescent is animated by that which it *can* or *could* do.

12. This process of expansion in all directions which the idea of action and being acted upon undergoes is not due merely to natural association which, led by similarity with primary and easily intelligible causal connections, completes the picture in imagination and traces the course of changes backwards and forwards. Just as the simplest idea of action is motivated by the thought of a ground of unity for the simultaneous changes of two things, so that which impels us to extend the idea of the causal relation, even where no perceptible event calls for it, is none other than the

need for connecting isolated events or states, of bringing unity into the changing course of things, of mediating sudden transitions from rest to movement and unexpected changes by finding a ground from which they proceed. The same motive from which we obtain the mastery over a particular succession of changes by the thought of efficient action, impels us to establish within the whole spatial and temporal universe a connection by which the particular is torn from its isolation. And this need becomes especially imperative when a sudden transition occurs which cannot be explained by the habitual behaviour of the particular thing. A change which proceeds continuously, as in the growth of plants and animals, or in the movement of stars and the flowing of streams, is much less likely to make us ask after the cause which brought it about ; we are satisfied in such cases by referring to the unity of the moving or changing thing itself, which seems to us to be animated from within. Sudden changes are less comprehensible, and we surmise an external cause for their origin all the more easily because we have often seen such sudden changes occasioned by intrusion from without. The Herbartian theory that the whole concept of change is contradictory, and that the concept of cause has been introduced to do away with this contradiction, is so far correct that every change in a thing which deviates from its own habits and from the behaviour of similar things is difficult to refer to the thing itself, and indicates its connection with others. A man in good health does not ask what makes him healthy, and streams may flow for long before any one troubles himself as to why they flow ; but when the streams are frozen, and the healthy man falls ill, curiosity arises and impels us beyond the particular to its connection with other events.¹

13. If we pause here, without at present taking into consideration the consequences which follow from consciously generalizing the effort to establish causal connection into the so-called principle of causality, we shall find that the idea of efficient action as used in ordinary popular thought offers difficulty enough to its logical determination if the thought contained in it is to be freed from all ambiguity and reduced to a fixed concept. We have in this logical determination to overcome an obstacle like that which we met in determining the concept of the thing, an obstacle the ignoring of which has brought hopeless confusion upon the treatment of this doctrine by the English empirical school from Bacon to Mill.

In our ordinary way of regarding events in the perceptible world, from

¹ Cf. Zeller, *Vorträge und Abhandlungen*, 2. Samml. p. 26.

the point of view of their causation, we fall back upon unities of which the limits are easily defined in time, just as in our apprehension of things we are guided by spatial unities. The blow which strikes a fragile object is one event; its breaking, until the fragments rest upon the floor, is the second event. The swallowing of poison is one act, the death which follows a few hours after is a second event; the striking of a match is one, its burning until it is consumed is another; the warm spring weather which begins to-day is one, the vanishing of the snow and the appearance of buds is another. Thus, in speaking of causes and effects, we ordinarily refer either to events which, because of their homogeneity, we can easily combine into a temporal whole or to especially striking changes; we do not concern ourselves either about the divisibility of every duration into smaller sections of time, or about the interval—indifferent to us—which lies between the two changes which strike us. This explains how it is that to our ordinary apprehension it is such a matter of course for the effect to be later in time than the efficient activity of the cause, that we find no difficulty in the fact that there is actually an interval between the event which we regard as cause and that which we denote as effect. It is upon this popular apprehension that the attempt is based to strip the causal relation of all meaning but that of a regular sequence of different events.

14. But here we are threatened with logical confusion, when we remember that after all it must be things, and not their changes only, which are causes, that these things exist and persist contemporaneously with those upon which they act, and that efficient action is primarily regarded as an activity beginning in time and directed towards a particular object. Now if we are to be accurate, we cannot speak of the efficient action of an *A* upon a *B* so long as *A* alone changes and *B* shows no sign of change. If the action consists in the production of the effect, the cause acts just in that it produces the effect, the action of the cause *A* and the production of the effect in *B* must be simultaneous; that which goes on in *A* before this we cannot call its (efficient) action, but merely an intransitive change which precedes this action.¹ The movement of the stone which I hold in

¹ Wundt observes (*Logik*, i. p. 540) that we can never infer from the logical relation between concepts to the temporal relations between the phenomena to which the concepts refer, but in this general form the statement does not hold good. From the logical relation between the concepts thing and attribute there certainly follows simultaneity in the existence of the thing and certain of its attributes; and from the concept of the position of different things in space there follows simultaneity in the existence of the things and of the relation between them. It is the same here. If effect consists in a change, and if we can

my hand does not follow the movement of my arm, but is simultaneous with it; the lifting of a load does not follow the movement of the lifter, as he contracts his muscles the load rises; the movement of the horse does not as a whole precede that of the carriage, but only continuing work produces the effect, the progress of the carriage, and when the action of the cause ceases, the change or movement which can be regarded as immediately effected ceases also. The saying *cessante causa cessat effectus* could find as much support in a host of popular instances as the view that the cause must precede the effect.

15. We find ourselves called upon by this antinomy to examine more closely in our conceptual determination of efficient action the temporal relation between cause and effect, and this the more imperatively because of the necessity of deciding whether that which we regard as a unity is really a unity, and must necessarily be thought as such, or whether the time-occupying event which we regard as the action of the cause, and that which we take as the effect, must not be broken up into a series of component events which themselves form another causal chain. The simple consequence of the concept of change, its divisibility into indefinitely small successive degrees of change, impels us to ask as to the ground upon which the second stage of a change must follow the first, and either to look for the ground of the progress of the change in the first cause of the beginning of the change, or to find another ground for it.

The bearing of these observations will be most easily seen if we remember that in mechanics Galileo's principle of inertia first threw light upon these difficulties, and laid the foundation for all further progress by giving a universal ground for the necessity of the persistence of a movement which has once been begun—a ground which was no longer found in a continued action of the cause of motion (*A*), but in the necessity of the persistence of the object *B* in the state of motion once induced, according to which every succeeding differential of the orbit must be traversed because the preceding one has been traversed, and a new cause is involved only by a change of movement as to velocity or direction. Uniform motion in a straight line now falls under the same point of view as rest, persistence in which is generally regarded as something quite natural, so that no one asks what is the cause of the rest of a body (cf. p. 98).

speak of causation only in so far as a change occurs, if, therefore, nothing is effected until the change occurs, then the action of the cause and the resulting effect must necessarily be simultaneous. Wundt's statement is true only of the abstract concept of dependence, which is no doubt applicable to different time-relations.

In this way we may distinguish three stages in time :—

1. There is the state of the cause (motion of the striking body) which precedes its action (*e.g.* a blow).

2. There is the time in which the cause acts, imparts a motion of a given velocity to another body, or changes the velocity or direction of an existing motion.

3. As soon as this action ceases there is the simple persistence of the new state of motion. In the wider and only mediate sense this persistence, which may possibly be of endless duration, can and must be regarded as the effect of the efficient cause; and thus every effect has an infinite duration in time; in the same way the preceding motion of the striking body is mediately the cause of the blow, which it strikes only by virtue of the velocity it has previously acquired. In a narrower sense, however, the effect of the cause is only the communication, change, or annihilation of the motion; the continuation of the state thus brought about is not the immediate effect of the acting cause, but only the necessary consequence of this effect, and thus we have simultaneity in the strictest sense between the activity of the cause and the realization of the effect.

Here, no doubt, we fall into another difficulty as to whether an instantaneous action, such as we think we perceive in a blow, really is possible and conceivable, whether a finite velocity can arise in a nothing of time, and theory inclines to assign to every mechanical action a period of time during which the body moved passes from a state of rest through all the intervening stages to a finite velocity. In the first unit of time a minimum of acceleration is imparted, which persists according to the law of inertia and to which the accelerations of the successive units of time are continually added, until the finite velocity is reached. We need not enter upon a more exact exposition of this problem of mechanics; all we have to show here is, that the attempt to grasp the relation of cause, action and effect with conceptual accuracy necessarily leads to a breaking-up of temporal wholes which we do not hesitate to treat as unities in ordinary language, and that it is necessary to analyse what takes place in the cause into its action and the process which precedes and prepares the action, and what takes place in the object into the immediate effect of the action and its continuation.¹

¹ It is characteristic that mechanics, in the fundamental views of Galileo, was forced to break up time into points within which action takes place and empty intervals in a way exactly analogous to that adopted by atomism in order to free the concept of

Of course even where the action, as in the interaction of atoms, occupies a period of time too small to be measured, or indeed none at all, the whole process which we bring under the category of causality is not crowded into this instant, but must represent a lapse of time; the blow is only possible through a preceding movement of the body striking which occupies time, and what takes place is the communication of a velocity to the body struck, which velocity must again manifest itself in a movement of some duration. Time cannot be altogether eliminated, in the first place, because what we originally apprehend as effects are changes which take place in time, and even states of equilibrium, when brought under the same point of view, can only be considered as persisting; in the second place, because even the action of the cause itself can at first only be considered with reference to a preceding state as the beginning of a new state, while, on the other hand, it must have for its consequence a change which takes place in time. Moreover, since an unlimited duration of efficiency is contained in the concepts of the so-called continuously acting forces, this efficiency may consist either in actual movement or merely in the maintenance of a given state of equilibrium or tension.

16. If we apply the questions which first made themselves clearly felt in mechanics, to the concept of change in general, the problem which confronts us everywhere is to distinguish in that which we are tempted to regard as the effect of a cause between the immediate change accompanying the action and the mere persistence, or further independent development, of a process once induced, and in this way to find a solution of the antinomy between the proposition that the effect follows the cause and the proposition that the action of the cause and the beginning of the effect must be simultaneous. When one man wounds another by stabbing him, his action in the strictest sense is limited to the time in which the dagger penetrates the body and tears its tissues; what follows is the further development (independently of external interference) of the state which has been brought about, of the wounding of the tissues, of the opening of blood-vessels, etc.; while the process which precedes, the seizing of the dagger, the movement of the hand holding it, the velocity thus communicated to

substance from the troublesome continuity of space. That manner of conceiving of the acceleration of a falling body, which thinks of it as brought about by a succession of instantaneous impulses which increase the previous velocity, shows the same endeavour to break up the continuous into the discrete, as the reduction of extended masses into indivisible points. On the objections of Benno Kohn to the above view see Appendix D.

the weapon, does not yet constitute an action upon the wounded man, though it can itself be analysed into actions and their consequences.

17. As soon as we are convinced that the action of the cause and the realization of the effect are necessarily simultaneous, and further, that in the effect itself we must distinguish between what follows from the mere faculty of persistence in the substance acted upon, and what is produced by the action of the acting substance, a further alteration in the popular ideas becomes necessary. In the first place, we find that what we began by regarding as the simple transference of the action of a substance *A*, to another substance *B*, is now grounded, at least partially, in the substance *B*, which changes because of the action, and that the ground of the connected event must no longer be looked for in the substance *A* alone, but in the substance *B* also. It depends upon *B* in what way it changes and continues the change when it has been introduced, and thus it ceases to be the mere object of the action, the mere passive substratum upon which *A* exerts its power, and becomes a part of the total ground from which its perceptible change proceeds.¹ Add to this, that whatever proceeds from this part of the total ground is already completely simultaneous with the action of the cause, and that the action of the cause exhausts itself in the production of the effect, by which, therefore, its course is determined (quite apart from any resistance which it experiences), and we see still more clearly that what at first was one-sidedly regarded as the action of the cause *A*, is the common action of *A* and *B*, and that its ground must be sought in both together. But since it is contained in neither by itself, it can only lie in the mutual relation of both, by virtue of which the action of *B* is guided by that of *A*, and, *vice versa*, the action of *A* is determined by the nature of *B*.

This relation cannot be thought of as one external to the two things, as a third cause in addition to the two things themselves which acts upon both, and thus produces harmony between their activities. This expedient, by which occasionalism, with its continual intervention of divine omnipotence, or Leibnitz, with his pre-established harmony, hoped to escape the difficulties involved in the idea of efficient action, only doubles the problem; the divine action which is exercised upon both *A* and *B* has now to be explained, or we are forced, in following Spinoza's doctrine, to relinquish *A* and *B* as independent things altogether.

¹ Bacon himself illustrated this thought by the example that the sun softens wax and hardens clay.

There is no way open to us but to find this relation in the permanent nature of both substances. We found ourselves obliged to think of the one-sided action of a thing as the manifestation of a force which is always present, in order to reconcile it with the unity of a substance persisting in time; and the same concept presents itself here, although it needs modification in a direction indicated by the considerations referred to above. Force can no longer be conceived as a quality, as if a thing by itself possessed the power of intruding itself into the sphere of other things, disturbing and changing them; it becomes of necessity a relational concept, expressing that the nature of a thing, the inner ground of its activities, cannot even be thought of if we regard it as absolutely isolated, that there is contained in it a relation to others such as under certain conditions gives rise to common action.

18. In proportion as the logical elaboration of the concept of substance constrains us to think of substance as something invariable, because in this way we most easily obtain a fixed and absolutely definite concept, must the force which belongs to it, *i.e.*, its essential relation to other substances as the ground of changes, be also thought of as something invariable. But the ground of changes cannot be found in that which remains the same, but only in something which varies, and it is only the relations of things, above all their spatial relations, which are variable; in these relations, therefore, we must find the ground for the varying manner in which forces become efficient. They contain the conditions under which constant forces can become active, that upon which it depends whether and what changes follow from the essential relation of things contained in the concept of force. In this way we distinguish between the ground of change which is contained in force, and is unaffected by differences of time, and the changing relations which are the conditions under which appear the changing consequences of this ground. The word "condition" is here used in its ordinary sense, not as the ground itself, but as something which makes its activity possible and the absence of which would annul this activity.

This explains why, as we make our concept of efficient action more precise, we can no longer regard things as, in the first instance, the ground of changing occurrences, but speak instead of their relations, and substitute for the concept of cause as the ground of a given event a sum-total of conditions upon which a result depends. It is when the name cause, contrary to its original meaning, is transferred to these changing relations

upon which depends the action of the forces that the proposition arises that this cause—*i.e.*, the sum-total of conditions—*precedes* the appearance of the change. But this is not the effect of the cause when conceived in this way in the same sense as it was the effect of the acting force. When the thread by which a heavy body is suspended is cut, we may indeed call this cutting the cause of the fall, and the cutting does precede the fall in time. But no one thinks that the cutting throws the body on the ground ; as *causa occasionalis* it merely brings about the condition under which the attraction of the earth can take effect, and it is this which is the *causa efficiens* of the movement. Thus the proposition that the cause precedes the effect presupposes quite another meaning for the word “cause” than we find in the proposition, that cause and effect (more exactly the action of the cause and the production of the effect) are simultaneous.

19. To illustrate the course of this alteration in the concept by a concrete example, we may take the most simple instance of an impulse, in which the action of *A*, the body striking, upon *B*, the body struck, seems to be immediately clear. As we first apprehend it, we find the ground of the whole phenomenon in *A*, the body striking ; it is it which drives the other body *B*, and the whole movement manifested by the latter until it comes to rest is the effect produced by the force of the striking body. More accurate analysis then teaches us that the body *A* cannot take effect until it has struck *B*, and a change appeared in *B*, and that it ceases to take effect so soon as *B* withdraws ; the efficient action is limited to the moment of the blow which imparts a movement to *B*, to be carried on by *B* in accordance with its own nature. But even in the moment of the communication of the movement both are active ; the ground of the result lies in the fact that it is as much a part of the nature of *B* to move when it is struck as it is part of the nature of *A* to occasion this movement ; thus, the ground is really that relation between *A* and *B* from which this form of event arises in accordance with the nature of both. We think of this relation as some sort of repulsive force, which is an essential possession of both bodies, and present in *B* as well as in *A*. But the repulsive force which is a permanent determination of both bodies becomes active only when they approach each other ; it always belongs to them, but produces change only when certain conditions are fulfilled. These conditions—the approximation of the two bodies at a given velocity, the absence of obstacles to the movement of *B*, etc.—are spoken of as introducing the change ; and what we now call the cause of the movement

is neither the body *A* nor the mutual repulsion which always belongs to them, but that contact of the body *A*, moving at a given velocity, with the body *B* which gives rise to the conditions under which alone their repulsive force can take effect. Here the movement of the body *B* does not appear until after the point of time at which *A* arrives at *B*.

20. From the alterations which the concept of force has had to undergo, it follows of necessity that its significance extends beyond its origin. This we found in the need of unifying the perceived and continuously connected changes of different things, the results to which this attempt leads necessarily involving also their state of rest with respect to each other. It depends upon the conditions whether forces become active in change, it also depends upon the conditions whether they remain inactive; thus rest itself now appears as emanating from the same forces to which change is due; it is grounded in an equilibrium of forces, in conditions which permit no single force to take effect. In this way it first becomes possible to apply the same conception to the whole duration of the existence of things.

21. What we have said will serve to show how those alterations in the concepts connected with the idea of causality which appear in the history of science may be simply explained by the logical need of distinguishing and making precise the elements contained in the popular ideas. But the final source of the firm hold we keep of the fundamental idea itself is the same as that from which in the first instance it sprang—the instinct which impels us to refer continuously connected Being and Happening in a constantly increasing extent to one fixed comprehensible ground.

22. From the same source spring also other elements which we have hitherto overlooked, but which must be connected with the thought of efficient action when we take it as a concept and develop the nucleus contained in it into all its consequences.

The most important point is that to which the empirical doctrine has attached exclusive importance, regularity in the action of causes. We have already dismissed the view that it is this regularity perceived in the succession of two events *a* and *b* which first gives rise to the thought of efficient action; we must, on the other hand, willingly allow that to a large extent our most ordinary experience manifests this regularity in the sense that when the same things come together or are brought together in the same way, then the same effect will be repeated. That drink regularly quenches thirst, that fire warms, that a blow hurts, that a stone

flies off when thrown, this is easily taught by experience, and the expectation grounded upon such observations guides all our action and treatment of external things. But none the less false and impossible is the empirical doctrine that simple observation of the course of nature shows us uniformity in the sequence of antecedents and consequents, in such a way that we obtain by an easy and obvious generalization from experience, not merely the proposition that under like circumstances like will occur, that the same antecedents are followed by the same consequences, but also the further and much more far-reaching proposition that everything which happens is preceded by antecedents from which it must inevitably follow. If we could appeal only to the results of our ordinary experience, then the impossibility of calculating what will happen, the freaks of chance, the occurrence of events for which we find no familiar ground of explanation as a regular antecedent, will be observed at least as frequently; throughout large spheres of observation changes succeed each other in the most varied diversity and our expectations continue to be bitterly deceived. If we had needed merely to open our eyes in order to see "uniformity in the course of Nature" everywhere before us, belief in the thoroughgoing constancy of the way in which causes act would not have been so slow to arise nor have been still only a scientific and not a popular belief; nor would the tendency to make capricious powers, demons and gods, responsible for what happens in the universe have been so deeply rooted. No doubt instances of the invariability of the effects of certain causes are frequent enough, even in an untutored experience, to bring us close to the thought that like things always act in the same way upon like things; but that thought becomes prominent not through its generality in experience, but through the *value* which regularity of effect has for our practical needs, and for our effort after knowledge.

If we disregard as remote from our present subject those views which refer to the possibility of general judgments concerning a plurality of similar things, then the value belonging to the regularity of the effects produced by one and the same thing upon the same other things depends, in the first place, upon the value attaching to the thought of invariable substances (see the previous §). In this alone the thought of the thing actually reaches its logically perfect conclusion, and since the concept of substance, when we grant it as capable of taking effect, involves the concept of force, there results from the invariability of substances the

invariability of forces; but it follows from the invariability of forces that given the same relations the same actions and effects, *i.e.*, the same connected changes, must present themselves. Thus the definition of the invariability of substances and their forces reduces itself to the establishment of a law, according to which given actions depend upon given relations. Thus law is the expression of force, the formula in which we state the one permanent ground of manifold changes.

In such laws the assumption of necessity as the ultimate ground of that invariability and regularity is expressed in the only comprehensible way. When we regard some particular event as an instance of efficient action the relation of necessity which obtains between the ground and its consequence first appears in the form of the constraint which the object of the action undergoes; it is not a matter of choice whether it will undergo the action and change itself, it is subjected to a power external to itself which does violence to it. But as we proceed in our logical development of the concept the meaning of necessity also gains in depth; when we look for the ground of their behaviour in the nature of the things acting and acted upon, the idea of external constraint vanishes, and necessity appears as such that both parts by virtue of their nature are equally subjected to it as to the inner coherency of their essential determinations. We think of the essence of a thing as invariable and strictly one, and necessity manifests itself in the invariability with which like effects appear under like conditions. It is here that we find a justification for extending the concept of cause even to changing relations of which it cannot be said that they act, but only that changes follow regularly upon them. The external relation at first posited between the things acting and acted upon passes into one which corresponds to the logical relation between the general law and its particular applications, and the necessity of the logical ground finds a perfect counterpart in the necessity with which a given mode of action follows from the nature of things as from a real ground.

23. The last point needed to complete our logical elaboration of the concept of causality refers to the quantitative determination of the relation of cause and effect. It follows from the concept of efficient action itself that the magnitude of the change effected is the measure for the action of the cause, for it is just this production of change which constitutes the action. The heating of a body 20° indicates greater efficiency in the cause of the heat than the heating of the same body

10°, the moving of a double load with the same velocity indicates a doubled efficiency of the moving cause. At first it is the effect actually produced by which we measure the actual efficiency of the cause; afterwards, assuming regularity in the causal relation and invariability in the things acting, we refer action to the capacity which things have of action and expect the same result from the same conditions, and then we frame a universally valid proposition in which the measure of the capacity for action under given conditions is determined by the magnitude of the possible future result. A mass moving with a given velocity possesses the power of imparting a given velocity to another mass with which it comes in contact; a compressed spring when released the power of lifting a given weight a certain height. This measure of capacity for work which is given in a particular state is called energy, to distinguish it from constant force. Though we speak in the first case of kinetic, in the second of potential, energy, we must remember that in both cases alike we have included a hypothetical element in our concept; even the moving body only works if it comes into contact with another, just as the spring only works if it is released, and so far the antithesis between kinetic and potential energy is not entirely correct. The real opposite to potential energy would be actual energy, and this is to be found only in actual work done; kinetic energy therefore is not actual energy, but is the capacity for work of a moving body as distinct from that of a body at rest. Only if we should think of the movement of a body itself as a continuous exercise of its energy would this be actual; but to think of it in this way is contrary to the principle of inertia.

A universal measure for the capacity for work presupposes that the actual or possible amount of the effect can be reduced to a uniform unit of measurement. The disputes between the schools of Descartes and Leibnitz as to the true measure of kinetic forces prove that it was no easy matter to measure the effect, which consists in movement, in such a way as to enable its relations with the capacity for work of the cause to be harmoniously determined. After this had been successfully done, the next undertaking was to reduce to the same measure effects of quite another sort, such as the heating of a body, to find the equivalents of different kinds of effects.

The proposition that the magnitude of the effect is the measure of the action of the cause seems to find another expression in the proposition upon which J. R. Mayer bases his treatment of the question: *Causa*

æquat effectum. By *causa* here is certainly not meant the thing endowed with force ; since the effect consists in a change, movement, heat, etc., it cannot be equated to a thing ; it is a measure not of the thing, but of the action of the thing. But Mayer's proposition has still another meaning which we cannot reach until after further consideration, and this meaning rests upon the assumption that in the sphere within which the proposition holds good we have to deal only with external causes and with invariable subjects and objects of action.

The action of a thing is, as such, not an object of perception nor of direct measurement ; we know it only by inference from perceptible change. But, if we confine ourself to the mechanical aspect, a body possesses capacity for work only in so far as it has a definite velocity, and as the capacity depends upon the amount of this velocity, it can be measured by the velocity ; and inasmuch as the body loses its velocity in imparting it to another body of equal mass, we are able to lay down the further proposition that the capacity for work is exhausted in the production of the effect—*vis agendo consumitur.* In the same way the distance of a body from some centre of attraction by virtue of which it attains a certain velocity admits of direct measurement ; here again it is the measure of the capacity for work, and here again it is exhausted by the actual fall, which in its turn can produce a further effect of equal magnitude, e.g., the lifting of an equal weight to the same height. On the other hand, it follows from the assumption of merely external causes that the capacity for work of a body cannot be exhausted except by the production of an effect ; but the effect itself represents a further capacity for work, so when a change is communicated from one body to another the amount of energy remains intact. When a state which we know to be capable of work, such as the movement of a body, vanishes, it must have produced an equivalent effect ; it cannot at one time, say in contact with an elastic body, elicit an equivalent motion by the loss of its own, and at another time, in contact with an unelastic body, have no effect at all. Its energy must be preserved in some form or another, and it is so preserved by being changed into heat, which in its turn may again be transformed into motion. It is of the capacity for work which is conditioned by given states of substances—motion, spatial disconnection, heat, chemical dissociation—that the proposition holds good "*ex nihilo nihil fit, nihil fit ad nihilum.*"

From what Mayer says in his first treatise (1842), we cannot doubt that he thought his results were obtained from truths of axiomatic validity,

and that in the mechanical equivalent of heat he merely proved empirically what followed of necessity from his principles. The starting point is the indestructibility of energy ; but then it has to be shown empirically what states or changes are to be regarded as energy, and what are the quantitative relations which obtain between them.

We shall have to discuss later on (§§ 97*b* and 100) to what extent the validity of the principle of the indestructibility of energy can be maintained, and what consequences it has for the methods of discovery of causal laws. Here we have only to point out that the principle of conservation of energy, which gives us a general principle by which to determine the causal relation between the successive states of different substances, does not contradict, but presupposes the concept of force as the permanent relation of substances. Only on this assumption can spatial disconnection or chemical separation be regarded as potential energy. Gravity is not exhausted by taking effect ; it is only the capacity of a heavy body to attain a constantly increasing velocity which diminishes with approximation to the earth, and vanishes when it falls upon it ; what vanishes is not gravity, but the space for falling, the condition under which gravity takes effect.

24. Our investigation of the idea of efficient action has shown us its close connection with the idea of the thing, and the similarity of the motives by which the two ideas are to be explained. In the one our aim is to combine in a unity elements which are presented as together in space, which appear in one and the same place, and which persist continuously in time or change without a break, thus thinking of the different elements connected in this way as proceeding from one ground. In the other we have to combine in like manner the spatially and temporally connected changes of different things, and to explain them by one ground of unity. The universality with which, as a matter of fact, this takes place, with which ideas of things are formed, and their simultaneous changes explained as the action of one upon the other, indicate a necessity grounded in thought itself ; and this necessity is ultimately based upon the fact that only in this form can we grasp the thought that what we perceive *is*, by finding in the given itself a ground which corresponds to the comprehension by self-consciousness of the manifold which is continuously extended in space and time.

25. So far the principle that "everything which is, is a thing with attributes and activities," is primarily only the expression of the internal

necessity by which we form the idea of things, and so far it is *a priori*; and the particular form which we give to this idea in dealing with the external world finds utterance in the principle that "two different things cannot occupy the same position in space." But how in particular cases the reference of perceptions to things is to be carried out cannot be deduced from this general statement, and we are therefore called upon to consider with respect to the particular nature of what is given whether and in what manner it necessitates one definite kind of reference. This is what we saw to be the methodological question as to the rules according to which the concept of the thing must be formed, if it is to be formed in a perfectly logical and unambiguous manner. We found that we should obtain perfectly definite concepts, such as would best meet the needs of thought, if we could refer everything to absolutely simple and invariable substances; but whether we can succeed in doing this is another question, and the necessity of such predicates cannot be inferred from the concept of substance. Indeed, our needs might be satisfied also, though in a less comprehensible manner, by things or unities possessing (like the monads of Leibnitz) the ground of their changes within themselves as an inner law of development; and only the nature of what is given can determine which of the two assumptions best serves our purpose, or whether perhaps some of the phenomena admit of the former explanation, while others admit of the latter only. We shall, however, be methodologically justified in endeavouring to make use of the most perfect form of explanation everywhere.

26. Still less is it possible to derive from the motives which lead us to the idea of efficient action any one sure guide for our methods, which will admit of only one possibility. Here is the place to discuss the meaning and truth of the so-called causal principle.

The sphere within which this principle is to hold good is certainly the existent, and it presupposes, therefore, that something should be thought as existing. It is in this way that its sphere is distinguished from that of the logical ground, which refers only to thought, no matter whether this thought is of what exists or not. Moreover, as we see it in the first place and immediately, thought is something original and ultimate; of an existent we can only speak in so far as we think it, and even if we regard our own thought as existent, inasmuch as it is an activity in time of an existing Ego, yet we do so only by means of a thought of which this existing thought is the object. Thus we have to do here with a

principle which does not apply to thought as such, but to that which is thought of as existing.

We need not discuss the question how the thought that something is arises;¹ we may confine ourselves to the fact that it is always present and also that what is perceived in space and time is always said to be. It is to this that the causal principle, however understood, refers.

As the principle is most generally understood, and as it is stated by Leibnitz, it tells us that for everything which is, there is a ground which determines why it is, rather than is not, and why it exists in one way rather than in another;² and by "ground" (*ratio sufficiens*) Leibnitz means nothing else than that which enables us to see that the existent is necessarily, and such as it is.

Leibnitz assumes here that the existent consists of substances and their changes. What he looks for is something which will enable us to understand that substances are such as they are, and that events occur.

But this tells us as yet nothing as to the direction in which the ground is to be sought, nor as to the nature of the necessary connection.

It appears from what we have said that the effort to understand the perceived as necessary, to find a ground from which the given follows of necessity, is always at work, and manifests itself even in the formation of the idea of a thing. It is true that we did not begin with the universal form of the principle, but with the procedure which starts from particular objects; but the motive of this procedure is none other than to find satisfaction by understanding the necessity of the given.

When we become conscious of this impulse, which acts at first instinctively and in particular cases, and when the satisfaction which follows it is taken as a universal aim, then we can derive from it the generality of the principle that we ought to understand everything as necessary by finding its ground; and as we cannot rationally seek a ground unless one is there, the assumption follows that everything has its ground, from which it proceeds of necessity.

What do we include in "everything"? Our analysis of the concept of

¹ Cf. I, § 12, 7, p. 72 sq. Schopenhauer's attempt to show that the assumption of external objects first arises from the principle of causality (an attempt of which we have already spoken from another point of view, I, § 48, 4, p. 320 sq.) moves in a circle in so far as it starts from the assumption that our sensations are subjective; this already involves the antithesis between an existing object and the subject, and all that can be done is to determine how such an object is constituted, not to infer its existence.

² Cf. I, § 32, p. 189.

necessity in the preceding volume¹ has shown us that the demand to understand "everything" as necessary involves an insoluble problem. Every ground which enables us to understand something existing as necessary breaks up into an existent which is assumed, and the relation of connection with its consequence which makes this consequence necessary; somewhere or other we must come to an end with a simply existent. Thus the "everything" needs restriction, and this we shall find by pursuing the way upon which we have entered. Permanent and variable perceptions are referred to things. These things themselves stand in spatial and temporal order; thus a ground of unity may be sought for their plurality, and for their spatial and temporal co-existence, and in this way we arrive at the one substance of which all particular things are parts or accidents. This one substance is absolute; in it is grounded everything having particular existence, things as well as their changes; it stands to the whole perceptible universe in the same relation as for our ordinary apprehension the particular thing to its parts, attributes, and activities.

The alternative is to maintain these substances in all their plurality and particularity; then their co-existence demands still more imperatively a ground which is sought in the World Creator of Leibnitz, or some kindred idea. But this ground simply is.

If we are to regard both views as issuing from the same causal principle, it is evident that the term "*causa*" merely presupposes being grounded in quite a general manner; it does not determine the manner of being grounded. Something may be grounded in the same way in which the connected existence of a thing is grounded by the unity of the thing, but it may also be grounded by a relation to an effective cause, which produces something external to itself.

The same distinction holds good even when we narrow the sphere of application, and accept substances as the simply existing, asking only as to the ground of that which they are. Here, again, we may find a ground in two ways; we may refer everything *either* to the nature of the substances, which develop a series of states, either permanent or varying, *or* to their relations, by virtue of which states or changes which appear together are necessary.

The proposition that every change has a cause cannot, therefore, mean

¹ Cf. I, § 33, p. 196 sq.

that every change of a thing must have an external cause, must depend upon the action of another thing. Nothing but actual experiment can show whether the one or the other kind of reference leads to a consistent view. Indeed, our analysis of the concept of efficient action has shown us that a ground which is merely external to a thing leads to contradictions. Thus in the course of these considerations the meaning of the words cause and effect and their synonyms has been extended still further. A cause means primarily a thing, in so far as it acts upon another, and what we regard as effect is a change which is produced in a thing from without by the action of another. Thus our remarks in this paragraph apply first (1-23) to the concept of the *causa transiens*.

But in the conceptual determination of this thought we found on the one hand that the ground of the effect cannot be entirely outside the object of the action, on the other hand that we were led to think of a thing as producing change by itself, in virtue of its nature; and thus the terms were extended to apply to a thing as itself the ground of its own determinations, and this in a two-fold sense. In the first place, the thing is the cause of its own activities in so far as they follow from its nature alone; thus Spinoza's substance is the cause of all its modes, but it is the *causa immanens*, and even *causa libera*, in so far as it cannot be determined to action by anything external; in the second place, however, a given state or action of the substance appears as the ground of following states or activities, which proceed from it of necessity, and of which it is therefore called the cause. To take Spinoza's example, the understanding is the immanent cause of the thoughts which it creates; but in another sense a thought may also be called the cause of another thought which necessarily follows from it. If in this latter sense we call one given state of a substance developing its activities from itself, the cause of the following state, we can still find the necessity of the connection only in the nature of the substance manifesting itself in this regular succession of its activities upon one another.

27. Neither the proposition that everything has its external cause (in the sense of *causa transiens*), nor the proposition that everything (in the sense of *causa immanens*) must be explained as the necessary development of its own nature, can claim to have the validity of a universal principle which is necessary for thought. Only the general stipulation that the given must be understood as necessary can make good its claim to be regarded as absolutely universal from the nature of thought, and this is

so only in the form of a postulate, of which the meaning finally proves to be that the plurality of perceptions must be combined according to principles of unity. Any more definite ideas of the rules according to which things and their changes must be referred to "causes" can only be verified by the material of our perceptions themselves. The conception of efficient action arises naturally as one form of this reference, on the ground of the already presupposed conception of things; logical elaboration may make it precise and clear, but how it is to be applied so that we may be certain of the necessity of what we are doing is a question which cannot be answered beforehand and from general principles. Its place is not in the investigation of conceptual elements, but in the investigation of the universal judgments through which we endeavour to know the perceived as it is actually constituted.

28. It is never possible—though the attempt has often been made since Wolff's time—to reduce the relations of ground and consequence, whether in the sphere of logic or of reality, to that of identity, if we are to understand this term in all the strictness of its meaning. In this strict meaning upon which is based the formula A is A , it signifies that what is, or is thought, is absolutely the same; but a ground aims at combining that which is different and distinguishable, whether in the form of the *thing*, which is the unifying ground partly of states which are only temporally distinguishable, having the same content, partly of attributes which exist together in time but differ in content, and partly of variable states which differ both in content and in time; or in the form of *causality* by virtue of which one determination follows as necessary from another. The proposition of J. R. Mayer, "*causa aequat effectum*," expresses no identity; he does not regard movement and heat as identical, but only as comparable; and this only with respect to their capacity for work. When in the sphere of logic the ground is obvious as necessarily producing the consequence, or when affirmation annuls the opposite negation, we have no repetition of the same, but necessity combining different acts. When the conclusion follows from the premises, the ground is the relation between the general and special concepts. And this again is a relation, not of identity, but of distinguishable objects of thought; this is most obvious when, as with number, the production of the general concept is necessarily accompanied by the whole series of its differentiations. The general concept of number is not identical with that of the particular numbers; each of these is formed in a particular manner, but according to one law.

To refer all necessity to the relation of identity, is to maintain the Eleatic doctrine, which denies plurality and becoming.¹

§ 74.

The analysis of PSYCHOLOGICAL CONCEPTS into simple elements presupposes the conscious reference of internal events and states to the one ego, and all it can aim at doing is to discriminate those given acts and affections which cannot for our consciousness be broken up into further distinguishable elements.

We must carefully distinguish between the problems of this conceptual analysis and those involved in the explanation of inner events from simple elements which are only assumed. To state the conditions under which certain psychical phenomena develop cannot help us in determining their content; it is only an expedient by which in our analysis of concepts we give reality to the idea of inner states, and are enabled to remember them.

In so far as in self-consciousness we refer ourselves in different ways to something which is different from ourselves, there is a ground for the distinction between the form and content (or object) of our inner activities.

An exact determination of those psychological elementary concepts, which appear as simple forms of our action, is rendered difficult by the complication of this action with the functions by means of which it comes into consciousness, and by the varying degree of energy of these functions. We shall find most help towards a complete analysis in considering the results of our psychical activities, since what is distinguishable in these indicates distinguishable activities.

The application of mathematics to psychology, and the utilization of mathematical methods within its sphere, is excluded by the peculiar nature of the subject.

In so far as particular psychical acts are regarded as arising out of each other according to law, and as being grounded upon each other, the general causal conception is applied to psychical life also; but the causal concepts which develop in this connection must be distinguished from those by which we connect material events.

1. The conceptual elements so far considered are those to which we are

¹ See Appendix E.

led by the analysis of ideas referring to the external world. Far greater difficulties meet us when we attempt to reduce the processes given in self-consciousness as our own inner action to simple and fixed concepts, of which we could be sure that they were the same for all. It has frequently been pointed out (and sometimes in an exaggerated manner) that the sphere within which inner events can be immediately perceived is inconveniently limited to the individual, and that even within this sphere there are many hindrances to the exact observation and accurate comparison of our mental states; so that there is no object in repeating these complaints. What we are here concerned with is the question what we have to look for in the course of our analysis of psychological concepts, and by what means we may hope to find it.

2. The first task in establishing psychological concepts must be to determine what we find in immediate self-consciousness in a way which may be unambiguous and the same for all, and in this way to obtain predicates with which to express the states and activities which we refer to ourselves as subjects. An analysis directed towards this end presupposes that we have in immediate self-consciousness ideas of ourselves, our states and activities, which show a manifold content; that in every moment of our waking life there is present to us not merely a surrounding external world, but clearly distinguished from this the self as the subject of states and activities of which we are immediately aware. Language, when we come to examine it, offers a surprising wealth of names for particular inner states and processes, as well as for the peculiarities by which individuals are distinguished in their inner life, for particular transitory events, as well as for more permanent dispositions of temper and mood, or for the varying degrees of intensity of our total energy—a wealth which seems at first to contradict the complaints alluded to. If such varied inner experiences have already found distinguishing names in popular language, and if the descriptions of inner events which are depicted by the help of these names are understood, and can give us intuitable pictures of an alien life without our being able to have an immediate intuition of it as of our own, then neither the limited horizon of the particular individual nor the difficulty of apprehending inner states can be such an important and insuperable obstacle to the development of manifoldly different ideas which shall yet be the same for each person. And yet, on the other hand, the state of scientific psychology, the wide divergence of different theories in the questions of classification of

psychical phenomena, and the absence of fixed concepts known by generally accepted names, prove sufficiently that the problem of establishing a system of simple and unambiguous conceptual elements, together with well-determined forms of synthesis, has not yet been solved, and that logic can therefore hardly be called upon to describe the methods by which psychological concepts are formed.

3. But logic both may and should undertake to warn us against a confusion of the problems towards the solution of which we need methods in psychology. The first and most fundamental task of psychology has often been neglected and confused with more remote problems, because psychological investigation has been more directed towards explaining phenomena than towards analysing them and reducing the object of self-apprehension to fixed concepts. It often happens that the aim which the psychologist has in view is to exhibit a process of development, in which composite phenomena are gradually produced from purely elementary acts of the simplest kind, as a plant is produced from cells or a tissue from threads ; and inasmuch as this development cannot be directly observed, the objects given to consciousness being already the results of a long process of construction depending upon innumerable conditions, a hypothetical method is introduced, and the hypotheses employed are generally determined by metaphysical propositions. In this respect the Herbartian psychology falls behind the views which it combats. Though it has done great service in accepting as the ultimate and proper aim of psychology the investigation of the causal connection of what actually takes place, instead of the mere classification of psychical phenomena, which was what Psychology as the doctrine of the faculties of the mind first aimed at ; and though it has established many valuable observations of particular facts, yet the method by which it undertook to construct that which we find from an assumed simplicity which is nowhere to be found, was entirely arbitrary. The forced interpretations put upon the processes of feeling and volition, and a construction of the fact of self-consciousness which, though acute, yet avoids the kernel of the question, were necessary consequences of a method which, swayed by metaphysical presuppositions as to the essence of the soul and the nature of its self-conservations and their relations to each other, proceeded by mere construction, without having first ensured the possibility of comparison between the results of the construction and the actual facts by establishing exact concepts for the latter through an analysis of the immediately given.

But the work of reflecting upon the genesis of mental processes, and of finding the laws according to which they follow upon external conditions or upon each other and combine together, is essentially other than that of analysing the contents of what we apprehend as consciously experienced at every moment and of applying clear concepts to distinguishable elements. The prism teaches us that white is a composite colour, and that the sensation of white arises when a combination of spectral colours in certain relations strikes upon the eye; but we cannot construct the sensation of white from the ideas of the spectral colours, nor would any analysis of the idea of "white" help us to discover in it the conditions of its genesis. White, as a fact of sensation, is something absolutely simple, co-ordinate to other colours, and the nature of the sensation is not made more comprehensible by a description of its genesis; all we can do is to establish the fact that under certain conditions it always arises. It is the same with the ideas of our inner states; the number of presuppositions upon which the actual coming to pass of an inner event depends does not involve the same number of conceptual elements in the idea of this event; the unpleasant feeling awakened by the dissonance of two notes or two colours is not a composite note or a composite colour, and the inner experience to which it gives rise cannot be resolved into colours or notes as its elements.

4. Nevertheless there is some ground for the tendency to substitute for an analysis of the ideas of mental events the enumeration of the conditions under which they arise, and hence (for example) to speak of feelings and acts of volition as relations of the ideas from which they result, instead of states and activities of the Ego. The peculiarly difficult circumstances under which we have to work in a logical determination of our psychological concepts makes it necessary to employ a statement of the conditions of particular events, as a help—not towards analysing the content of the idea, but to enable us to rouse the idea itself and keep it before us, and to ensure agreement as to the use of terms. What I immediately experience at any moment is but little, and even memory can only give me a successive intuition of previous experience, the familiar and frequently occurring being more easily recalled than events which are rare and which deviate from the daily routine. Before these can be recalled at all, and compared with others, there is no other way, where the term denoting them has not yet been sufficiently defined, than to remember the conditions under which the event is wont to occur, thus exciting the

reproduction of what has been experienced, unless, indeed, as sometimes happens we can succeed in directly producing the actual event, as in experiment. We generally choose the latter way where it is possible to elicit definite psychological effects by voluntarily bringing about the external conditions. The process by which we compare and conceptually determine sensations depends upon our knowledge of invariable external conditions which will regularly give rise to the subjective events of sensation ; and in order to maintain our concepts and to subsume new perceptions under them, we are always obliged to actually produce the sensations before we can compare them with confidence, as when we bring together a given note and the note of a tuning-fork, or a given colour and a colour of the spectrum. In the same way we produce at will the pleasant and unpleasant feelings of consonance and dissonance, or give rise by the stereoscope to the illusion of the third dimension. These expedients correspond to demonstrative experiment, which is merely intended to bring before intuition events which do not present themselves at every moment in the course of nature. Allied to them are the experimental methods, which help us to analyse given facts of consciousness into their simple elements, and make us realize our tendency to accept as simple, facts which conceal a plurality of distinguishable elements ; such, to recall the most familiar instances, are the methods of analysing a chord into its constituent notes, or the proof that certain supposed sensations of taste are compounded of tastes and smells—by a process of isolation we separate the elements.

5. But throughout a large part of our mental life it is impossible, from the nature of the case, to evoke our inner states at will, and so compare what they contain. The feelings of liking and dislike, of love and hate, or the excitement of resolution to important actions are not the simple results, always recurring in the same way, of a few circumstances which can easily be brought about. If they have ever been experienced at all they can be recalled intuitively only by recollection of the whole state of the individual ; the idea can be re-animated only by a return to the conditions of the individual. In this way, no doubt, the investigation of causal connections in our psychological life serves to clear up the meanings of current expressions, and a general statement of the conditions under which certain events occur enables us to survey more easily the sphere which is occupied by different modifications of a general concept ; but such a statement of conditions is not conceptual analysis, and cannot be sub-

stituted for it. When I say "the circle is the line described by the movement of one end of a straight line when it is moved in a plane around the other end," then the intuition of a circle arises within ideal space, and I see its growth under the stated condition. But when I say, "fear is the feeling which accompanies the expectation of an evil likely to occur," the feeling of fear does not proceed from the idea of this theoretical expectation as something already contained in it; my proposition, in the guise of a definition combining various characteristics, lays down a general rule of what happens, and states that the expectation of future evil always has for its consequence that definite, and perhaps unanalysable feeling which I call fear. From a logical point of view such a proposition resembles definitions like "white is the colour which is formed from the mixture of the spectral colours," or "sleep is the state which follows upon taking a given dose of chloral." Only if we assume that mental states are causally connected in all individuals in the same way, and that every one is conscious of a similar association of their memories, can such formulæ serve to include all similar states; but these states themselves are not brought into thought, nor is it in any way determined whether we have in them simple conceptual elements or not. The definitions of Spinoza, again, in the third book of the *Ethics*, are not statements of facts themselves, but of their conditions. For instance, the definition of *amor* as *lætitia concomitante idea causæ externæ* merely states the generic concept and the relation which gives rise to the particular nature of *amor*; the specific determination of love itself is not given in the definition, but merely developed from it afterwards as its consequence. Such statements are like the formulæ of chemistry, which express water as H_2O ; we are told merely what water consists of, not what it is; it is assumed that we already know this.

6. What we must substitute for this causal explanation in our analysis of psychological concepts is, first of all, the discovery of those ideal elements of our inner life which can no longer be regarded as a plurality of distinguishable parts, as composed from heterogeneous factors; and the investigation of the manner in which these combine themselves into composite phenomena for consciousness.

From one point of view we have attempted such an analysis. In trying to find the conceptual elements which resulted from the analysis of our idea of the external world we had to bear in mind that this idea of an external world, with all its component parts, is also a fact of our consciousness, and expresses one of our modes of thought; we succeeded in breaking

up the objects of thought into simple conceptual elements only in the sense that we found in the activity of thought itself distinguishable acts which, though never taking place apart, are yet different and not identical for reflective consciousness. What we were first called upon to do was to observe what *we do*, when we have any given idea. Here the content of the object of thought, which we could keep before us, served to guide us in our distinctions; in so far as it contained a plurality of distinguishable elements we were constrained to reduce them to a plurality of mental acts, which combine together to produce the idea of a composite whole. In this way we discovered the activities which distinguish and re-combine, the functions which are active in the ideas of space and time; and thus we gained also psychologically elementary concepts, which took the form of activities, always connected in the same way with the most varied content of that which is discriminated in thought or represented as existing in space and time.

7. But the canon which led us here to refer differences in the objects of thought to differences in thought itself cannot be similarly applied throughout the whole sphere. When, for example, we have to deal with simple sensations, it does not seem as if to every difference of content there corresponded an equivalent difference of activity. Colours, notes and smells are incalculably manifold; but in ordinary language, as in scientific, we speak of seeing, hearing and smelling as if these modes of activity remained the same however varied the colours, notes and smells. Although we never have seeing in general, but always the seeing of something, never hearing in general, but the hearing of some particular sound, we find nevertheless something common to all seeing or hearing. We do not think that the hearing of a deep note is another hearing than that of a high note, the seeing of a bright colour other than that of a dark grey; we see something other in either case, but we do not see otherwise, *ἕτερον*, not *ἐτέρος*. We are accustomed to express this distinction by saying that the form of our activity is the same, that only its content or object differs; and the same distinction recurs throughout the whole sphere of psychology in the concepts of thought, will, etc.

If, however, we try to think of this common and identical element by itself we are met by peculiar difficulties, for we cannot imagine a seeing without also thinking of a visible which is seen, nor of a hearing without some sound which is heard. If we say that seeing and hearing are simple relational concepts, and denote the manner in which the visible and audible

come into consciousness, the conscious reference of the ego to the seen and heard, then all distinction between seeing and hearing as the subjective form of our relation to the object would disappear, and only the general relation of becoming conscious would remain ; the peculiar characteristics of particular spheres of sensation would be dependent only on peculiarities in the objects, seeing would be the relation of our consciousness to colours, hearing to sounds. But, on the other hand, particular colours would yield no common element other than that of being seen, particular sounds none but that of being heard ; and thus we seem to move in a circle. Nor can it avail us to point out that the thought of seeing is accompanied by the idea of the eye, of hearing by that of the ear, and that certain sensations of touch or movement are referred to the eye or the ear ; it does not seem difficult to set aside these subsidiary ideas, and the distinction between seeing and hearing clearly does not consist merely in the fact that the one takes place through the eye, the other through the ear, else must the sensation of pressure on the hand be specifically different from that on the foot.

We cannot, then, be content either with the view which accepts merely an identical relation to various objects, for then seeing and hearing differ merely by their object ; nor with the view which relinquishes the common element in all seeing and assumes as many kinds of seeing as there are colours. There remains only to find the identical element in all seeing in a similar excitement of the subject, which announces itself immediately as such, and which is immediately perceived as different from the excitement which is common to all hearing. And it is just this purely subjective element which is indicated in language, where seeing and hearing are treated as active and transitive verbs, and thus acknowledged as definite modes of *action* ; what is expressed is not merely a coming into consciousness, but a taking up into consciousness. In this way language itself contradicts the theory that we can be conscious of the sound only, not of hearing it. The physico-psychological theory, which describes sensations merely as the effect of an action exerted by external objects through our sense-organs upon consciousness, is directly contradicted by the view embodied in language which always finds in sensation an act, issuing from the subject, and directed upon the object. The simple fact that to see and to hear are used in the imperative mood indicates our consciousness of a volitional element, and it is some time since a careful psychological analysis pointed out that attention is one condition of fully con-

scious seeing and hearing. In this way the external reference to the object must be supplemented by certain forms of our purely subjective mode of being before we can complete the whole of that which we regard as our own share in the process.

8. This example should serve to show that the distinction, so familiar in psychology, between the form and the object of our activity does not straightway solve all the difficulties which beset our attempt to find identical and perfectly determinate conceptual elements; that it can only be justified by showing that, notwithstanding difference of objects,*there really exists for immediate consciousness a homogeneous activity. This difficulty did not occur with respect to the functions of distinction, counting, etc., because these referred to a content already given, and had for object only the recurring relations of this content.

9. In the sphere of presentations, again, differences of presented content facilitated our investigation of the distinguishable acts of the subjective activity at work upon it; but this help is also withdrawn when we come to aspects of the inner life which cannot be directly conceived as ideal relations to an object distinct from ourselves, but appear at first as merely varying states of our own being, varying activities of our life, which are indeed conditioned and accompanied by that manifold of presentation, but can never be resolved into it. So far as concerns these phenomena of our inner life, we can only attain our purpose by observation of what we *do*, and by analysing what we *find* at any given moment. The terms used in ordinary language must serve to help us as clues, and a comprehensive consideration of the results of our psychical activities, so far as in the forms of external actions and products they belong to the external world, must guide our attention to the source from which they spring; this is especially the case in spheres of social activity. A deficiency was recognised in the ordinary treatment of psychology when the psychology of nations (*Völkerpsychologie*) was initiated as distinct from the psychology of the individual. What was meant by it was that man is never found in isolation, and that the psychical events which we actually find in him are on the one hand variously conditioned by community with others, through which they first arise; while on the other hand they themselves constitute and maintain this community and give to it its particular character; and so far it was a decided gain to reclaim for psychological treatment the spheres of language, customs and law. But the antithesis which appears is not happily expressed by the antithesis

between the mind of the individual and the mind of the people or the common spirit, and the division between the psychology of nations and that of the individual is untenable. All psychology is of the individual, for it can speak only of what takes place and is present in consciousness, and this consciousness can only be that of an individual of himself. But in the stirrings of the individual life we must undoubtedly attend most carefully to those events, and investigate especially those moods and efforts which determine the relation of man to man, for upon them depends the historical life of mankind. Here again, what we have to guard against is the assumption of purely hypothetical causes for community of action amongst men, which are not to be found in our immediate consciousness; fundamental psychological concepts can only be concepts of activities or events which we actually discover by reflection upon ourselves.

10. The first result of such reflection will be a number of qualitatively distinguishable states and events, which we are unable to analyse further, and which therefore cannot be further described, but must be experienced in their simplicity and denoted by the current term. The distinction already noted between form and content recurs here, accompanied by the same difficulties; we desire the most varied objects, but the inner movement which we call desire seems to be homogeneous; our will is directed towards the most manifold ends, but the relation in which we place ourselves towards the future state, which we take as an end, appears to be always the same.

But it is not principally here that we find the greatest difficulties in our analysis of psychological concepts. They consist rather first in the complication of the functions through which our own activities come into consciousness with the activities themselves; and, secondly, in that relation between the different functions interconnected in one psychical state which prevents us from accepting them as a mere sum and from being satisfied with an analysis of the state into its particular components.

11. With reference to the first point, the question by which all psychological investigation is burdened is: whether the degree of consciousness is the degree of the conscious process itself. On the one hand, it appears a matter of course that psychical events exist as such only in so far as they are conscious, and that their distinctive character lies in just this being conscious. We can form no idea of a seeing of which we know absolutely nothing, a pain of which we are in no wise aware, an act of volition

which takes place without our being able to notice it in any way, because the seeing which we know is given to us only by means of consciousness, and this consciousness cannot be separated from the thought of seeing. It is impossible to get determinate concepts of psychical activities or states unless we presuppose that we are conscious of them and thus have ideas of them, that we can consciously distinguish them from each other and recognise particular instances as of the same kind.

On the other hand, it is just as indisputable that what we call consciousness of an inner event does not always take place in the same way, that there are different degrees and modes of becoming conscious, and that therefore actual events may also come into consciousness in different ways. When we speak of consciousness of our actions, we think first of cases in which we say : I see a red light in the sky, I hear thunder, I feel a touch on my arm, I feel a sharp pain in my chest, I know who stands before me, I distinguish different colours in this stream, I remember this name, I am pondering over this problem, I am trying to lift this weight, I want to be quiet, I will carry the message, etc. In such instances what takes place is a subsumption of the particular event under general ideas, by which the form of judgment is determined ; and, indeed, if the given fact is to be fully expressed, not merely is the particular form of happening subsumed under a psychological concept, but the object to which it refers is also stated, for there is no seeing or willing in general, but only seeing or willing of this or that. It is cases such as these which Herbart has in view when he finds the essence of self-consciousness in self-observation ; this explains how it is that he can regard the subsumption under already present ideas as the essential point, and can represent the process as the taking up of the new idea into an ideal mass already present. But this is not the primary and original process. Before the subsumption can take place the element to be subsumed must be given, and it must be given in an immediate apprehension upon which the further process of subsumption supervenes. Just as it is not necessary to the conscious seeing of a colour, or hearing of a noise, that for every colour and every noise there should be a concept already fixed in form and provided with a name, so it is not a necessary condition of a consciousness in general that general ideas of psychical events must be forthcoming. Such general ideas, indeed, can only be formed gradually from particular unnamed perceptions. I use the names of colours to say *what* I see, not to say merely *that* I see something ; and I use the concept of seeing, pain, etc., to say *what* goes on in

my consciousness, but not to get an immediate knowledge of some particular distinguishable mental content.

But even with reference to what comes into consciousness in this way without reflection it may be said that it is at least *indispensable* that there should be a discrimination of the particular elements which co-exist at every moment, accompanied by some notice of them, before we can speak of consciousness. At every waking moment there are given at least sensations of touch and sight, and connected with these slight feelings of comfort and discomfort, as well as memory-images, and it would seem that we can hardly speak of a consciousness of all this until these different components have been actually noticed and distinguished, at any rate so far as to enable us to perceive them as different. This is a necessary condition to their being afterwards subsumed under concepts.

But closer consideration forbids us to limit even so far as this the concept of consciousness in its widest signification. What is to be distinguished must already be given in some way; we can create nothing by making distinctions, we can only draw boundaries within what is already given. Before we distinguish in the field of vision different objects, or even colours, the whole field must already be somehow in consciousness, the visible must make a total impression, which we break up by a process of distinction into its component parts. And we cannot say that what we do not thus specially notice is therefore in no sense conscious. We do not generally, for instance, notice the slight pressure of our clothes, or even of the support upon which we lean, in the sense that we are conscious of them as distinct elements of our given state; but should the pressure suddenly cease we should certainly be aware of it, and notice a change. But the cessation of what is absolutely unconscious could make no impression, and could not be noticed; the cessation of waves of sound which had not entered consciousness at all could not evoke a change in consciousness. In cases where such a change takes place they have been heard, but not especially noticed; they have merely formed a part of the unanalysed background on which stand out the elements which are distinguished.¹

It is, then, indubitable that in the manner in which we become aware of psychical events there are perceptible differences which justify us in

¹ On the questions discussed above cf. Stumpf, *Tonpsychologie*, I., § 1, and the treatment of them by H. Cornelius in his articles on *Verschmelzung und Analyse*, *Vierteljahrsschr. f. wiss. Phil.*, xvi. 4, xvii. 1.

distinguishing in every mental process between the function through which we become conscious of something and that of which we are conscious, although we can never actually separate them from each other; and although the actually perceived intensity of a sensation or a feeling is not altogether independent of the amount of attention which we direct towards it. The view that in self-consciousness we have immediate apprehension of an actual event generally represents this actuality as independent of the varying vivacity and determinateness of self-consciousness; and inasmuch as the clearness and energy of inner perception can diminish from a maximum to zero, it is supplemented by the further conception of unconscious mental activities, unconscious syntheses, unconscious inferences, etc. It is true, however, that the distinction is not always carefully made between that which does not come into consciousness at all, in any sense, and that which merely does not come into consciousness in such a way that we distinguish it within the whole complex presented at any moment, pay special attention to it, or even subsume it under any given concept.

As a matter of fact, in order to bring some connection into mental life and to explain the constitution of our ideas, we are forced to assume functions of which only the result comes into clear consciousness, while they themselves take place unnoticed, certainly without being distinguished and observed. Already in the course of our investigation we have had occasion to speak of spatial, temporal, and other syntheses which had taken place unconsciously, because we found them presupposed as a condition of the complex content present in consciousness; and we were undoubtedly justified in this in so far as we were able in particular instances, by fixing our attention upon them, to bring such functions into clear consciousness, or so far as we could show that they were at first consciously, and even intentionally, performed, and that it was only gradually and by force of habit that attention was dispensed with, the functions themselves not disappearing altogether since they continued to manifest themselves in their results.

This is not the place to investigate how far we are justified in making use of this assumption. Our object is to obtain definite psychological concepts and conceptual elements, and for this we must keep to those modes of consciousness in which we are able to perceive clearly definite forms of psychical process, to distinguish them from one another, and to maintain the distinctions between them; we must limit the proper signification of psychological terms to that which we can grasp in this way. Not

until this has been done can we proceed to hypothetical assumptions as to similar processes taking place, although we are not conscious of them in the same way and with the same clearness.

12. Thus the methods at our disposal cannot do more than help us to get a firm grasp of definite, well-characterized events which easily impress themselves upon consciousness. We cannot reduce to conceptual determination all the variation in intensity with which they enter consciousness, nor yet the imperceptible transitions between them, because we have no measure either for intensities or for the qualitative differences of similar phenomena ; nor is it possible, as with sensations, to bring together at will similar processes or proximate degrees of the same action, and refer them to an objective scale of stimuli having approximately the same result at all times. At the most all we can do is to erect a scale of just perceptible differences for the strength of the volitional impulse through which we act upon our muscles in order to lift a weight or exert a pressure, and compare it with the magnitudes of the results attained, and even this we can only do within the limits laid down in § 70, p. 69 sq. For the intensity of our joy at the success of our effort, for the strength of the will which manifests itself in attention or makes itself known in a resolution, we have no analogy, and however certainly we know that differences are there, we are yet unable to say *how great* they are.

Thus throughout a large part of the inner life the psychologist must content himself with bringing into clear consciousness the differences between characteristic modes of activity in cases which are simple and easily accessible to memory, and with establishing them as *types* which recur with a greater or less degree of similarity, through such analysis discovering and correcting the confusion or ambiguity of verbal expressions. He proceeds in much the same way as language does in dealing with sounds, giving names to the most striking, most easily remembered, and most frequently recurring, and then ranging the others around these fixed points; or as the physiognomist, who maintains definite types of race or family characteristics without being able to reduce them to fixed concepts. The ground for giving one instance precedence over another is, in the last instance, an æsthetic impression. What shame is, or anger, or pity, we all know from instances in which we have been conscious of unmingled and unmistakable emotions, but we cannot reduce all the shades of the feeling of shame to order, nor draw up a table of the degrees of warmth of our compassion. The specialization of such concepts can be indicated by

remembering various occasions, but it cannot be carried out, and their limits can no more be determined than can the limits between growling and roaring, between whistling and hissing. But although our consonants are indefinable noises, and our vowels unanalysed sounds, which are uttered by every individual in a slightly different way, yet we can speak so as to be understood ; and we can imagine a development of the elements of psychological concepts which would maintain and reproduce essential differences so far as needed for scientific purposes. We must, however, turn our attention from the unattainable ideal of a mathematical psychology to that which is attainable, and must relinquish the fancy that physiology may be forced to yield the key to a region which, on the contrary, alone enables us to interpret the physiological processes of the nervous system and to guess their significance. Physiology has given a most desirable impulse to psychology, but in passing beyond the limits of psychophysics has raised more questions than it has answered. The tendency to assume that for every psychological event there must be a definite change of some part of the brain, and to explain it by hypothetical events in the ganglionic cells, has often stood in the way of a clear, careful, and purely psychological analysis by withdrawing attention from objects which could be directly investigated, and turning it towards uncertain constructions within the region of the unknown.

13. Any application of mathematics, or of spatial construction in any but a purely figurative sense, is made futile also by the second of the above-mentioned points (p. 130), *i.e.*, by the relation in which elements stand to each other which are given contemporaneously to consciousness as distinguishable parts of our life. No moment of life in which we can discover a plurality of distinguishable acts and forms of activity can ever be regarded as a mere sum of elements, or represented figuratively as a spatial whole consisting of different parts ; it is much more true that the manner in which the particular co-exists in consciousness is something in itself which cannot be reduced to a compound of its constituent parts. To take the simplest instance : to see a picture is, of course, to see the particular figures in it, and the whole thing seen contains the particular parts of the picture ; but then, to see the whole in one comprehensive act of vision is something more than to see the parts, and includes the seeing of the parts in a way which cannot be expressed in any external figure. It is in the same way that in a melody we hear the particular notes and mark the interval from one to another ; but the same particular notes and their

intervals might be heard by a number of men, each of whom heard only two successive notes, and any one hearing the melody would hear more than all the others together, although they would amongst them have all the particular parts. Thus every combination of distinguishable activities produces a special effect for consciousness, comparable to the sense of harmony or discord in music, which is something more than the mere sum of the notes that are sounded ; and it is just the highest and most important developments of our psychical life which depend upon such functions of the second degree, functions which have their ground in the particular form in which elements are combined without being absorbed as in a mixture.

14. One more point remains which prevents an application of mathematics in the psychological sphere which is often attempted—it is the relation of psychical functions to time. It is true of the infinite divisibility of space that it must also be applied to that which *occupies* space, and of motion that for every portion of time, however small, a portion of space must be traversed ; but the thought ceases to be applicable when we turn to events of the inner life. Our functions succeed each other in time, and inasmuch as they occupy time continuously they must have some duration ; but there is no sense in investigating the temporal magnitudes of particular acts which we must regard as simple, or in conceiving them as in some way analogous to motion and coming to pass successively, or in applying the divisibility of time to the thoughts which arise in time. When we count, of course we need time to get from 1 to 100, but yet the thought of each particular number is indivisible in time. It is just this which forms the problem, how it is we are able in time, by means of functions which occupy time, to annihilate distinctions of time itself, and to comprehend a temporal series of moments in the indivisible Now of thought.

15. The difficulty of psychological analysis obscures all concepts which can only be thought of as grounded upon psychological events, especially those which express a relation to our feeling and will. We can only tell what “good” means, and whether the word is ambiguous or not, by referring back to the elementary functions which take effect in our will, and to the feelings in which it is reflected ; what we are to understand by an “end,” and whether the relation expressed by it between our will and the object thought of is simple or composite, can only be answered by psychological analysis ; and the extension of such concepts depends upon the correct delimitation of the elementary psychological

concepts. But here again it is true that one chief help to this analysis lies in surveying and carefully distinguishing that which emerges most clearly from the purely inner life into the objective world which is common to all. For we are accustomed in knowledge and action to attend much more to anything which confronts us as a tangible and permanent object than to the manifold of inner events, and we often show first in our estimate of things, and in the way we treat them, the inward impulse towards them.

16. So far as the thought of causal connection can be applied at all to conscious inner events, it is so in a two-fold sense. Inasmuch as sensations appear as the effects of stimuli, we get the concept of transient causation (§ 73); the psychical subject is affected by the change of another, and responds to this affection by the conscious process of sensation. But even within the course of conscious events we affirm a connection in virtue of which one psychical event of a particular kind depends upon another psychical event of the same or a different kind; certain ideas excite other ideas according to the so-called laws of association, or ideas excite feelings of pleasure or pain, or feelings give rise to efforts. It is obvious that this relation of real dependence between our functions cannot be subsumed under the same causal concept which has for its content the action of one thing upon another, the dependence of change in a thing upon conditions contained in its relation to other things; we prefer to say that one activity of the subject is the ground from which another activity of the same subject proceeds without our being obliged to have recourse to an external cause for its explanation. The relation is the same as when, by virtue of the law of inertia, we regard the movement of a body from *A* to *B* as the ground for its further movement from *B* to *C*; we do not generally call the movement in any preceding moment the *cause* of the movement in the subsequent moment, but merely look or ask for a cause when there is any change of velocity or direction. But in the inner life we have to do, not with the simple distinctionless continuation of a given event, but with changing and newly introduced functions and states, and with the question as to how these are grounded; there is therefore no objection to affirming a causal relation here also when we know that we are applying the expression in a sense essentially different from that of the transient causality familiar to natural science, which depends upon a relation between different subjects (cf. § 73, 26).

At first sight it seems to be a matter of course that within the sphere of the inner life *events* only should be regarded as causes in this sense, and that we cannot therefore speak of efficient action in the sense developed above, in so far as we do not derive inner events from external stimuli, but consider them as determined by preceding facts of consciousness alone. The task of psychology would then be to establish the laws according to which a given conscious state or event—e.g. hearing the words of a description—is followed by another event, the idea of the object described, or the sight of a wounded man is followed by the feeling of compassion, and this again by the wish to help him.

There is no doubt that if psychology is ever to pass beyond mere descriptions, it can only be by regarding a subsequent event as determined by preceding events; the scientific endeavour to know what is given as necessary must in the first instance work in this direction. But this does not mean that psychology can only know events, and cannot get beyond regularity of connection amongst these events; that because the object of inner experience is only a changing succession of events, therefore the assumption of a single subject for these events is unfruitful or impossible. Nor is it proved that psychology could manage to make even this concept of causality sufficient for its purposes, or that it could resolve into mere activity—whether transcendental apperception or volition—all that is meant by the word “I.”

In immediate self-consciousness all particular events are referred to one subject; we cannot express consciousness except in the form “I am thinking this or that, I feel pain, I will do so-and-so.” It is perfectly true that we have no definite intuitive idea of what we call “I,” such as we have of external objects, or even of particular, temporally-limited events in consciousness; we cannot even make our own ego completely object, for there must always remain that subjective activity by which the object is thought. But is this any reason why we should abstract altogether from a single permanent subject of our psychical activities? Can we really think of our feeling as mere event without anything which feels, of our will as mere event without a subject which wills? Is it possible to conceive of our whole psychical life as we ordinarily do, say, of a storm, as a series of occurrences the definite subject of which remains concealed, and which we describe by saying it storms, it lightens, it thunders, it rains, it hails? Is it necessary, in order to speak with scientific accuracy, to speak impersonally—it thinks, it feels, it wills? Even the psychologist,

who demands a psychology of mere events, always speaks of *our* will, *our* ideas, *our* psychical states ; in the mere description of the fact he employs a symbol for a subject of which ideas, feeling, or will are the activities or states, and I doubt whether any one could seriously think of a pain which hurts no one, a will which is no one's will, or, further, of an inner perception of these events in which there is no perceiver, but only the object of the perception, the subjectless will or feeling itself. No one can seriously think that there is a stream of impersonal thought connecting the particular events of seeing, hearing, feeling, and their memory-images, and incessantly, at every moment, creating the illusory thought of a subject to which these events are referred ; and that connected with this there is the further impersonal event, by which is thought the identity of this subject throughout the successive moments, but which in truth only produces a phantom for thought which has no subject. I doubt whether any one can consistently think that what he calls "I" is merely the incomprehensibly connected sum total of these events (to which must also be added the event which thinks of this sum total as such). We may, indeed, with Hume, be influenced by certain presuppositions to affirm that we find within us nothing else, but we can only do so by ignoring the "find within us" and taking account of the occurrences "found" alone ; there is always latent the ordinary way of thinking which inevitably assumes that the perceived objects of consciousness belong to the one perceiving subject.

If it can be shown that the idea of persistent and durable things is only possible on condition that we know ourselves to be identical in time ; if the thought of the identity of the same object necessarily presupposes consciousness of the identity of the thinking subject, then we cannot eliminate the latter and yet maintain the identity of objects. If it is true that the primary certainty of being does not refer to objects, but is expressed in the proposition "I am," that we can be sure of any being only in so far as we are first sure of our own being, then all knowledge is annihilated if we are to say that there is nothing more than an impersonal process of thought positing an imaginary subject. It is true that if we yield to the undeniable utterances of self-consciousness and accept a subject for all conscious activities, there arise difficulties for our further consideration as to the relation in which this subject stands to the totality of objects which we distinguish from ourselves ; but great as these difficulties may be, we cannot get rid of them by introducing the impossible thought of subjectless activities. The difficulty that we can never think ourselves merely as objects (cf. Lieb-

mann's *Aphorisms*) is not decisive; inasmuch as we must assume other subjects, like ourselves, we are constrained to think of self-conscious subjects, which for us are mere objects.

17. But even apart from the question, how we are to interpret the facts of self-consciousness, we are prevented from conceiving of psychical events as related merely to one another as cause and effect by the nature of the events themselves; for when thus limited, we cannot apply the relation of a sufficient ground to its consequences, and we are unable to explain subsequent stages of consciousness as necessarily following from preceding stages. If it is true that the idea of the thing is not merely a sum of the particular sensations, if—as Wundt (*System*, p. 314) has excellently pointed out—a new act of consciousness is called for which contains a kind of creative synthesis, if the pleasure arising from a harmony is not merely the sum of the elementary feelings accompanying the particular sounds, if, therefore, what we call effect contains more than the cause assumed, if something new supervenes, which is not yet contained in what is given, whence shall we say that this more, this new element comes? Under these circumstances the subsequent cannot really be explained by the precedent, by itself and regarded as mere event; it can be explained only by assuming a subject in the nature of which it lies to produce upon the occasion of certain activities other activities of its own, the ground of the new element is really to be found in the capacity of development of the subject to which both the preceding and subsequent activities belong as functions. In the physical sphere the constancy of forces enables us to carry out to a certain degree a causal connection between mere events; the principle of the conservation of energy, the equivalence of motion which precedes and which follows, causes physical events to appear as a uniformly flowing stream in which the power of work present at a later point of time is the same as was present at an earlier point, only in a different form; nothing new occurs which was not already contained in the conditions. But in the psychical sphere this new element appears wherever there is a development in the true sense of the word; and if the principle of equivalence between cause and effect does not hold good, then the preceding state cannot be regarded as the one and sufficient ground; what we now call cause in a wider and inaccurate sense merely means a condition under which the psychical subject produces new activities. For this reason the much-despised because misapplied conception of “faculty” nevertheless really expresses the relation before us; the

term denotes, when rightly understood, that nature of the psychical subject by virtue of which it produces from within itself, and upon certain occasions, activities which are not merely continuations of those which preceded, and by virtue of which it develops itself in time, and so realizes what is contained in its disposition. A psychology, such as the purely empirical and sensational, which regards the whole content of consciousness as only a sum of sensations which are excited from without, and which finds nothing more in all later stages than the summation of these simple effects of sense-stimulation, can represent the successive states of consciousness as cause and effect; it is done also by the Herbartian psychology, which disengages ideas from their ground, and, maintaining the invariability of the soul, transfers all activity to the ideas which come and go, combine and resist, as independent things. According to this view, the real cause of psychical events lies quite without the subject, which is merely the passive theatre for influences from the external world. But if this view fails to satisfy us, then there is no other way than to recognise that successive sensations are not by themselves, and in the full sense of the word, the cause of all the further "events" which arise out of them, that the presence of two or more sensations does not necessarily involve their being distinguished or counted, that a feeling by itself contains nothing more than a state of pleasure or pain, and that it would never develop into anything further were it not the nature of the feeling subject to be impelled thereby to the further activity of desire and will.

18. Whether or not the subject of psychical events, which is indispensable if we are to speak comprehensibly, is to be placed under the concept of substance, depends upon how we define substance; the dispute is mainly one of words. If we start from the concept of substance which has been developed in natural science, if we take as its essential characteristics invariability of forces and modes of action, so that under the same conditions substance always undergoes and produces the same changes, then, indeed, the soul is not substance. But if we merely mean to express by this term that we are compelled by the nature of our thought to add to the varying events always combined in one consciousness a subject which explains the connection of these events, and which, remaining one with itself, forms a common ground for the continuously successive changes, then the subject of self-consciousness must be called a substance. Not indeed a substance which has an existence apart from its activities; it *is*, in that it is active, but it is not the mere momentary activity; its existence is not

exhausted in the particular activity any more than the existence of material substance is exhausted in its momentary state, but outlasts them as the one ground of succeeding activities. The materialist may try to identify the subject of psychical events with the collective unity of the brain, endowing it, in addition to its chemical and physical powers, with the faculty of producing ideas and feelings; but if we decline to do this, we cannot think of mental processes as a mere happening without any subject. We may attempt to substitute for the individual soul a universal ground, but then, this must always be the *res cogitans*, and particular conscious states proceed from the *infinita potentia cogitandi* of the infinite substance. Such metaphysical supplementations of the immediately given do not at any rate attempt to think of a happening without something in which it happens, a knowledge which no one knows, a will which is no one's will.

19. Here, again, the general idea of cause may be variously applied. In one direction the causal concept has already been used in so wide a sense as to be applied—as by Spinoza—to the relation of substance to its activity; the understanding is called the cause of its thoughts as plants are called the cause of their own growth, and the falling body the cause of its fall because the falling is its activity. But we shall do better to avoid including in the concept simple action as such. In mechanical science the concept is limited to the relation between different unities; it is the *causa transiens*. We apply the same concept in psychology, when we refer inner events to a cause distinct from the subject of consciousness, when we refer sensations to stimuli proceeding from the external world. The incomparability of the two events and their subjects is not in itself an obstacle to the application of the causal concept; the proposition that like can act only upon like would, if strictly interpreted, render every application of the causal concept impossible; and we are bound by the actual connection of phenomena which constrains us to find a ground for what takes place in a relation of cause and effect.

When we come to the relation between particular psychical functions, the causal concept finds another application; we are here dealing with the states or changes of a subject which by virtue of its nature has other states for a consequence of these, and in this sense an idea is called the cause of a feeling, a feeling the cause of a desire. In so far as what we mean to express is the regular connection obtaining between states of the same subject we use the word in the same sense as when the lowering of the temperature of water below zero is called the cause of its freezing; by

virtue of the nature of water the one change is necessarily accompanied by the other.

But in the psychical domain the causal concept gains a still more specific meaning in the relation of the will to other functions; not only because volition appears in voluntary movement as the direct cause of a material event—states of feeling also appear as causes of the most varied material events,—but because even within the sphere of events of consciousness themselves we are conscious of directing our volition towards the realization of certain acts, and find within us the power of producing or at least guiding them. It is so when we fix our attention upon perception or thoughts, when we voluntarily excite a series of recollections, or follow out a logical chain of thought. It is not a necessity independent of consciousness and known only by its results, which connects that mental act which is cause with that which is effect, as when feelings are excited by some object of thought. When we realize what we have thought of, we are immediately conscious of determining ourselves by our own activities; and this immanent self-determination passes into the sphere of external causality through the dependence of our bodily movements upon conscious volition.¹

¹ Cf. here my treatise, *Der Begriff des Wollens und sein Verhältniss zum Begriff der Ursache*. *Kleine Schriften*, II. 115, sq.

CHAPTER II.

THE SYNTHESIS OF CONCEPTUAL ELEMENTS INTO COMPOSITE CONCEPTS.

§ 75.

AN exhaustive analysis of our ideas into SIMPLE ELEMENTS which are completely determined and the same for every one, and an investigation of the different forms of their SYNTHESIS, constitute the condition under which alone it is possible to determine our composite ideas in a way to satisfy the demands of logic.

In the construction of composite ideas itself, it is necessary on the one hand that we should carry out the syntheses of the conceptual elements to the extent called for by the aims of thought; on the other hand, that the construction of the particular composite groups should be accompanied by their arrangement in a definite system of relative subordination, co-ordination, and disjunction.

In so far as our ideas are of such a nature that definite principles of synthesis are given together with the elements, composite concepts are generally formed by a free construction according to those principles of all possible combinations. But in so far as the synthesis of certain conceptual elements is only required by their actual co-existence in objects of experience the formation of composite concepts is limited to reconstruction of the given, and these concepts are principally arranged by the method of abstraction.

In the construction of final concepts both methods are combined.

1. In the preceding section the analysis of our ideas into simple elements showed the impossibility of getting constituent parts of ideas in complete isolation. Everywhere we found inseparable functions determining the content of thought, and the particular elements were always connected with definite forms of synthesis. Unity itself cannot be thought without plurality, nor line and point without space; the concept of the thing has no meaning except as the synthesis of different elements, and

concept of causality would be nothing if it did not connect the states and changes of things. According to the different classes of ideas these syntheses differ entirely in import; the relation between the unities in a number is incomparable with the relation between points in space, and this again with the relation between qualities and between these and the thing to which they belong.

2. It follows from this that we cannot find any method of combining conceptual elements into composite concepts which shall be absolutely universal and apply to all concepts in the same way.

No doubt if we start with a purely formal denotation of conceptual elements and their synthesis, as when the content of a composite concept A is expressed by its characteristics $a\ b\ c\ d$, it is natural to apply to the sum total of simple elements obtained by analysis the method of combination of different elements, in order to find all possible combinations and so to obtain the whole series of possible concepts and their relations. Then even the principle according to which they would be arranged would be that by which we should be guided in obtaining the combinations; we should start from some series of characteristics, and we should take the same series as the basis for an enumeration of the combinations. Thus from the characteristics a, b, c, d , we should get the series of binary combinations ab, ac, ad, bc, bd, cd , of trinary combinations abc, abd, acd, bcd , and finally the combination of all the elements $abcd$; and we are at once struck by the remarkable phenomenon that there are more of the general concepts ab, ac , etc., than of the more specialized abc , etc., and that all these genera have only one specific concept under them, for the doctrine presupposes that the arrangement of characteristics in a concept is indifferent, and that $abcd$ is the same concept as $bcda$.

The principal difficulty in the simple application of this theory of combinations lies in the fact that characteristics are not all compatible, but that many of them are so related that the affirmation of one excludes a number of others; thus by a wearisome process a great number must always be excluded as impossible from the combinations found by the calculation. If, however, the process were otherwise correct, and ensured a certain result, we should have to submit to this inconvenience; aiming merely at complete determination of the relations of incompatibility. For this it would not suffice to discover the simple incompatibilities, for it sometimes happens that an element a would be compatible with b or c each by itself, but not with both together; equilinear is compatible with

rectangular by itself (in the square), and with triangular by itself, but not with both together.

But apart from these difficulties the method of combinations in this general form is not applicable to the relations of conceptual elements. As generally understood, it presupposes an equivalence of elements and a single meaning for the union between them, which is not true of the elements of our concepts; and important as it is when properly applied, it has proved unfruitful as a general logical method for the construction of concepts, whenever, that is, the attempt has been made to revive the art of Raymundus Lullus. In all attempts to reduce logic to an arithmetic with simple elements for units, it is overlooked that arithmetic is always based upon homogeneity of elements and of their syntheses, a presupposition which is all the less applicable here because the particular elements do not appear in isolation, but are dependent upon each other in the most complicated manner.

3. If we are to proceed methodically in the formation of our concepts, we must be guided on the one hand by the natural conditions of thought, on the other hand by the aims towards which we are striving in thought.

With reference to the former, our investigations in the previous section have brought out characteristic distinctions. In the sphere of numerical concepts we came across an activity proceeding according to conscious and comprehensible laws, by which the objects and their relations were created and which can be continued independently of all external relations, while in dealing with qualities of sensation we are constantly dependent upon external conditions, and can neither produce them in their manifoldness according to any simple law, nor yet submit to one comprehensible law the relations between qualities belonging to different senses. In the former case one general law suffices to give rise to the whole manifold; in the latter an indefinite manifold is given without fixed limits, and the general law which should govern it cannot be found.

So far as concerns the aims of our conceptual construction we have on the one hand to obtain fixed and definite predicates for our judgments, and to keep them ready for judgment as extensively and as well arranged as possible; while on the other hand we must apprehend what is given to perception as exhaustively as may be, and represent it in logically ordered concepts. On the one hand the tendency is to multiply and

specialize our concepts as far as possible, to construct a comprehensive system of combinations of conceptual elements ; on the other hand this tendency is limited by the fact that we form our composite concepts only so far as we are called upon to do so by observation of given things and their relations, and have no occasion to form those concepts for which there is no corresponding object in experience. Here the foreground is occupied by given things, of which the synthetical form is the concept of substance. The idea of every particular thing, however, is compounded from an incalculable multitude of elements, and the co-existence of these elements is the less to be derived from definite general rules because the concept of substance is completely indifferent towards its content.

Thus we have two opposed starting points. In the sphere of concrete, real things, we have given to us a manifold of the richest combinations, incalculable in number ; our task is to determine the whole content of the perceived, to exhibit similarities and differences in logical arrangement ; in other words, to classify what is given ; we cannot, however, discover from the characteristics themselves how they are combined, we can only learn it by their actual co-existence. On the other side, we find the ideal elements, with which is given according to their nature the law of their synthesis, and for which there is the possibility of development. Here our task is to carry out this development to its fullest extent, and this takes place most markedly in the sphere of mathematical concepts, which are developed in particular directions much further than is called for by the claims of immediate perception.

4. In calling this formation of concepts from their elements CONSTRUCTIVE, logic has borrowed a term from geometry. The production of some figure from particular spatial elements, lines and angles, which can take place either in pure imagination or by the external help of drawing, seemed the most fitting image by which to represent a process in which the composite idea is produced for us, while the voluntary variation of elements independently of external perception, and the combination of them together, gives the construction the character of a free activity guided only by the nature of the ideas themselves.

It is less appropriate to contrast with this process of construction that of ABSTRACTION, as the process by which concepts are formed from given perceptions. Abstraction in the sense of separating the ideas combined in the unity of a thing, so as to present the particular qualities by

themselves, is one of the expedients used in the analysis of ideas into simple elements; but abstraction in the other sense, as expressing the formation of general concepts from special ideas by the omission of different characteristics, cannot be the general process by which we are to reduce the given according to its determinations to conceptual expression, for to do this we should first need to note all the differences which distinguish particular objects from one another, and to establish an exhaustive formula. A process of abstraction, leading to the grouping of different specific concepts under one higher general concept, could not be made use of until we wished to reduce the ideas already gained to logical order, and to exhibit the higher concepts under which they could be subsumed. But since we have no other familiar term at command, the word may be used in its wider sense for the sake of brevity.

In a more detailed exposition it will have to be shown that neither process can take place alone; that in the sphere of construction, abstraction must be called in, while the classification of the given which is begun by abstraction cannot be completed without construction.

5. The FINAL CONCEPTS, which are the objects of our will and guide our voluntary action, occupy a peculiar position.

On the one hand they share in the reference to reality which prescribes definite rules of formation to the concepts of real things. That which is really willed, and not merely dreamed in idle wishes, is always a real state of ourselves and of other men or external things, which is thought, indeed, as future, but which, if realized, can be realized only in the concrete definiteness which belongs to all objects of the real world; that which is to be effected by our will is a change of real things which must be latent in their nature, and leads to a state which constitutes a part of the real world and is governed by its laws.

In so far, then, as our final concepts are frequently formed upon the ground of previous experience, and our will is simply directed towards the repetition of that which previously pleased and satisfied us, their content seems to differ in no way from that of the ideas by means of which we always think of the real, and to be formed in just the same way; it is only the subjective reference to our will which makes a final concept of the concept of any given existing thing. The water which we want to drink when thirsty, the fire which we desire to light when cold, does not differ because thought as a final concept from any other fire and water. We

find also the same relation between special and general concepts in our aims as in the classification of the given, and this in a twofold way. The will may be directed to a particular object, to a momentary satisfaction ; but in the final concept we affirm only a general idea under which fall the various specific objects, a general idea, however, which is abstracted from our experience of common attributes in a number of things. Thus the thirsty man wants drink in general, and the cold man combustibles of any kind. On the other hand the final concept may be general with reference to the will itself, when the will is directed not to some particular object or state as the means of momentary satisfaction, but to a plurality of objects or states which all yield a similar satisfaction. The will to gather supplies for the winter, or to injure a hated person, is directed towards a general idea ; but this general concept itself can only be abstracted from experience ; my knowledge of what will serve as supplies, or of what will injure a man, can have been learned from experience alone. And I could have formed the same concept without any reference to the will by a comparison of particular relations of the given. The fact that the relation which was at first only an object of experience is now an object of the will can make no difference in the way in which the concept is formed ; given states or events are taken as objects for our will just as experience has taught us to know them, and the more accurately they are known, the more our concepts are complete and correspond to the real state of affairs, the more secure we are in volition, since we shall have no unexpected results to fear.

6. Nevertheless a closer analysis shows even in these cases one aspect which distinguishes final concepts from the purely theoretical concepts of experience, even when they have exactly the same content. When the thirsty man desires water, or the cold man fire, the immediate object of his will is not the external thing as such, but the cessation of his discomfort ; thus the ground upon which he desires these objects is the definite relation in which they stand to him as the means of satisfaction, and it is only because a certain object is the only one known or attainable in the real world that his will is directed, not towards the general, but towards the definite and concrete. When various objects possess in the same degree the quality of satisfying the want which gives rise to the will, then the generality of the final concept appears ; and when we actually and consciously separate what is really willed from its accompanying determinations, then the final concept thus formed becomes distinct from

the general concepts gained by abstraction, for then the general precedes the special and serves as a clue by which to find the means of satisfaction in the real world.

This is at once obvious when the objects to which the will is directed are not forthcoming, but must be *invented* and made. The final concept now appears as the problem: how to produce a thing having a given quality or manifesting given relations. Only what corresponds to the end is determined by it, everything else is undetermined; and what we have to do is to change the things given, or to combine a number of them, in such a way that the result may have the desired qualities. A consideration of the processes by which the end is realized in a concrete thing, as well as of the movements of thought in its activity of invention, must come later on. At present we have merely to point out how in the sphere of ends a formation of concept takes place which is relatively independent of the things given, and how the results of this process have at first the character of general conceptual formulæ, which await their specialization and determination from a consideration of the means by which the characteristics contained in the concept can be realized in the real world. (Cf. § 42, 8. I., p. 273 sq.)

§ 76.

The constructive formation of concepts has for its first and fundamental form a process of combination which starts from the simple elements and forms from them in their appropriate synthetical forms (of number, space, etc.) a complete series of lowest groups, and then combines these again according to the same methods.

This process is accompanied in part by another which, starting from given forms of combination, develops them through all the differences of which their elements are capable, and is characterized as logical division; in part also by a statement of general conceptual formulæ, which express certain conditions to be fulfilled by a number of more special concepts, and which present themselves as problems and therefore as final concepts.

When the number of elements and differences is unlimited, these methods cannot be actually carried out; completeness of conceptual construction can only be indirectly expressed either by general symbols, representing an infinite series of differents, or by negative formulæ.

The process of abstraction is a mere expedient, by which we determine the concepts of certain combinations in cases where, owing to the infinity

of possibilities, no definite way is prescribed to free construction. It is supplemented by imaginative invention.

1. We find the simplest example of a purely constructive formation of concepts in numbers themselves, and in numerical combinations and combinations of numerical combinations; it is in them, therefore, that we may most easily trace the simple characteristics of the constructive process. First of all units are combined in a series,—two, three, etc.—according to the simple, recurrent synthesis of counting, and in this way we get the concepts of simple numbers. But even here we find that the process of combination cannot be fully carried out, owing to the absence of limit, and we substitute for the incomplete and infinite series the *letter*, used as a general numerical symbol.

This first and fundamental combination, in which the perfect equivalence of all the elements admits of no difference beyond that of number itself, is followed by further developments. The same form of synthesis being applied to the combinations already obtained, there arise the forms of combination contained in the modes of calculation known as addition, multiplication and involution. From the concept of the sum, again, follow sums arising from two or three or more terms; and inasmuch as each addendum may be of any value we get an infinite series of combinations of which, for practical purposes, only the twofold addition of the numbers from 1 to 10 is generally carried out one by one, since the decimal notation of numbers in language and writing reduces all addition to the addition of numbers between 1 and 10. In the same way the concept of multiplication leads first to the multiplication table showing all the combinations of the same numbers in products of two terms, but admits also of an infinite series of products resulting from two, three or more factors.

Thus the numbers combined by methods of calculation become themselves the subjects of new combinations, and there arise, according to the same principle of combination, sums of differences, differences of sums, products of sums and differences, sums and differences of products, etc.; and these general concepts are again differentiated by the fact that each one contains the possibility of any number of terms, and each term may be of any value. Just because the principle itself is so simple and so easy of application, there is no need to actually carry out all the combinations or to provide special names for the more complicated forms; the infinity of possibilities, moreover, would make it impossible to get further than the beginning of any actual statement of all combinations.

2. This principle of the complete combination of given elements into composite forms is accompanied by another which in one aspect develops from it, and this second principle starts from a given combination and extends it beyond its original limits by an exhaustive enumeration of all particular cases. The concept of multiplication arises first from the repeated addition of equal numbers; instead of $2 + 2 + 2$ we say 3×2 . But when we take the general expression of the product $a \times b$ and substitute for a and b all values in the numerical series—including as the result of counting backwards 1 and 0 (p. 31 sq.)—the concept of the product is extended beyond the limits imposed by its original meaning. Both $1 \times a$ and $0 \times a$ are brought into the concept as limiting cases which from one point of view may be formally included in the concept of multiplication, though in such a way that its original meaning is no longer applicable, and is indeed fundamentally altered by the new interpretation which must be given to the formulæ. If, notwithstanding this, formulæ such as $1 \times a$ and $0 \times a$ are not to be regarded as self-contradictory concepts, it is because they form part of the same series and proceed according to the same law as the true products, and according to the same law have as definite a value. In the same way we extend the concept of difference by thinking of the general formula $a - b$ as applying to all values of a and b , and the concept of involution by introducing negative and fractional exponents. The extension of the concept over the whole series of possible combinations leads us to include under the same name numerical combinations which no longer admit of the same original interpretation, though their meaning can be inferred from this.

It is obvious here that in these cases pure construction gives first only the narrower concept, and that we get the wider concept only by an extension of the synthesis already given.

3. The importance of the latter process appears still more clearly in geometry.

In the first place, what we may call the geometrical synthesis of simple elements is determined by no such simple law as in the formation of numbers. That which determines the synthesis is space, but in space we cannot think of a plurality of elements without introducing relations of situation and magnitude. The number two never varies; but two points can only be thought of in connection with some distance between them, and with three points appears the question of their mutual position. Thus in geometry all synthetical conceptual construction is concerned with the

genesis of the idea of spatial figures, variations of the general relations of position, magnitudes and distances of points and lines.

An inexhaustible field is here opened to construction ; but it is a field in which from the first we are without definite guidance, so that the construction must be purely inventive. Just because the elementary and conceptually determined ideas of the straight line, the angle, etc., can only be gained by a discriminating analysis within space, the total idea of space is indifferent with respect to all combinations. Whether we begin by forming systems of points, or investigate the position of two lines in space, or imagine a continually increasing number of radii issuing from a point, there is no internal necessity to determine this production of manifold geometrical images, in the same way as the concept of number itself produces the numerical series, sums and products.

4. But however the construction may begin, it depends, on the one hand, upon certain fundamental assumptions as to the simplest constructions and their possibility¹—such as are contained in the *αἰρήματα* of Euclid—and, on the other hand, certain rules are prescribed to it by the nature of space. From the combination of a few elements—lines or points—it proceeds further in space to the combination of more ; at first two straight lines, then three, then four, are drawn and brought into different combinations of position. Thus from the beginning number is needed as a guide, and here again we find that the construction of arithmetical and geometrical concepts cannot be regarded as co-ordinate, inasmuch as the geometrical synthesis presupposes the arithmetical. The continuity of space, again, permits of an infinite number of possibilities of position, e.g. of two straight lines with respect to each other ; for this reason conceptual construction can never proceed by an exhaustive combination of a fixed number of elements, but can merely draw limits within the continuum, leaving it to intuition to fill up the space between these limits. We cannot enumerate the particular distinguishable cases in which the position of two lines to each other differs ; but by turning and moving them in all directions we get certain definite cases which can be determined by simple characteristics, with reference to which all others can be determined by negative characteristics alone (§ 43, 5, I., p. 280 sq.). Two lines in space either coincide and have all their points in common, or they do not coincide ; if they do not coincide, then they either have one point in common and

¹ Cf. Zindler, *Beiträge zur Theorie der mathematischen Erkenntnis* (*Sitzungsberichte der phil.-hist. Classe der Wiener Academie*, vol. 118, 1889, ix. p. 32).

intersect, or they do not intersect ; if they do not intersect, then they are either parallel or not parallel ; it is in this form that we must develop the closer determination of the combination which begins by bringing two straight lines together as elements. The concepts are here formed by disjunctions, of which the one term is a definable case from amongst an infinite number of co-ordinate cases, while the other term is the mere negation of this case, merely borrowing a significance from the general intuition of space.

Or suppose we investigate three lines in a plane ; all three are parallel or two are parallel and one not, or all three intersect, and intersect either in one point, or in three points so as to form a triangle, and then either so as to form three equal angles, etc. Everywhere we find the same method of dividing the infinite manifold of relations which are possible between certain elementary parts, by means of a disjunction (a and non- a) into a particular case and an infinite number of others.

5. Geometrical construction has shown a preference in its development for bounded figures : triangles, squares, circles, etc. ; and this points to an empirical origin. What is first given to sense-perception consists in images of bounded bodies and surfaces, and imagination also begins by drawing particular figures ; in distinguishing these and endeavouring to determine them conceptually our thought is occupied with a quite definite intuition. But when this intuition is thought of by means of a few general predicates only, apart from the fixed dimensions of the particular case, or when figures agreeing in certain characteristics are classed together by a process of abstraction, we have begun the process of extending a given figure by variation of its elements within the limits imposed by the fixed general predicates. The concept of the triangle is certainly due rather to the sense-intuition of particular cases than to the attempt to see in how many ways three straight lines may lie in space ; but then it has passed beyond the particular given figures, beginning with a comparison of different triangles, and including all possible positions and dimensional relations of sides and angles. Here again the limiting case appears, when the three lines coincide in one line, or the three points in one point ; but within the infinite variety we find certain fixed forms which are determined by equality of sides or by the possession of a right angle.

This process of abstraction, as employed in construction, is distinguished from that which is ordinarily so called by the fact that the concepts obtained in this way pass at once beyond the fortuitous beginnings of abstraction and find their limits in their own characteristics.

6. Whatever is thus produced by combination of points or straight lines is determined by the nature of space in general, and this same nature of space prescribes limits to the concepts, inasmuch as the variation of the particular elements can be limited by their synthesis. From this point of view the process of construction itself makes us conscious of the limits imposed upon it in the form of axioms of space-intuition ; the proposition that two straight lines cannot include a space shows by its negative form that it has been obtained through the attempt to combine two straight lines in all possible positions, and from the perception that as soon as they cease to coincide they continue to diverge from the point of intersection. To this extent knowledge of these axioms contains an empirical element ; their truth first becomes manifest after an exhaustive survey of all possibilities, but they attain unconditional validity only from the consciousness of the invariability of space, by which is excluded the possibility that if we were to try again we should get another result.

7. In the syntheses treated of above we have already seen the importance of the movement of points and lines in space, as the only means of surveying exhaustively the possibilities included in a general concept ; and this importance becomes especially obvious when spatial images are such that our ideas of them must have their origin in movement. In dealing with straight lines we have our elements, as it were, ready to hand, but curves can only be geometrically constructed by movement according to a fixed law ; the synthesis leading to the concept already contains this element within itself. It is in this way that we get our ideas of the circle, of rotating bodies, etc. ; and as lines and circles, planes and curved surfaces, intersect in free combination, constantly exhibiting new relations, we get an indefinitely increasing variety of constructions. Everywhere, however, we are called upon in the same way to survey the whole area of possibilities affirmed by a conceptional formula, and thus to obtain limiting cases and by means of division to draw limits within the area itself. When we construct the cone by the rotation of a right-angled triangle about its cathetus, it is the nature of space which in the first place determines what will result ; but the extension of the concept is fixed by varying the triangle through all the terms which intervene between the two limiting cases in which either the axis or the radius of the base is O , in which the cone passes either into the circle or the straight line. In the same way the general concept of the conic section has its origin in voluntary construction when we imagine the cone as intersected by a plane ;

but when once the concept is grasped, it is extended to all possible cases, and develops into its subordinate species.

8. There is in geometry another kind of conceptual construction which is essentially different from the above ; it is construction by formulæ which precede intuition and contain problems as to how geometrical figures may be produced which shall correspond to certain conditions, and which are therefore analogous to final concepts. When, for example, I propound the concept of a line having all its points equidistant from one point, then an attribute of the line is given which by itself is insufficient for the carrying out of the intuition ; even if I were to begin to draw any number of points at an equal distance from another point, I should never succeed in constructing from them the continuous line. Moreover, as thus stated, the concept is left quite indeterminate, since any line drawn on the surface of a sphere would satisfy the conditions ; even if we added the further determination that the line must be in a plane, our formula would not at once give us the intuitable idea of the shape of the desired line. The mere relations of magnitude contained in the formula would indeed suffice to distinguish the already constructed circle from all other lines, but in itself it contains in the first place a *problem* to be solved by the geometrical construction of a continuous line, and we are bound to ask whether it can be solved in one way only. All the equations of analytical geometry fall under the same point of view ; they contain the problem of finding a line of which the points will satisfy all the dimensional relations of abscissæ and ordinates expressed in the equation, and which finds in the equation the determining law of its construction.

While the true geometrical synthesis creates spatial figures for intuition, and the conceptual formulæ in which it is expressed at once give rise to the intuitable idea and show how far it is determined and how far it is capable of further variation, the analytical formula necessitates an investigation as to the position in space of the points which satisfy the equation, and as to whether it can be satisfied in one way alone or in several. Assuming that the intuition can be realized, then the problems of determining the limits of the concept recur ; for example, the equation of the ellipse includes the possibility of varying the relation between the greater and the smaller axis from equality to the vanishing of the one or the infinity of the other, and thus of passing through all excentric figures from the circle to the straight line or parabola. Whether, indeed, we may infer from this that the straight line is included in the concept of the ellipse and may

be treated as an ellipse of which the one axis = a and the other = o , is doubtful, since according to the original meaning of the equation the values of the two axes represent actual lines. This meaning is contradicted when one of the axes disappears altogether; it is only the continuity of the transition which enables us to regard the straight line as an ellipse, in the same way as we regard $o \times a$ as a product.

9. The latter method of geometrical construction passes without fixed limits into the sphere of those problems which instruct us to produce particular figures which will fulfil certain quite definite conditions as to position and magnitude. Every geometrical figure represents a general concept in so far as it may be repeated in the same way at every point in space, every position being merely relative, and hence particular figures which are fully determined may be regarded as merely the most specialized concepts, although they can never be developed by division because of the infinity of differences in magnitude. The logical import of the problem to construct an equilateral triangle with a given side, or a circle with a given diameter, is just the same as that of a conceptual formula which merely states the conditions to be fulfilled by a species of figure; in both cases we are dealing with a kind of final concept, to which we have to find the corresponding real intuition. The difference lies in the fact that such a problem of construction involves conditions which do not admit of an unlimited variety of objects, but can be fulfilled only by a single spatial figure or by a limited number; the question which arises here is: how far we must proceed in the determination to exclude all indefiniteness, and how far we can proceed if we are to avoid affirming incompatible determinations. A right-angled triangle with a hypotenuse = a is an indeterminate problem, *i.e.*, the formula represents a concept which includes an unlimited variety of right-angled triangles; a right-angled triangle having the hypotenuse a and the perpendicular $b > a$ is a contradictory concept, and a right-angled triangle with the sides a, b, c , is a contradictory concept unless $a^2 = b^2 + c^2$.¹

¹ Zindler (*loc. cit.*, p. 33 sq., p. 53 sq.) speaks of existential propositions in mathematics, and maintains that what is ordinarily held to be derived from a definition really follows from an existential proposition which is in thought but not expressed (cf. J. S. Mill's *Logic*, bk. I, viii. 5). But I should like to avoid applying the concept of existence to purely mathematical objects. The question is not at all whether these have any existence independently of our thought, but only whether a conceptual formula which is stated in general characteristics can be realized in intuition, or whether (as in arithmetic) it can be actually carried out by our thought. $\sqrt{4}$ "exists" only in the sense that I actually carry out the operation required by the root-symbol, and can state a number which corresponds to the condition; $\sqrt{-1}$ does not exist, the conditions cannot be satisfied in the sense in

From this it is clear that conceptual construction, which does not start from the intuitable synthesis, must be guided by general propositions deciding the possibility of the combination of characteristics, and that in so far it is secondary (§ 84); just as in the formation of real final concepts it is knowledge of the real causal connections which enables us to distinguish possible from impossible syntheses. The original syntheses, on the other hand, are governed only by the rules which we have called axioms of conceptual construction (§ 48, 3. I., p. 317 sq.), which must come into thought together with the consciousness of the ideas because they merely express the relations obtaining between our elementary ideas as such, and upon which it depends how far in the specialization of a concept variation in one element determines variations in other elements. Thus simple intuition teaches us that variation in the relative magnitudes of the sides involves variation in the magnitudes of the angles in the triangle, but not in the parallelogram; and we obtain the distinction between characteristics which are mutually dependent and those which are independent.

10. It is only when we are developing an already established general concept by differences of independent characteristics that there is room for a safe and really useful application of a kind of calculus of combinations, without fear of coming upon the impossible as well as the possible (cf. § 43, 10, I., p. 285, and below, § 85).

11. We cannot here pause to show how the same methods find application also in the form of the synthesis which is contained in the concept of motion; for the particular concepts relating to motion depend partly upon geometrical concepts of orbits, partly upon relations of magnitude between space and time.

§ 77.

When we come to the classificatory formation of concepts no simple and direct rules can be established for the synthesis of the elements which unite to form the concepts of real things and their real relations; we can, indeed, proceed here only hypothetically and provisionally.

which they are stated. Zindler himself allows (p. 33) that to "exist" in mathematics generally means to "be possible." He is, however, quite right in saying that a mathematical definition is only legitimate when it involves no contradiction or incompatibility amongst its characteristics, and again, that there are axiomatic—*i.e.* immediately evident—propositions concerning the possibility of certain objects expressed in conceptual formulæ; but these are essentially of the same kind as those which Euclid calls *αἰτήματα*.

The beginning of classificatory formation which accompanies language starts from the assumption of fixed forms and sharp distinctions between them, but it can have for its foundation neither extensive completeness of the things to be classified nor exhaustive knowledge of the particular.

The methodical completion of what is thus begun must also do without this foundation, insistence upon which would render any classification impossible ; but it must aim at being founded at least upon the most exhaustive knowledge possible of known particulars. The conceptual formulæ are complicated by the necessity of including in them causal relations and laws of development, and we are thus forced to substitute diagnostic definitions for complete statements of the content of the concepts.

But these are themselves possible only when grounded upon a comprehensive induction, which again presupposes that a classification has already taken place. From this it necessarily follows that every classificatory formation of concepts has a provisional character. On the other hand, we are driven to hypothetical construction by the difficulties which result from the gradual transitions between things, and which render uncertain our formation of the higher specific concepts.

The formation of concepts of things is connected with the formation of the complex concepts of attributes, activities and relations ; and this again, in so far as the concepts are intended to express only the actual, is carried out only so far as the given affords occasion, and lacks a fixed principle for the necessity of its syntheses.

1. We turn now to the other side of formation of concepts, that which starts from perceived things and aims at establishing general concepts under which they can be subsumed with their relations, and at forming these concepts in such a way as to represent the essence of things themselves and to express one ground of unity for the co-existence of distinguishable characteristics. We have already indicated above (p. 144 sq.) why it is impossible here, starting from the analysis of conceptual elements, to proceed by constructive combination. The concept of substance is an empty form determining nothing by itself, in which we combine the most manifold given content ; in itself it tells us nothing as to the characteristics to be combined in it ; it gives us neither a law by which to distinguish compatible from incompatible, nor a rule according to which one characteristic would depend upon another. Only the most general and formal determinations as to the spatial and temporal relations of

presented qualities and changes are contained in it, while no general principle prescribes what definite sense-content we shall combine in it, and how ; and even when it is supplemented by the concept of causality this again is much too indefinite, with its thought of necessary connection, to say beforehand what forces we must ascribe to a thing, and how we are to determine its mode of working. Attempts, such as have been made by Spinoza or Herbart, to derive from the mere concept of substance or of the existent its definite characteristics or attributes, lead by latent assumptions alone to a result which is not contained in the general concept of substance.

Here, therefore, we are thrown back entirely upon experience, which shows us which of the attributes given to sensation stand in such spatial and temporal relations that we have occasion to apply the thoughts of substance and causality as a form for their combination, and which has already been elaborated in this way by unpremeditated psychological processes.

2. It is upon these considerations that the doctrine is based according to which all concepts arise from comparison of the given, and abstraction of that which is common to much of the given ; but to proceed methodically with this abstraction we must be conscious of the aims to be pursued by conceptual construction. Language, to which this doctrine looks first, is no sure guide ; it does not aim at the highest ends of knowledge, but serves primarily the immediate necessity for mutual understanding, and its terminology is not grounded upon an exhaustive analysis of given objects, but has in view a few easily noted characteristics, often neglecting finer distinctions. In its general terms, again, it combines whatever seems to agree in some specially important attribute, but does not consciously aim at forming those species which will make possible the greatest number of general propositions.

Scientific method, on the contrary, starts from particular things in their most concrete determination, and no aspect which they present to perception may be overlooked or neglected ; it must begin with the whole extent of knowledge which is attainable with reference to particular objects, and which will include purely individual traits, or at least absolutely definite degrees of more general qualities. This accurate knowledge alone can be the ground for any conclusion as to which resemblances should guide us in forming general concepts, and what extent of individual differences in the particular may be overlooked to enable us to

form a general concept comprehending objects of the greatest possible similarity. For what judgment demands is certainly not mere reconstruction of the purely particular, such as would be given by the exhaustive description of the particular object, but general concepts which will be applicable to a number of objects; such concepts alone yield predicates by which the particular can be determined and distinguished, and enable us by the abbreviation of general propositions to survey the endless plurality of actual perceptions.

3. But assuming this beginning to be made, we find ourselves threatened with failure at the first step of our procedure because of the unlimited nature of our material. There is no limit to what can be said of the particular thing when we take into account all its relations, and we cannot exclude them, since all which we know of it rests finally upon relations; and there is no limit to the extent of the particular within which our comparison must seek for similars and determine the value of differences. The method which demands that the whole material of particular things shall be surveyed before beginning the formation of concepts is logically correct if all that exists is to find a place in our conceptual system; but it can never pass beyond the preliminary steps to the beginning of its real work.

As a matter of fact the formation of concepts from the perception of actual things has never proceeded by this direct way; it has always begun within a limited area from incomplete knowledge of the particular. Its mode of proceeding has been—guided here again by language—on the one hand, to group into lowest species objects the most similar for its knowledge; on the other hand to divide into higher kinds objects which seemed separated by specially important and widely spread differences. To form the concepts of man, horse and sheep was as natural as to separate the animate and inanimate halves of the living into the wide classes of animals and plants. In this way scientific reflection began not only to appropriate the formation started by language, but also to adopt the assumptions latent in this process—assumptions which are in part due to and confirmed by those objects which are attainable, and in part grounded upon the needs of our thought when it is directed towards the knowledge of things.

4. These assumptions have reference, first, to the nature of the generality of the concepts thus obtained. The Socratic definition of the meanings connected with words undertakes to fix the content of the thought accom-

panying the word man, horse, gold, in the form of a concept, which, as identical with itself and strictly determined, would form one stone in the edifice of our knowledge; and in so doing it assumes that objects themselves manifest a corresponding constancy, remaining the same in the future and recurring in instances which never vary—in other words, that there are fixed forms in nature. The value of a concept does not consist in the information that such and such characteristics have been common to a certain number of things, but in the fact that it presents a type to which the particular will always and everywhere conform, that it shows the die with which nature stamps her objects. In the Platonic doctrine of ideas and the Aristotelian doctrine of forms we have a simple and clear expression of this view, which supposes an inner necessity combining definite characteristics and prohibiting arbitrary and lawless change, or the constant flux of things.

A second assumption is that there are clearly marked distinctions between these forms which enable us to fix their limits with certainty. Within the region in which conceptual construction begins we find unmistakable divisions between gold and silver, between oak and beech, between horse and ass; differences in the particular objects form gaps sharply separating similar objects of one kind from those of another species. The same necessity which binds together certain characteristics within narrow limits of variation holds others apart; the distinctness of forms is only the negative aspect of their fixity, the importance of the *διαφορὰ εἶδοποιός* is necessarily given with the doctrine of fixed forms.

5. Starting from such assumptions, the way seems open to a complete classification of the given. But we can only learn by actually carrying out our attempt whether these assumptions hold good in the sense in which we affirm them, whether with their help we can complete a classification which will be simply a continuation of the nomenclature begun by language and will exhibit fixed and distinct forms, and whether the superficial and, as it were, mechanical rules which bid us compare the objects to which language gives the same name, or which present themselves in some other spontaneous way for comprehension into one concept, will always suffice to yield a firm basis for classification. Although we must certainly begin with such processes in order to get a preliminary standpoint within the multiplicity of phenomena, yet we find grave difficulties when we attempt to establish concepts within the whole extent of the given, which, as *infimæ species*, shall express it in its full determination,

and so that none but insignificant differences peculiar to the individual, and easily separated from those which are important and specific, shall be overlooked.

6. We are first confronted by the variability of objects themselves, which forces us beyond the statement of a fixed complex of perceptible characteristics, and obliges us to include causal relations or laws of development in our enumeration of the attributes by which one class of things is distinguished from all others. The two-sidedness of the concept of substance, by virtue of which it refers to one and the same ground, not only that which co-exists at a given point of time, but also the changing in time, complicates the conceptual formulæ ; for if they are not merely to serve to fix an idea obtained under certain circumstances, but are to express the nature of the thing as manifested in the course of time in different phenomena, it must take into account the variability of things themselves. Quicksilver seems to admit of a simple statement of characteristics by means of which its attributes are expressed in a combination which belongs to no other object ; but it is only at an ordinary temperature that it is such an easily recognisable object, it evaporates in heat and becomes solid in cold, it combines with other metals to form amalgams, and with sulphur to form cinnabar, and not until we have included these transformations in our concept can we claim to have stated what quicksilver is. Moreover only experience can teach us whether and how these changes depend according to laws upon certain external circumstances. Finally, even the most immediate predicates resolve themselves into causal relations, so that the concept we were looking for turns out to be a system of laws expressing the way in which an *X*, which cannot be immediately communicated in any simple expression, is related to us and to other things.

It is the same with organic beings. When we construct their concepts it is generally the permanent forms of their maturity which we have in mind ; but if these concepts are to express the definite nature of the particular organic forms, then all the stages of development from the earliest germ must also find a place in the expression, and here again the concept seems to give way, and to be obliged to include laws of development, and even the dependence of development upon external conditions.

But in proportion as the concept strives towards completeness it becomes evident how impossible is the task of preserving the conceptual formula as a comprehensible unity. The nature of a metal, a plant, or an animal

proves inexhaustible when we try to include all its relations in the conceptual expression; and this is even more the case when the innumerable causal relations, in which a thing alters its modes of appearance, cannot be deduced as necessary consequences from other attributes, but form elements which for our knowledge are independent of them and of each other. From the attribute of fluidity in quicksilver we may, of course, derive a number of attributes which depend upon the general laws to which all fluids are subject, but these laws themselves were only empirically discovered; in other aspects, *e.g.* in chemical relations, there are hardly any but special laws between which there is no known connection. Thus the task of establishing exhaustive concepts for the lowest species seems impossible, and even were it possible such concepts would lose in usefulness by their richness of determination; they would be too cumbersome to be manipulated or to serve for the subsumption of particular things as they appear to perception under established conceptual formulæ; it would be necessary to renew the whole round of investigation with every object before it could be safely subsumed.

7. For this reason the formation of concepts in this province always aims at abbreviations, which enable us to accept a part of the characteristics as representatives of the rest, and so to obtain formulæ which may be briefly called diagnostic definitions.

Such an abbreviation of the conceptual formulæ may be carried out by substituting for a group of characteristics which always occur together a combination of a few, or even a single one, amongst them, which occurs only in this group, and which is therefore a sure sign of the presence of all the others. In direct opposition to the method recommended by the ordinary doctrine of *conceptual formation* from particular ideas, that of combining what is common to all the objects of a group, scientific classification aims rather at selecting the characteristic attributes which distinguish the group under consideration from all others. It is obvious that this necessitates a very extensive survey, and an investigation as to which characteristics may be accepted as always occurring together because connected by natural necessity, and this investigation presupposes the methods by which we obtain general propositions from particular perceptions, *i.e.* the methods of induction. We find the same dependence upon induction in the fact that in establishing the conceptual characteristics we are always obliged to employ causal relations which are only to be arrived at by induction.

It must be left to our consideration of the inductive process later on to

show what difficulties it has to overcome, and how far it is possible at all to make sure of the fundamental point, that a given characteristic occurs only in connection with a certain group of others, and is therefore characteristic of this group. For the present the conclusion we have reached is that a formation, which begins by combining particular things, can only be provisional, and must reserve the right of future correction.

8. From still another point of view we may see the impossibility of starting upon what we may call the direct way from the construction of infimæ species to obtain serviceable classificatory concepts. The teachings of Darwin have especially emphasized the gradual nature of the transitions in the sphere of organic forms which intervene between the clearly distinguished and limited classes, which at first sight we seemed able to take as a basis. The gradually increasing differences in space, slow changes in time taking place by hardly perceptible steps, seem to mock beforehand all our endeavours to obtain a fixed and sharply defined complex of invariable characteristics as the expression of the permanent nature of a connected group of organic individuals, and the attempt to establish in this way a concept under which a plurality of similar individuals may be easily subordinated, while others are unhesitatingly excluded. The greater the extent of observed facts from which we start, the more arbitrary it appears to draw the circles of our concepts within the stream of differences; if we proceed to specialize, then the number of concepts becomes incalculable, and yet we do not escape the difficulty of finding intermediate forms always presenting themselves upon the boundaries of our concepts; and if we disregard subordinate differences and pause only where the course of known phenomena actually shows a gap, yet this gap is itself fortuitous, and we can never proceed with the consciousness that our concepts express a law contained in the nature of things, nor determine what elements belong together once and for all, and what we are forbidden by the laws of being to combine.

9. The same difficulty presents itself in singular and rare deviations from a form which is well defined and recognisable in a great number of instances; what can we make of men with six fingers, or less than 32 teeth, of plants whose stamens are wanting or change into petals? From the point of view of the conceptual construction, which merely compares the given, such abnormal cases have as much right as any others to be the ground for specialization, for the rarity or frequency of a form is in itself no determining ground for our construction; that they are not so is only because we generally introduce presuppositions concerning normal laws by

which our classification is guided, we frame certain types which we regard as ideally perfect, and by which we test particular instances. Only a renewed and comprehensive comparison of the particular, and an investigation of the laws of its genesis, can really teach us what we are justified in regarding as normal and what is fortuitous and abnormal.

10. If what seems the easiest step, the obtaining of the *infimæ* species, which appears to be immediately forced upon us in the given, is so beset with hindrances, how shall we be guided in our progress by means of ascending abstraction to the higher classes? For if we start from the given, and attempt to arrange it according to similarities and differences, we are left free to choose what we will take as the determining common characteristic of a higher class; it is as if we were called upon to classify the words of a language according to the letters which they have in common. Shall we begin by grouping together all which have one letter in common, or those which differ by one letter, and which letter shall it be in either case? If we find the combinations *abcd*, *abcf*, *abfg*, shall we combine the two first as *abc*, or the two last as *abf*, and how shall we dispose of the third? The most serviceable arrangement is doubtless that which groups together things which have most in common; but in order to achieve this we must have complete knowledge of all which can be predicated of the particular kinds, and inasmuch as they manifest their nature chiefly in the causal relations, this again presupposes a previous investigation of the causal relations.

11. From what we have said above we may at least gather that, so far as concerns those concepts which are meant to correspond to the existent, no simple and fundamental direct rules for the combination of elementary characteristics can be given, which follow from the nature of these elements; that although some objects submit to classification more easily than others, yet even with these there is no obvious necessity constraining us to form certain concepts and prohibiting others; and that our formation of concepts has no certainty except when based upon knowledge of general laws stating that certain groups of characteristics are always combined and others never. If we consider, moreover, that general laws themselves presuppose a preceding formation of concepts by which alone we can obtain subjects for them, it follows that we are moving in a circle between abstraction and induction. The only method which can actually be pursued, and which has been actually pursued in the history of science, is characterized as hypothetical; starting from the general assumption of the possibility of a

classification, it begins at first with inadequate means, proceeds then to inductions of the causal laws, and modifies in accordance with these such of its first attempts as cannot be confirmed by them. But as we shall see upon closer investigation, we need the help of construction also before we can finally gain our highest and most general laws.

12. Connected with the formation of those concepts which are to serve for the classification of given things is the formation of composite concepts of qualities, attributes and relations; and these, again, are generally developed only so far as occasion is afforded by the nature of the given.

Owing to the nature of the relation which exists between the attribute and activity on the one side, and the thing on the other, the ground of the synthesis between concepts of independent attributes or activities is to be found ultimately in their common relation to a thing. "Coloured" and "extended" naturally belong together, because only the extended can be thought of as coloured; but in the concept "metallic" determinations are combined which, when thought of in the abstract, have no necessary relation to each other; their bond of union is their common appearance in a class of perceptible things. The psychological concepts, by which we express total states and complex activities, show a more complicated structure; for the most part they contain not only the thought of a mutual dependence between the particular elements, but also the relation mentioned on p. 135, by virtue of which the co-existence of certain elements in consciousness gives rise to new processes in which they themselves are absorbed. In addition to this they denote in part actual processes taking place at a definite time, such as the concepts of the emotions, in part mere dispositions and faculties, such as the predicates, quick-witted, sanguine, passionate. These concepts, again, are held together in the last instance by reference to the one subject, as activities or states of which their elements appear.

Relatively independent of substantival concepts are those relational concepts which contain the thought of a complex relation between certain points of reference; a spatial and mechanical relation, as in "support," or a relation of homogeneity and causality, as in the relation of the offspring to the parent. Here again psychological relational concepts show the most complex combination of relations; how many distinguishable references are combined in the concept of friendship? Here belong more especially all concepts of relations within the province of law, which are compounded from actual states and references to an end which depends

upon the will ; but these we cannot enter into more closely here. From one point of view they fall into the sphere of classification, in so far as their syntheses are carried out only to the degree demanded by actual relations ; from another they contain a constructive element in so far as constructed from the thoughts of ends.

§ 78.

The forms of synthesis, when we are dealing with the concepts expressing perceptible things, differ according to the kind of unity by means of which each of the things to be classified is thought of as one.

That unity of conceptual elements which is contained in the concepts of different kinds of MATTER is distinct from the unity contained in the concept of the INDIVIDUAL FORM. This latter unity is either merely CAUSAL, or also TELEOLOGICAL.

Collective concepts contain a synthesis of individual unities ; this again may be either merely causal, or also teleological.

1. Although the methods of classification can only be fully considered in connection with the methods of induction and cannot be presented as direct rules for the synthesis of the simple conceptual elements of perception, it is at any rate possible to state in general outlines the forms of synthesis which are applicable in this sphere. Logic has until now neglected too much to discriminate between them ; as, for example, when it treats of the concepts of different kinds of matter and the concepts of organisms in the same way, although the sense in which we speak of conceptual characteristics is quite different in the two cases.

If we start, as all method must start in this sphere, from the endeavour to reconstruct given ideas of things, we are at once confronted by the differences of the processes which lead us on the one hand to the concepts of known kinds of matter, on the other hand to the concepts of things which we regard as individuals. The elements are the same, but the meaning of their combination is different.

2. No explanation is needed to show that an easy and obvious abstraction is sufficient to form the concepts of different kinds of matter from a number of things which differ only in their geometrical form and are alike in everything else, and which in the course of nature, or as the result of human interference, manifest variability in form while their sensuous qualities and properties in general remain the same. Water, glass, iron, etc., never appear to us without some form, but the forms they

happen to take disengage themselves from that which takes the form, because within the same thing every part makes the same impression and shows the same attributes to our senses ; thus the attributes which a thing has are independent of magnitude as of form, and division, however far it may be carried, affects nothing in them but quantitative relations. This seems to make the synthesis of attributes independent of spatial extension ; a piece of gold is gold in every point ; the attributes of gold are present in the same manner. But this is only apparent ; only so long as the parts are perceptible, and therefore extended, can they possess the same attributes as the whole, and amongst the attributes of the whole belong also the relations of its parts to each other—cohesion, hardness, flexibility, etc. Thus the attempt to fix the concept of any given kind of matter necessarily leads us to homogeneous parts, but is unable to fix any definite limits to the magnitude of these parts, and cannot therefore represent any given matter as a definite sum of homogeneous units. Matter, therefore, as the object of perception, falls under the concept of the continuous, and is similar in its relations to spatial magnitude. If then the immediately given is to be expressed conceptually, without the aid of hypothetical ideas, such as atoms and molecules, we have need of a standard of measurement which, without referring to actual units, can nevertheless express the relations of magnitude between different wholes, or between a whole and its parts. This measure is in the first place spatial extension, taken in connection with the assumption that space is occupied homogeneously ; and with reference to particular kinds of matter it is always assumed that (*e.g.*) two cubic centimetres of the same water contain twice as much matter as one cubic centimetre. Descartes attempted to employ extension as the measure of the quantity of matter in general, but the standard proved impracticable, partly because of the variability of the volume of the same body, partly because of the impossibility of reducing to harmony by its means the mechanical differences of matter and its modes of working. The desired standard has finally been found in a causal relation, in weight ; and the general standard for the quantity of matter, the concept of mass, is determined by the number of units of weight with which a given body is in equilibrium. We cannot, however, show in detail upon how many assumptions as to the causal relation of things which cannot be regarded as self-evident this standard depends ; the whole calculation is based upon the mere assumption that two units have twice as much effect as one, so that we are here dealing with a

system of laws which are grounded upon and conditioned by each other, and which attain certainty only by the fact that they enable us to point out harmonious regularity on the part of phenomena. The logical import of such propositions can only be made clear by the theory of induction; meanwhile, this method also falls back ultimately upon the original spatial standard, since the weight of a cubic centimetre of water of a given temperature is taken as unit.

Thus we see that it is with the concept of the synthesis expressed by the word "matter" as it was with the conceptual determination of the simple sensuous qualities; it can only be determined by means of a number of assumptions which we make concerning the causal relations, and of which only experience and observation can give us an exact understanding. Here again, then, we find that formation of concepts depends for its completion upon induction, which in its turn presupposes that formation has already begun.

3. Essentially different from the synthesis which produces the concept of matter is that which is active in the concepts of *INDIVIDUAL FORMS*. In the former case magnitude and form are indifferent and are abstracted from in the construction of the concept as fortuitous and changing, the number of the parts and their mutual position being determined by no rule; while in the latter this relation of the parts to each other ranks as a part of the concept itself, and combines as a constituent element of the synthesis with the characteristics which merely express the kind of matter.

4. In the first place, it is the merely geometrical form of a spatial limitation which gives a definite unity to the objects and excludes indefinite divisibility from the concept. Crystals, for example, appear to us as such unities; in their concept definite form is included in addition to the attributes which characterise the matter from which they are formed; and this is due in the first instance entirely to the fact that the same kind of matter is presented in a number of cases in the same form. Then, because the same forms occur in different kinds of matter, abstraction from differences of matter takes place, and a general concept is constructed which contains *only* a certain geometrical form as the form of many different kinds of matter.

The concept of form gains a further meaning when it includes also the composition of a whole from heterogeneous parts connected in a definite spatial position. Here again it is primarily grounded upon the geometrical form of the whole, which must be established before we can speak of its parts and determine the relation of these parts to one another;

the formation of the concept is at first guided only by the external co-existence which manifests itself in cohesion and community of motion. Thus in the perception and discrimination of organic forms the element by which we are guided is the spatial image, as it might be given in a drawing, with its definite arrangement of differently formed parts; the conceptual synthesis by which we reconstruct these forms is therefore similar at first to the geometrical construction which directs us to construct squares upon the three sides of a triangle, the determining characteristics of the concept refer to the definite spatial combination of parts formed in a certain way. The only difference is that here the limits of variation are imposed not by some *a priori* law, but by the actual occurrence of the forms. When we add the special material nature of the particular parts, the synthesis is complete.

But it is still only a synthesis corresponding to a merely anatomical process, which first analyses a composite body until it comes to homogeneous parts, and then consciously determines their relative positions. The question always remains how this relation of the parts to one another in this form is to be explained, and whether the mere fact of their spatial co-existence is not grounded upon an inward necessity.

5. Since every definite form which is not only casually induced from without indicates certain relations of the parts to one another, we must look for some principle for these relations in our synthesis of the concepts.

The first principle which offers is that of a causal relation between the parts; the connection between a certain form and a certain kind of matter is explained when the parts of matter assign to each other their respective positions by their mutual action, and thus give rise to a form of the whole which is circumscribed according to a definite rule. In this way we explain the globular form of the drop of water or quicksilver by mechanical laws, and assume even for crystals a similar necessity, which gives rise to the form of each particular crystal. The only difficulties in the way of our construction here lie in the fact that in matter itself no units are given which we might regard as the ultimate subjects of these effects, and that we are obliged to assume these units hypothetically.

Where, however, heterogeneous parts combine in one form, which we cannot explain as determined by the nature of the matter according to general laws, another principle for the unity of synthesis is found in the concept of the end.¹

¹ Cf. with what follows my *Kleine Schriften*, II. p. 24 sq.

What we have to say about this concept is at present only from the point of view that in it is affirmed a ground for the unity of a whole consisting of different parts, and we are led to consider it by the fact that since Aristotle the unity of organic individuals has generally been expressed by just this concept.

6. There can be no doubt that the conception of an end has its origin in our own voluntary action. When we act, we first imagine a state of ourselves or other things, towards which our will for some reason directs itself; as this purely subjective thought, as a purpose proceeding from some need or wish, the end contains as yet no synthetical principle. But this purpose is to be realized, the willed state is to be brought about in the actual world, and this can only be by means of movements of the body which are immediately subject to the will, and which produce a real change in external objects corresponding to the purpose; that is, a real causal relation is introduced the result of which, as the end, stands over against the producing cause as the means. It is upon this causal relation that the saying is grounded that to will the end is to will the means, and involves the actual carrying out of the means.

7. When we are dealing with particular ends which are realized at a given point of time, the means consist first of all in the action of some cause, or of a chain of causes, in a change therefore which takes place in time, and ultimately brings about the desired state; the means consist primarily in the action, derivatively in the acting thing; but the relation between end and means is transitory. When, in order to move a load, I take the nearest stick and use it as a lever, or when I break a nut with a stone, then the movement of the stick or the blow from the stone—and hence the stick or the stone itself—is the means by which I attain my end; but the relation is momentary and cannot form the ground for a conceptual relation, by which I can determine the thing employed as means. The qualities which it has, as such, permit of its being used as a means, but it is not determined by this final relation.

It is different when I shape and prepare things for a given particular or general end, when I make *tools*. In these the form given to a stone, to a piece of wood or metal, is determined by the end which they are to serve as means; the end of cutting determines the handle and blade in the knife, the end of striking the handle and head in the hammer. As in the simplest tools the form is determined, so in the more complicated the combination of differently formed parts is again determined by the end,

which thus appears as a synthetical principle of unity. This is still more apparent when what is constructed is not a dead tool, which awaits the motive power of the human will before it can act, but a machine working by natural forces ; the result which is brought about by the combination of parts of a certain form under certain conditions is the ground of their form and of their combination. The end makes of the parts a whole which loses all meaning if any part is wanting which is a condition of the result ; and the unity of which consists in the possibility of the co-operation of the parts towards a definite result.

8. At this point it is clear that the end to be realized, when thought as the ground of the form of a thing and of the unity of a whole consisting of parts, is not opposed to the causal concept, but includes it. When a thought is to be realized in the external world, it can only be done by the existing causal relations, by the forces through which things act upon each other, by the causality of our will upon the movement of our limbs, by the action of our limbs in changing other things. The form of the tool is certainly determined by the end ; but it is so only through the causal relations, which ordain that only a sharp blade shall overcome the cohesion of bodies, and that only a heavy, hard, and rapidly moving body shall break a stone. The movement of thought when seeking the means starts from the end, to find from its knowledge of existing causal relations that form and material nature for the means which will produce the intended result. Appropriateness to an end, the fitness of a means to produce the desired result, depends entirely upon those qualities of the things by virtue of which they can take effect ; it is only the presence of the means or tool, when it does not occur of itself in the course of nature, which depends upon conscious purpose.

Leaving now on one side these conditions of the genesis of the means, let us look only at the relation of a given means to an end, as manifested, for example, in a machine which presents itself as a finished product ; what we first find is a purely formal application of the final concept as a synthetical form of unity. A purely causal treatment would start from particular elements of action, and inquire what must result from this or that combination of them, according to natural laws and by virtue of their material nature and their form ; it would regard the movement of the paddles of the waterwheel as a consequence of the flow of water, and would know from the form of the wheel and the position of the axis that the former must revolve about the latter ; other combinations of the same

elements would have another result. A treatment from the point of view of the end, on the contrary, would start from the result, and would ask what combination—or what sort of combinations—of causes would bring about just this result, what it is which must be if the result is to appear. Thus the treatment of the end is to the treatment of the efficient causes much as division is to multiplication; if we start with simple numbers, the multiplication table will show us what products result from the combinations of any two numbers; but if we take any number and regard it as a product, the question which arises is: what are the factors by which it might be produced? The causal treatment states that 6 times 6 is 36; the final treatment that 36 might arise from the multiplication of 4 by 9, as well as of 6 by 6. As in this example, so it is also in nature; absolute necessity permitting of only one method does not hold good when we look backward. The same effect may be produced by different combinations of causes, and when we start from the end we often obtain a disjunctive judgment stating that either this or that combination of elements is necessary to a result; within each of the combinations, however, each particular element is a *conditio sine qua non* of the result, an integral part of the complex of means.

9. If we had complete insight into the causal connection of the world, these two modes of treatment would coincide completely; and in so far as we have insight the connections may be represented in either way. When we subject the given masses and orbits of the planets to a calculation, and find that their mutual disturbances always compensate each other, and produce oscillations in the orbits only within certain limits, the stability of the solar system appears as the necessary result of given causes; and this is the causal treatment. If, on the other hand, we start with the solar system as a stable whole, and ask how this stability is brought about, the uniform persistence of the relations of its parts now appears as the end, and we inquire as to the conditions under which it is possible; various possibilities may perhaps present themselves, amongst which one—incommensurability of times of revolution—is realized. This is the formal teleological treatment.

The two modes of treatment are similar also in that, when estimated from the standpoint of necessity, they both contain a hypothetical element. The causal treatment assumes a plurality of elements acting upon each other; it says that when *abc* are present in this combination the result *d* follows, when in another combination the result *d'*; but that just these

elements should be given in this combination is not a matter of necessity, but merely of fact. The teleological treatment reverses the process, and takes the result for its starting point; it treats it as something to be realized, without, therefore, having to demonstrate necessity why it must be realized, although if it is to be realized this combination of conditions is imperative under the natural laws known to us. Thus from the point of view of the end just that is necessary which the causal treatment must take as merely given.

10. It is easy to see that in application to empirically knowable things and events we find motives to employ sometimes the purely causal view, sometimes the teleological, in order to obtain the synthesis of a manifold in a unity. When a number of known things, acting according to known laws, produce persisting results in persisting combinations, we can explain their connection in a whole, without further help than the thought of their active forces; in this way we explain the spherical form of the earth by the laws of the attraction of its parts, and because of these constant relations it is for us a whole. When, however, the combinations vary, and give rise to varying results, no occasion presents itself for forming the concept of a connected unity, because the relations are only transitory.

It is different when we see combinations of different parts to a whole in the process of formation, without understanding what causes are producing them according to general laws, or finding any necessity which might be explained by general laws, according to which the particular parts take shape and co-operate, although there actually is a persisting result. This is the case with organisms. The maintenance of individuals and of species is a constant, always recurring result; but by what necessity different kinds of matter combine in organic forms, and the particular limbs develop and differentiate, we are unable to explain from the general qualities of matter. Here, therefore, it is natural to explain these phenomena by starting from the whole and its constitution, and inquiring as to the means which will produce this actual result. The whole being taken as an end, there follow a certain connection and mode of action between parts formed in such a way; the relation of each part to the unity is understood when we have decided what contribution it yields towards the preservation and maintenance of all the other parts in their combination. Physiology has, as a matter of fact, never been able to guard itself from this point of view; and it was a superfluous polemic which combated every applica-

tion of the final concept, with a view to carrying out a purely causal and mechanical treatment of the animated universe. The final concept does not contradict the causal treatment, but insists upon it; it is a guiding principle for the discovery of causal relations; and since we certainly cannot refer in particular cases, as in the explanation of a machine, to some power acting like a human being, there is no limit to the investigation of causal relations called for by the final concept. Its importance rests only upon the fact that it expresses the unity of a system of parts which are such that when taken in isolation we are unable to deduce this particular combination from their nature. It follows also that the end can never be taken as ruling absolutely; the realization of the end depends upon the means at hand and their modes of action, and this involves that the same means which realize the end have also incidental results which cannot be subordinated to the end (*e.g.* the vulnerability of the animal tissue), and also that the end can only be attained within certain limits. The enumeration in the so-called Dysteleology of everything which does not seem to subserve an end, or seems hostile to an end, makes claims upon this idea of subservience which are rendered unattainable from the first by its connection with causal laws.

11. The unity which is explained by the end leads to a special view of the concept of the individual. We found that the first motive for discriminating definite unities in the world offered to our perception, and for distinguishing them from each other as particular things, lay in a given spatial limitation; what we denote as one particular thing is given in the first instance by the fact of a permanent or continuously changing spatial outline. But the attempt to fix this unity was wrecked by the question whether anything within a spatial area was necessarily to be regarded as a definite unity, and from this question we were led to the thought of the atom which alone has a unity of which we can speak as necessary. But as soon as we take into consideration the causal relations between atoms, we get a new point of view from which we can speak of the unity of an extended thing; where and so long as these relations continue to subsist between the same atoms, even though it be only from external causes, there we have a ground for combining them into a unity, and for distinguishing things as particular. A stone which retains its form and magnitude by the cohesion of its parts, or a piece of wood which does not of itself change shape, counts for such a unity because of this permanence of the causal relations between its parts. But this unity is only one of

fact, and has no necessary connection either with the nature of the particular parts, and their relations to each other, or with their number; from this point of view it is to a large extent a matter of chance how large or of what shape a piece of wood or stone may be; there is no fixed connection between the matter, the shape and the size. Such unities are particular pieces, but not individuals; they might as well be, or have been, parts of a larger unity, just as they may be divided into smaller parts without any change in the meaning of the unity of these.

The term "individual" first begins to be applicable where a definite relation exists between the unity of the whole and the plurality of the parts, where the parts necessitate a certain circumscribed form for the whole, or the end of the whole necessitates a certain combination of parts; where, therefore, the unity is not arbitrary and casual. The fixity of the form of the crystal justifies us in speaking of individuals; because the matter itself prescribes its own form, which can be fixed in a strictly geometrical concept, the unity appears as one determined by the parts themselves, and whenever a crystal is destroyed this form is modified, and the law of its construction contradicted; the destruction annuls the concept, the fragments of a crystal are no longer crystals in the same sense. But when to the definite form is added the unity of the end, we get a still richer concept of individual unity, and this unity disengages itself in its ideal and formal character still more decidedly when, as in organisms, identity of matter is replaced by continuous change of the particular material parts, only the form of the parts as of the whole, and the mode of interaction between the parts remaining constant. When thought of in connection with time, the identity of the organic individual depends *only* upon the form into which there enter successively new parts, and upon the continuous development of the form of which the particular stages can be combined to unity from the point of view of the end.

It is this which justifies the Aristotelian view that the form, both the geometrical and that determined by reference to an end, is that which makes a thing into a definite particular thing, a *τὸδε τι*; and from this it follows that the concept of the atom and the concept of the form mutually supplement each other, the first as the limit of analysis, the second as the principle of conceptual synthesis. It follows also that no objection can be urged against the logical justification of the concept of the individual from the fact that we are confronted with difficulties in its application to real objects,—difficulties such as appear in the botanical dispute as to

whether the whole tree, or the single twig, or the ^{*}cell is to be called the individual, or in the argument that the animal individual has no fixed limits in its temporal existence because it has broken away from a parental organism, nor yet in space because the air in its lungs passes over without fixed limits into the surrounding atmosphere. From the point of view of the end those points in which the exclusiveness of the individual yields so as to admit connection with the surrounding world in no way contradict its concept, for it is implied in this that it exists through universal causal laws and becomes what it is by a process of gradual development. The logically perfect concept is an ideal concept, constructed as the strict consequence of one principle, and the impossibility of finding a completely corresponding counterpart to it in perception does not prejudice its logical justification and usefulness any more than the ellipse or the parabola are made futile because no body can show an outline, no planet or comet an orbit, which is a strict ellipse or parabola in the geometrical sense. Such objections only prove once more that mere perception of the given with its gradual transitions could never lead to any concepts at all, because from this point of view all limits and distinctions would be ultimately arbitrary; only by the productions of spontaneous thought can we check the stream of differences and solidify into clearly defined figures the fluid mass of phenomena.

12. We have still to investigate the more comprehensive synthesis which leads to the so-called COLLECTIVE CONCEPTS. All collective concepts in the sphere of things are concepts of a whole consisting of a plurality of discrete parts which are themselves thought of as unities, of a whole formed from pieces or individuals. The concept of a given mass of matter, of a quantity of water or iron, is not a collective concept so long as no reference is made to discrete unities, but merely the possibility of this reference is present. The parts are still indiscriminate in the whole, the opposites which appear separate in the collective concept are as yet blended in unstable unity, while we speak of divisibility but not of definite ultimate parts; it is the atomic theory which first makes the concept of a visible body a collective concept.

The unity affirmed by the collective concepts passes through stages analogous to those of the unity of the particular thing. First there is the external and causal unity, corresponding to the "piece"; when we speak of a heap of sand, a bundle of wood, a group of trees, a range of hills, and so on, the spatial co-existence in this number and grouping of the particular

pieces or individuals is not determined by any necessity contained in them, and there is no other relation between them than might exist between any other things whatever.

Other collective concepts have for the basis of their unity a causal relation (it may be dependence upon one cause, or it may be interaction), which connects the particular discrete unities, whether spatial limitation of the whole be also affirmed or not. Thus the collective concept of the solar system has gradually advanced from the mere unity of the sum-total to causal unity; thus the merely genealogical concept of the family is based entirely upon the causal relation of descent from a common ancestor; and thus we can introduce into the collective concept of the wood the mutual dependence of its parts in their vegetation as a causal element.

Finally there appears the teleological unity. It is most obvious where the end is immanent in the particular members as conscious thought, or at any rate as instinct; such is the case in all the relations of human society, or in swarms of bees, or in herds of gregarious animals. Here we find again, as with organism, that so soon as the unity consists in the reference of a plurality of parts to a common end, the identity of the whole consists in the form in which its parts are related, and is independent of the identity of any particular parts; a State remains identical although all its members change within a given time, so long as those who take their places do so in the same combination of subservience to an end; its unity consists in the institutions which are not the mere results of the forces of particular co-existing individuals, but which rather as final concepts determine the manifestations of these forces. Here, again, it is true, moreover, that the teleological view of society and of the State does not exclude the causal, but on the contrary insists upon it. Just because the teleological view ceases here to be merely a formal logical principle, and finds its justification in the fact that the State lives and persists only by actions of the individuals, which are consciously led by ends, we are forced to ask how the individuals come to have such ends, and what are their motives for maintaining and pursuing them; the State and all similar unities must also be explained as total effects of individuals thinking, willing, and acting according to psychological laws.

13. From our investigation of collective concepts, and of the forms of unity of individual things, it follows that the syntheses expressing them can be regarded in two ways. On the one hand, they may be taken as concepts of substance, and it is as these we have so far dealt with them; on the

other hand, they pass of themselves into relational concepts, where the permanent is to be found only in the relation of a number of definite elements, not in these elements themselves regarded as particular things. The concept of the State can be included amongst collective concepts in so far as it always presupposes a number of persons who are combined by definite relations into a unity ; it may also be taken in the first instance as a composite relational concept, as a system of relations between persons. As such it certainly presupposes, as does every relational concept, that the elements combined by the relation should be present in a general sense ; but it does not call for the existence of definite permanent persons before it can be carried out.

CHAPTER III.

THE DIRECT METHODS OF FORMING JUDGMENTS; DEDUCTION AND PROOF, AND THEIR PRESUPPOSITIONS.

WHAT we ultimately aim at in our construction of judgments is to reach perfectly certain propositions, of the grounds of which we are conscious. Our investigations in §§ 45-48 have shown the conditions under which such judgments are possible ; we are further called upon by the general aims of thought to carry out our perfect construction of judgments to the greatest possible extent.

If we now ask as to the ways in which these ends are to be attained, we find that the immediate utterances of self-consciousness, which were dealt with in § 46 (1), fall beyond the reach of methodical investigation as soon as the concepts have been obtained under which we may safely subsume the particular facts of self-consciousness. With reference to other judgments, in which it is possible to have either immediate certainty or syllogistic inference from immediately certain judgments, the methods differ according to the movement of thought in judging. Either the ground is already contained in the way in which the judgment arises from its presuppositions, and it appears from the first as the necessary consequence of its presuppositions ; or else the conception of a judgment as a hypothesis and the discovery of its ground fall asunder, and what first arises is a question or surmise, as to the validity of which we have to decide.

The former is the direct development of judgment, the DEDUCTIVE PROCESS, which is divided into the establishment of immediately certain judgments and the syllogistic development from these ; the latter is the process of proving a statement which is at first suggested hypothetically.

The process of REDUCTION is the reverse of deduction ; it finds premises for given propositions from which they might follow deductively, and it serves to bring into consciousness the highest starting points of deduction.

PROOF presupposes the propounding of hypotheses which first appear in the form of questions ; it thus necessitates heuristic methods, which constitute the art of asking questions or of finding hypotheses.

In addition to the determinate questions, which call for Yes or No as an answer, there are determining questions which ask for the completion of an indeterminate term of a judgment.

If from the given presuppositions we cannot attain to a definite affirmation or denial, but are yet able to exhaust the number of possibilities in disjunctive judgments, then we can proceed to the calculation of the PROBABILITY of a judgment, which is a peculiar form of deduction from disjunctive judgments.

I. DEDUCTION.

1. *Development of Concepts.*

§ 79.

DEDUCTION first appears as the merely analytical development of concepts. Its positive fundamental form is the so-called *SORITES*.

1. The simplest form of deduction is the purely analytical development of concepts. In so far as it deals with the explication of an already constructed conceptual system, it takes place when developing the *content* partly in the simple unfolding of the particular attributes combined into a concept, which when fully carried out in a conjunctive judgment yields a definition by means of simple elements, partly in the form of syllogisms which proceed from the next highest concepts contained in a concept to those which are more remote; when developing the *extension*, in divisive judgments which describe the logical extent of a concept by actually affirming the differences which are possible, starting from one general characteristic. In both cases it merely repeats what must have already taken place in the construction of the concepts themselves, which cannot have come into existence except through active judgment.

What we have already said about this in §§ 43, 44, 53-55, needs only a brief supplementation in so far as the attempt to make the development complete gives special forms to the operations concerned in it.

2. The simple progress in the subsumption of a given concept under successively higher concepts produces first the so-called *sorites*, the premises being arranged in the order known as the *Aristotelian* :—

A is *B*

B is *C*

C is *D*

D is *E*

Therefore *A* is *E*

As the means, however, of complete development this construction needs to be expanded.

3. If, that is, our object is to find a definition which will unfold the complete content of a concept in its ultimate elements, we should have a series of definitions by the genus proximum and *differentia specifica*, in which each subsequent term would define the genus of the preceding definition ; a series, that is, of the form—

$$\begin{array}{l} A \text{ is } a \ B \\ B \text{ is } b \ C \\ C \text{ is } c \ D \\ D \text{ is } d \ E \end{array}$$

Therefore A is $abcd \ E$

a system of equations in which for every simple term a composite one is substituted in the next premise.

4. If, however, we aim only at a complete enumeration of the judgments which proceed from a concept, it must not be overlooked that we can proceed from every concept by itself to several higher ones, and the sorites instead of pursuing a simple line will branch off in different directions.

$$\begin{array}{l} A \text{ is } B, \ C \\ B \text{ is } D, \ E \quad C \text{ is } F, \ G \\ \text{etc.} \end{array}$$

A is D , is E , is F , is G

5. When the conceptual relations are followed out in the direction of specialization of the extension, progress to the lower species gives us first the so-called Goclenian sorites :—

$$\begin{array}{l} D \text{ is } E \\ C \text{ is } D \\ B \text{ is } C \\ A \text{ is } B \end{array}$$

Therefore A is E ¹

¹ If we compare the two forms of sorites—

Aristotelian

$$\begin{array}{l} A \text{ is } B \\ B \text{ is } C \\ C \text{ is } D \\ D \text{ is } E \\ \hline A \text{ is } E \end{array}$$

Goclenian

$$\begin{array}{l} D \text{ is } E \\ C \text{ is } D \\ B \text{ is } C \\ A \text{ is } B \\ \hline A \text{ is } E \end{array}$$

But here again a methodical treatment demands completeness of development not merely in *length*, but also in *breadth*, and we get—

$$\begin{array}{c}
 D \text{ is } E \\
 D \text{ is partly } B, \text{ partly } C \\
 B \text{ is partly } M, \text{ partly } N, C \text{ is partly } P, \text{ partly } Q \\
 \hline
 \text{Therefore } M \text{ and } N \text{ and } P \text{ and } Q \text{ are all } E
 \end{array}$$

a development which may be called a chain of divisions.

6. The value of such deductions becomes of more than merely didactic interest in two cases only. The first is when a concept which is newly constructed from definite attributes is to be connected with an existing conceptual system by making clear its relations of subordination and exclusion; the second, when we wish to make use of a definition consisting of characteristic attributes, for the subsumption of a particular object. When, for example, a chemist is enabled by some reaction manifesting a characteristic attribute to determine a substance as iodine, he ascribes to it by the subsumption all those other attributes of the element which constitute its full concept, and the process of inference takes the following form:

$$\begin{array}{c}
 S \text{ has the attribute } B \\
 \text{Whatever has the attribute } B \text{ is } C \\
 \text{Whatever is } C \text{ has the properties } D, E, F \\
 \hline
 \text{Therefore } S \text{ has the properties } D, E, F
 \end{array}$$

which again is what is called an Aristotelian sorites.

7. The relations of exclusion lead, like the positive relations of super- and subordination, to series of inferences which limit the single concepts on all sides, or prohibit the subsumption of an object *A* under a concept *B*. Since inferences from merely negative premises are impossible, nega-

it is evident that they differ only in the arrangement of their premises. Every process of inference which employs not one but several middle concepts is naturally capable in itself of being presented in either order, and from the point of view of ordinary syllogistic doctrine the distinction between the two forms is quite barren and superficial; moreover, it is misusing the name of Aristotle to suggest that he overlooked the arrangement of the premises which was afterwards pointed out by Goclenius. The distinction between the sorites and the simple syllogism, and the distinction between its different forms, has no meaning except from the point of view of method; here, no doubt, the first form is natural when we start from a given proposition and develop its predicate, the second when we proceed by subsuming more specific concepts under the subject-concept.

tive judgments yield no connection analogous to the sorites; and we may omit to investigate particular possibilities of connection between negative and affirmative premises, since their methodological value is rendered very small by the secondary character of the negative judgment, and by the impossibility of completing the series of negations in the same way as the developments in an affirmative sense¹ may be completed.

8. A special process of subsuming a given concept under a presupposed conceptual system takes place where a chain of divisions is employed for definition, by means of exclusion on the ground of disjunctive judgments. The form of this would be:—

$$\begin{array}{l}
 A \text{ is partly } B, \text{ partly } C \\
 \text{Every } A \text{ is either } B \text{ or } C \\
 X \text{ is } A, \text{ but not } B \\
 \hline
 \text{Therefore it is } C \\
 \\
 C \text{ is partly } D, \text{ partly } E \\
 \text{Every } C \text{ is either } D \text{ or } E \\
 X \text{ is not } D \\
 \hline
 \text{Therefore it is } E
 \end{array}$$

from which we finally get :

$$X \text{ is an } ACE.$$

The familiar example of the angler in Plato's *Sophist* (220A foll.) runs, at any rate partly, in this form.

2. Deduction from Synthetical Propositions.

§ 80.

When deduction passes beyond merely analytical conceptual relations and takes up synthetical propositions—which generally affirm relations—then the syllogistic process can no longer be exclusively represented in simply progressive sorites, but assumes more complex forms.

The possibility of progress here depends upon the development and transformation of concepts, enabling us to make new applications of the fundamental propositions.

1. Deduction first becomes really fruitful when it has for its ultimate premises not only definitions, but can combine with the definitions other

¹ Drobisch (*Logik*, § 105 sq.) has the credit of having investigated all possible combinations of inferences, including those with negative and particular judgments.

propositions which, by their synthetical character, bring about a necessary connection between the attributes of a concept and other predicates; this is more especially the case when the judgments which are developed deal with relations. It is this kind of deduction which we have in view when we speak of deductive sciences, and instance mathematics as the type; and it needs for its foundations axioms as well as definitions (cf. § 55, 5, I., p. 362).

2. Suppose a number of definitions and axioms to be given: the art of the deductive process consists in combining these in the greatest possible completeness, in such a way that they will be premises of valid syllogisms; in finding for every conclusion of a syllogism a new premise which, with it,—either as major or minor—will form the ground for another syllogism; and in thus developing from the fundamental propositions in chains of valid inferences the whole series of consequences.

3. Postponing for the present the question whether the definitions from which the deduction starts are supplemented by real axioms, or only by assumed propositions, and confining our attention to the form of the process, we find that it depends upon the nature of the fundamental propositions what syllogistic forms the deduction will take, and how the single propositions can be combined. One essential condition of this combination will be the finding of middle members. Here again the simplest way of proceeding is the sorites, but instead of the sorites in categorical judgments we now have the more general form of the hypothetical sorites which connects successive propositions by the necessary link of ground and consequence. It depends upon the movement of thought whether the premises naturally arrange themselves in such a way that the second premise which is added to the judgment forming the starting point is one which states the more remote ground, or one which states the further consequence; and this is true again of the successive conclusions.¹

¹ If we have three propositions of the form:—

- I. If *A* is true, *B* is true;
- II. If *B* is true, *C* is true;
- III. If *C* is true, *D* is true;

we may begin with the first two and draw from them the conclusion:—

If *A* is true, *C* is true, and then add the third—
 If *C* is true, *D* is true, so as to obtain the conclusion
 If *A* is true, *D* is true;

The nature of the hypothetical inference often necessitates a *πρόσληψις* (§ 50, 1, p. 330), and when the premises express relations the *πρόσληψις* refers sometimes to one, sometimes to the other term in the relation; such intervening substitutions of a definite for an indefinite subject interrupt the uniformly progressive character of the inferences, which then assume more complicated forms.

4. If, for example, we examine the beginning of Spinoza's *Ethics*, we find that first of all from the definition of the *modus* (*per modum intelligo*

in this case the process is to more remote consequences which are connected with the original ground.

Or we may begin with III. and II. so as to obtain from them the conclusion:—

If *B* is true, *D* is true, and then add the first—

If *A* is true, *B* is true, obtaining the conclusion

If *A* is true, *D* is true;

in this case we pass from the original proposition (III.) to more remote grounds.

It is in this form that we see most clearly the distinction which is expressed in logic with reference to categorical inferences by the terms *prosyllogism* and *episyllogism*; the *prosyllogistic* or *regressive* process supplies grounds for the premises of a given syllogism, which is taken as starting point, the *episyllogistic* develops consequences from its conclusion by the addition of further premises.

If we take a chain of categorical inferences in the first figure, with the premises in the usual order:—

- $$\begin{array}{l} \text{I. } D-E \\ \quad C-D \\ \hline \text{II. } C-E \\ \quad B-C \\ \hline \text{III. } B-E \\ \quad A-B \\ \hline A-E. \end{array}$$

then starting from the second syllogism, the first appears as *prosyllogism*, because it contains the ground for the major premise of the second, while the third appears as *episyllogism*, because by introducing a fresh minor premise it develops the conclusion of the second.

If we reverse the order of the premises to the so-called *Aristotelian* we get:—

- $$\begin{array}{l} \text{I. } A-B \\ \quad B-C \\ \hline \text{II. } A-C \\ \quad C-D \\ \hline \text{III. } A-D \\ \quad D-E \\ \hline A-E. \end{array}$$

Here the *prosyllogism* proves what is ordinarily known as the minor premise, while

substantiæ affectiones sive id, quod in alio est, per quod etiam concipitur) and the definition of substance (*per substantiam intelligo id, quod in se est et per se concipitur, hoc est cujus conceptus non indiget conceptu alterius rei, a quo formari debeat*) there is derived the proposition: *Substantia prior est natura suis affectionibus*. This can only be done, however, by the help of a proposition not expressly included amongst the axioms which would run: *Id in quo aliud est, hoc natura prius est*, and which would therefore express a relation, resulting from one attribute of the definition.

It is, however, impossible to construct from these propositions, as formulated by Spinoza, a simple syllogism or sorites of the ordinary form; the inference would run as follows:—

Id in quo aliud est, hoc natura prius est, to which proposition, by means of a two-fold *πρόσληψις*, substituting definite concepts for the two points of reference of the relation “in aliquo esse,” is added—

Modus est in substantia, from which follows: *Substantia prior est modis sive affectionibus suis*.

The major proposition is an axiom expressing the connection of two relations; the minor proposition follows from the definition of the *modus* which contains the first of these relations.

the episyllogism adds a new major premise to the conclusion $A-D$; the proposition which it introduces last of all is that which contains the most general concepts, and from which we started in the first arrangement.

From this we see that if we start from any chance arrangement of inferences, the distinction between prosyllogism and episyllogism is also a matter of chance and cannot represent any essential distinction in the movement of thought.

It would rather appear that the essential point of distinction is whether the prosyllogism proves the major or the minor premise. In the latter case it can merely insert intermediate terms; it is only in the former that it takes us back to a higher and more comprehensive ground for the conclusion, and is really regressive. It is the same with the episyllogism. If it adds to the conclusion of a given inference a new major proposition, it thereby places its subject under a general rule, and by means of this it derives a new predicate from the one already given, it moves by way of the development of predicates; but if it adds a new minor premise, it specializes the subject of the conclusion, and applies a given rule to the particular instances; it is only in the latter case that it is progressive.

Or in the hypotheticalal form; if to the inferred judgment

If A is true, B is true, we add

If B is true, C is true,

then we pass to further consequences, that which was originally consequence becoming the ground for a new consequence. But if we add: If X is true, A is true, then we move backwards to higher grounds; that which was originally ground is presented as consequence of a further ground, and that which was consequence as a derived consequence. The latter process is regressive, the former progressive, although in both cases we are dealing formally with a so-called episyllogism.

From the proposition : *Substantia prior est affectionibus suis* it is then inferred in the proof of Proposition V., again by means of a proposition not expressly set forth, and in the form of a two-fold πρόσληψις :

Quod natura prius est alio hoc deposito potest considerari ;

Substantia natura prior est affectionibus suis,

Ergo depositis affectionibus suis potest considerari.

On the other side the positive definition of substance : "*Per substantiam intelligo id, quod in se est et per se concipitur,*" yields first the negative conclusion which is appended to the definition and denies a relation : *cujus conceptus non indiget conceptu alterius rei, a quo formari debeat* ; and when as a consequence of this it is tacitly assumed that *substantia conceptum alterius rei non involvit*, and further added that *quod conceptum alicujus rei non involvit, nihil cum hac re commune habet* (the converse of Ax. V.), the proposition follows that *duæ substantiæ nihil inter se commune habent* (the clause *diversa attributa habentes* is only justified if it is intended only as an explanation of "two"). Thus we should get the following chain of inferences issuing from the definition and progressing by means of hypothetical propositions :—

Per substantiam intelligo id quod in se est et per se concipitur ;

Quod in se est et per se concipitur, ejus conceptus non indiget conceptu alterius rei, a quo formari debeat ;

Cujus conceptus non indiget conceptu alterius rei, id conceptum alterius rei non involvit ;

Quod conceptum alterius rei non involvit, nihil cum hac re commune habet.

Ergo substantia nihil cum alia re commune habet ; from which, by a πρόσληψις (substitution of *substantia* for the indefinite *re*) we get :

Substantia nihil cum alia substantia commune habet ; and finally, by a merely verbal transposition, based upon the mutuality of the relation :

Duæ substantiæ nihil inter se commune habent. The further development attaches itself to this predicate "*nihil inter se commune habent,*" from which by Ax. V. it is concluded that one substance does not include the concept of the other ; according to Ax. IV. the concept of the effect includes that of the cause, whence it follows—*modo tollente*—that one substance cannot be the effect of another.

We may find another example in the *Phædo* (78 B sq.). The inferences developed there can be expressed in the following propositions :—

- I. The soul knows ;
That which knows is of the same kind as the known,
Therefore the soul is of the same kind as that which it knows.
- II. That which is known by the soul as such is ideas ;
Therefore it is of the same kind as ideas.
- III. Ideas are unchangeable ;
Therefore the soul is unchangeable.
- IV. The unchangeable is simple ;
Therefore the soul is simple.
- V. The simple is indissoluble ;
Therefore the soul is indissoluble.
- VI. That which is indissoluble is indestructible ;
Therefore the soul is indestructible.

If we examine this deduction, we shall find that to some extent it really does take the form of a simple chain of inferences, especially in the latter half, but that sometimes the inference proceeds by means of substitutions—as in II.—which are occasioned by the introduction of relational concepts (known, of the same kind as). The example from Spinoza shows further, in the first place, to what an extent propositions concerning relations are employed ; and, in the second place, that the whole deduction can naturally only be carried out in hypothetical inferences, because it is only in these that we can operate with only negative determinations which are related as ground and consequence.

5. The same considerations hold good in the sphere of mathematics. Here the relations of equality and inequality form the predicates of most of the propositions, and the axioms referring to them the major premises from which the inferences proceed. The art of deduction consists essentially in constantly introducing as major premises propositions according to which one equality follows from another, and mathematical substitution, as distinct from the logical *πρόσληψις*, which replaces a general by the special included in it, is ultimately no more than an abbreviated syllogism, of which the major proposition is one of the constantly recurring axioms that equals added to equals, or equals multiplied by equals, etc., give equals.

An examination, moreover, of the forms in which these deductions take place shows us that the ordinary examples of prosyllogism and episyllogism, chains of syllogisms, and sorites, in which it is assumed that syllogism follows upon syllogism in such a way that the conclusion of the preceding syllogism becomes the premise of the one following, represent only the

most elementary combinations. In actual practice syllogisms are connected in a much more complicated manner; when, *e.g.*, two chains of syllogisms starting from different points converge towards a single result through one of them yielding the major, the other the minor premise of a syllogism; and still more, when the nature of the subjects of the judgments being expressed by conjunctive propositions, these give rise to parallel chains of syllogisms the results of which are again combined by a new syllogism, in such a way that its major proposition contains a copulative or conjunctive judgment, under the different terms of which the preceding conclusions are subsumed.

This is particularly obvious in geometrical deductions, the subjects of which can only be determined by their particular parts (two triangles are congruent when their particular parts are equal). It is impossible that all this should be reduced to the form of a simply progressive chain of syllogisms.¹

The elements of particular propositions may also be transformed in many ways; a magnitude, *e.g.*, may be regarded as either the sum or the difference of two others. It often happens, again, that only the expression of a proposition need be altered in order for it to serve for the continuation of the deduction.

6. Even more important than to see how incalculably manifold are the

¹ In the very beginning of the first book of Euclid (Prop. 4) we find this complicated form of deduction, which may be represented as follows:—

The subjects with reference to which the deduction takes place are first determined; the hypothesis of the proposition (If two triangles have the two sides AB, AC equal to the two sides DE, DF , each to each, and if the angles formed by them are also equal) is from a logical point of view equivalent to a constructive definition. By the help of superposition each part of this definition is made to yield its special consequences, *i.e.* :—

I. From placing D upon A and DE upon AB it follows, because equal lines when placed upon each other must coincide at all points including the ends, that E falls upon B ;

II. From the equality of the angles it follows that DF falls in a line with AC ;

III. From the equality of DF and AC it follows that F falls upon C ;

IV. From I. and III. together it follows that the line EF falls upon BC ;

From this last coincidence and that of I. it follows that the angle $DEF = ABC$;

From II., III. and IV. that angle $DFE = ACB$.

The principles employed in this deduction are partly such as to contain two or more conditions; hence the minor proposition by which they are applied consists of several links, of which each must itself be proved. Thus particular conclusions are combined in the most varied manner, to get the composite minor premises of new axioms.

The process is the same in the numberless cases in which we have first a deduction that $A = B$, then a second that $C = D$, and then the combination of the two that $A + C = B + D$.

combinations admitted by particular forms of inference is it to recognise that the creative power of progress in deductions lies in conceptual construction and development, which gives us new subjects to which we may apply general principles. It is customary to call deduction progress from the general to the particular, but this progress is possible only in so far as concepts are differentiated, and a wider sphere thus opened up for the application of a general proposition.

The Euclidean geometry is not really woven out of the few definitions, axioms and postulates by which it is introduced; it obtains material for its deductions by the inventive construction of figures and their relations, and by the specialization of general concepts. The syllogism of itself can do no more than enable two propositions to produce *one* new one; only concepts are really fertile.

II. PROOF.

§ 81.

The **PROOF** of a proposition is its syllogistic derivation from other propositions, which are known to be certain and necessary, ultimately therefore from **DEFINITIONS** and **AXIOMS**.

To this extent every deduction from definitions and axioms is also the proof of the conclusion reached by it.

When a distinction is made between proof and deduction, the proof is regarded as the problem of deciding as to the truth of a hypothesis, of confirming or refuting it. The proof of an affirmation is the refutation of the negation, and *vice versa*.

I. An affirmation is derived either from simple categorical or hypothetical premises, and by means of categorical or hypothetical syllogisms *modo ponente* (direct proof); or from a disjunctive judgment by negation of the other members of the disjunction (proof by exclusion or indirect proof).

The negation of a proposition follows from the fact that it conflicts, either in itself or in one of its consequences, with a true proposition. In the latter case the *deductio ad absurdum* takes place.

II. The discovery of the proof of a proposition consists in the discovery of middle concepts, and in the case of affirmative propositions these are found by development of the content of the subject and the extension of the predicate (in the case of hypothetical judgments, in the discovery of mediating propositions by means of the development of the antecedent into its consequences, and resolution of the consequent into its grounds).

In this way we get first the substitution of one demonstrandum for another.

When convertible premises can be introduced, the proof may be discovered by developing the demonstrandum syllogistically, until we arrive at a true result, from which the proof may be constructed by a reverse process (analysis in the sense of the ancients).

1. The form of the deduction is the same whether it starts from merely assumed propositions, which are to be developed into their consequences—we may, *e.g.*, investigate deductively what would happen if masses attracted each other in the inverse ratio of the cubes of their distances,—or whether it works entirely from propositions of immediate certainty, from axioms and definitions.

If the latter, then the formal correctness of the deduction includes the necessity of the results at which it arrives; it becomes proof. For to prove a proposition is to derive it syllogistically from other propositions, which are known to be certain and necessary; the new proposition itself obtains unconditional and necessary validity.

In such cases the derived proposition and its proof are discovered together.

2. If, however, the thought of the validity of a proposition has arisen in some other way, so that it is first presented as a hypothesis, then the problem arises of deciding as to its validity; and from this point of view of a problem the process of proof becomes distinct from direct deduction. The proof may then be merely didactic, when the result of the deduction is first imparted to the hearer and its derivation given afterwards, though the teacher discovered the proposition by deduction; or it may be subservient to research itself, when some proposition is first merely constructed in the form of a hypothesis, and its validity investigated later by the question whether it can be shown to be a necessary consequence of recognised principles. These are the ἀρχαὶ τῆς ἀποδείξεως, the *principia demonstrandi*; the proposition itself is τὸ πρόβλημα, *demonstrandum*; the ἀπόδειξις, *demonstratio*, consists in the syllogistic derivation of the *demonstrandum* from the *principia demonstrandi*. The problem of proving a proposition calls upon us, therefore, to show it to be the necessary consequence of true and necessary propositions which are recognised as such; to show it, therefore, to be the necessary consequence of im-

mediately certain propositions, or of propositions which are deductively obtained from these.¹

3. If we leave for the present the question as to the way in which a hypothesis needing proof may arise, and assume some supply of original or derived propositions from which the proof may be drawn, then the first question to be dealt with is : How are we to find the proof?

The method prescribed by the nature of syllogism of arriving at a proof of the proposition A is B , is by finding a middle concept (in the hypothetical form a mediating proposition) between A and B . If there were a concept X of which we could say from the beginning that A is X and X is B , then the proof would be given in the syllogism :—

$$\begin{array}{l} A \text{ is } X \\ X \text{ is } B \\ \hline \text{Therefore } A \text{ is } B \end{array}$$

Suppose the proposition to be proved to be that virtue is capable of being taught ; then the question is whether there can be found a middle concept, which is a predicate of virtue, and has the predicate "capable of being taught." Such a concept is knowledge ; virtue is knowledge, knowledge is capable of being taught, therefore virtue is capable of being taught. It is assumed that the two premises are acknowledged as true.

If the proposition to be proved is the negative A is not B , the proof may be obtained either by finding a middle concept which belongs to A but excludes B (or is excluded by B), or which does not belong to A and is necessarily connected with B ; or, again, by developing a consequence from A which is a ground for denying B , or which is contradictory to a consequence of B .

Suppose we wish to prove that an equilateral triangle is not rectangular ; equality of angles is a property of the equilateral triangle, but equality of angles excludes the possibility of one of them being a right angle ; or to put it in another way, equality of angles contradicts the necessary consequence of being rectangular, which is that the angles are unequal.

4. It is evident that when an affirmative proposition is to be proved in the first figure (*modo ponente*) such a middle concept must be found

¹ Aristot. *Tóp.*, I. 1 : "Ἔστι δὴ συλλογισμὸς, ἐν ᾧ τεθέντων τινῶν ἑτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει διὰ τῶν κειμένων ἀπόδειξις . . . ἐστίν, ὅταν ἐξ ἀληθῶν καὶ πρώτων ὁ συλλογισμὸς ᾖ, ἢ ἐκ τοιοῦτων ἀ διὰ τινων πρώτων καὶ ἀληθῶν τῆς περὶ αὐτὰ γνώσεως τὴν ἀρχὴν εἰληφεν. Cf. *Anal. post.*, I. 1.

amongst the concepts which belong as predicates to A , and also amongst those to which B belongs as predicate; thus what we have to do is on the one hand to develop the content of A , on the other hand to review the extension of B , and to ascertain whether there is any common element. Or, when applied to hypothetical inferences, we must, on the one hand, develop A into its consequences; on the other hand, ascertain the grounds from which B would follow according to known principles; and in this way discover a middle term to be the consequence of A and the ground of B . If, however, we wish to prove negative propositions, we must develop the exclusions resulting from A and B , and in this way find a common term which will make the syllogism possible.

5. Only in the simplest cases will a single middle concept suffice to help us to recognise propositions as starting points for our proof. Generally our analysis will have to be applied to several terms before the desired concepts are found, and the process becomes proportionately complicated until it becomes the trial of different possibilities.

Take, for example, the proposition A is B ; analysis of A shows that X , Y , Z belong to it, but we know of no proposition to the effect that either X or Y or Z has the predicate B . The question arises, therefore, whether any one of the propositions X is B , Y is B , Z is B can be proved; the same process begins anew of seeking a further middle concept by the development of X , Y , Z on the one hand, and a survey of that to which B belongs on the other.

It often happens that in this way we merely succeed in reducing one proposition to another in the sense that if the latter were proved the former would follow from it. A is B —in the above example—would be proved if it were proved that X is B , because it has been found that A is X ; in the same way the proposition A is B is proved when it can be shown that A is M , and we know that M is B .

Suppose, for example, it is to be proved that in an isosceles triangle ABC the straight line AD drawn from the apex A to the point D bisecting the base is at right angles to the base; all I need prove is that it makes equal angles, for equal adjacent angles are right angles; but it is proved that it makes equal angles if the two triangles ABD , ACD are congruent, and thus the problem is reduced to proving that the construction before us gives two congruent triangles. Here the reasoning works back from the predicate of the demonstrandum (or from the consequent) to its nearest ground; by the introduction of a universally known

proposition, for the final conclusion itself (that the angles are right angles) is substituted its ground (that they are equal adjacent angles). If we had begun by developing from the construction that it would give rise to triangles having three sides of the one equal to three sides of the other, then the proof would resolve itself into the problem of showing that when the triangles ABD and ACD have the sides of the one equal to the sides of the other, then the angles ADB and ADC must be right angles; the reduction would start from the subject of the problem, or from the antecedent.

This replacement of the original demonstrandum by another, from which it can be derived by a simple inference, was called by Aristotle (though in a narrower sense) μεταλαμβάνειν, and to the proposition, to the proof of which the problem is reduced, he gives the name of τὸ μεταλαμβανόμενον.¹

6. Such a change in the original demonstrandum may take place not only through simple development of the given; geometry can show many examples in which, with a view to discovering the proof, the subject or predicate of the demonstrandum is so changed by a special operation that the syllogistic middle terms may be found, *e.g.*, by representing the given lines or angles as sums or differences of others. Here inventive construction is at work, producing the conditions under which alone a development of the subject or predicate concept which will lead to the desired end is possible. If, for example, it is to be proved that in a quadrangle $ABCD$ inscribed in a circle the sum of the opposite angles A and C , B and D , always $= 2R$, the proof cannot be carried out without further data, the figure shows nothing from which the relations between the angles could be derived. It is only when the diagonals AC and BD are drawn, dividing each angle into two parts, that we get conditions under which these parts can be compared; in place of the original subject, the sum of two angles, there now appears the sum of four angles. The predicate is also replaced by another; the sum of the opposite angles is $2R$ when it is half of the sum of all the angles of the quadrangle, when, therefore, the sum of the angles A and C is equal to the sum of the angles B and D , and the μεταλαμβανόμενον is now: that the sum of the four angles lying on the one diagonal AC is equal to the sum of the four on the other diagonal BD .

¹ *Analyt. pr.*, I. 23, 41a, 39. For an explanation of this expression see my *Programm*, p. 4, note.

Or suppose the proposition to be proved to be that the three lines bisecting the angles of a triangle intersect in one point; for this proposition is substituted another, stating that the line drawn from the point of intersection of two bisecting lines to the apex of the third angle bisects this third angle. In this case we are first led to transform the predicate by the consideration that the bisecting line of the third angle must pass through the point of intersection of the two other bisecting lines if a line drawn from this point to the apex of the angle bisects the angle, because there cannot be more than one bisecting line; what now has to be proved is the equality of two angles instead of the passing of a line through a given point.

The expedients employed in this substitution of problems are manifold, and it is difficult to establish rules of method where an inventive combination is at work within a large range of possibilities; nevertheless, the fundamental character remains the same in these more complicated operations as in the most simple. The process always starts, on the one hand, from the subject (or the antecedent) of the problem, and develops it into its predicates, or the proposition into its consequences; on the other hand, from the predicate, or from the consequent of the demonstrandum, and works back from it to its nearest conditions, from which it proceeds, in order to ascertain whether these two ways coincide at any point.

7. In one case we may substitute for a process which assumes the character of mere experiment, in proportion as the number of terms to be discovered increases, one which leads directly to the end; when, that is, the premises leading to the conclusions are all convertible propositions, either categorical or hypothetical. When, to take the simplest instance, a proposition A is B can be proved by means of a middle concept X in such a way that the syllogisms

1. A is X	or	If A is true, X is true;
2. X is B		If X is true, B is true;
3. A is B		If A is true, B is true;

are valid, and the premises are convertible, then the first proposition (in ordinary terminology the minor proposition) can also be represented as the consequence of the second and of the conclusion, the second (or major proposition) as the consequence of the first and of the conclusion; that is, the following syllogisms will be valid:—

I.	2. B is X	If B is true, X is true
	3. A is B	If A is true, B is true
	<hr/>	<hr/>
	1. A is X	If A is true, X is true
II.	1. X is A	If X is true, A is true
	3. A is B	If A is true, B is true
	<hr/>	<hr/>
	2. X is B	If X is true, B is true

Here we may find a ground for that method which the ancients called analysis; the method which, taking the demonstrandum as starting-point, constructs a syllogism by introducing a second acknowledged premise, either as a minor or major premise, and then proceeds to develop the conclusion in the same way until a proposition is reached which is known to be true independently of this derivation. As soon as this point is reached, the process is reversed, and by conversion of the premises a chain of inferences is established, in which the last conclusion is the demonstrandum.

The deduction from the *Phædo*, quoted on page 190 as a proof of the proposition that the soul is indestructible, might (if we assume the propositions forming it to be convertible) have been found by some such analysis as the following, which starts from the demonstrandum:—

1. The soul is indestructible;
2. The indestructible is indissoluble;
3. The indissoluble is simple;
4. The simple is unchanging—

Therefore the soul is unchanging.

5. The soul as knowing is of the same kind as that which is known by it;
6. Therefore that which is known by it is unchanging.
7. The soul knows ideas;
8. Therefore ideas are unchanging.

Supposing this result to be true in itself, in consequence of a definition, as here, or as an axiom, and, therefore, capable of being used as a *principium demonstrandi*; then if all the intervening propositions are convertible, the whole deduction may be read backwards; the proof would now begin with 8, and read as follows:

8. Ideas are unchanging.
- 7, 6. The soul knows ideas, therefore it knows the unchanging.

5. It is of the same kind as that which it knows, therefore it is unchanging.
4. The unchanging is simple ;
3. The simple is indissoluble ;
2. The indissoluble is indestructible ;
1. Therefore the soul is indestructible.

We may here see what importance attaches, not only to the convertibility of simple universal propositions, but also to those relations which are either entirely reciprocal, such as equality, or which belong together as correlates, such as right and left, sum and difference, etc. For the proposition *A* is greater than *B* can always be converted by virtue of this correlative concept into the proposition *B* is smaller than *A* (though the proposition *A* is the ground of *B* cannot always be converted into the proposition *B* is always the consequence of *A*). Mathematical syllogisms frequently progress partly by means of the relation of equality, partly by means of such correlative concepts as larger and smaller, sum and difference ; and for this reason the method of discovering proof which we have just described is chiefly available in the mathematical sphere.¹ Of course the discovery of the proof, *i.e.* the derivation of a true proposition from the demonstrandum, is not to be called the proof itself, or even a kind of proof ; for the truth of the conclusion would not follow from the truth of the premises unless the further condition of convertibility of the premises were fulfilled (§ 59, 3).

8. A distinction is generally made between proof and refutation ; but this is incorrect if it is meant that they are different ways of procedure in thought, and if the distinction does not merely refer to a belief or an inclination to accept the proposition as true, which is present independently of the proof. In itself that which is still to be proved is uncertain, both its affirmation and its negation must be held to be possible ; only a subjective disposition can cause us to advance an ungrounded proposition with the expectation that it will be valid, and to feel the demonstration of its opposite as an overthrow of something already established. Apart from this disposition every demonstrandum is a question which calls for Yes or No, and for which we seek an answer ; every proof of the Yes is a refutation of the No, every refutation of the Yes a proof of the No. Thus the means of refutation are absolutely identical with the means of proof.²

¹ Cf. Duhamel, *Des méthodes dans les sciences de raisonnement*, I, p. 39 sq.

² ὁ γὰρ ἑλεγχος συλλογισμὸς ἀντιφάσεως. Aristotle, *Περὶ σοφ.* εἰλ., 6. 168a, 36.

It is however natural that the question as to the ways in which positive propositions are proved should occupy the foreground, as the positive proposition is the original, and the negation has no meaning except as the denial of a positive proposition, and thus the question as to the means of proving positive propositions, with which we have so far been chiefly concerned, does certainly fall apart from the question as to the means of proving negations. We have already indicated these means in a general way; they consist in the discovery of relations of exclusion which, when we are dealing with simple syllogistic forms of the first or second figure, can yield the middle terms required. But by far the most important and fruitful method of establishing the falsity of a proposition consists in the development of its consequences by the introduction of true premises; these consequences leading ultimately to contradictions amongst themselves or with recognised truths, the falsity of the conclusion taken in connection with the truth of the other premises disproves that one which was taken as a starting point. Thus, in the familiar example used by Aristotle, the commensurability of the diagonals of the square with the sides is refuted because, from it, it would follow that an even number would be odd.

This example itself shows a special form of refutation by the invalidity of the consequences, the process, that is, through a divisive or disjunctive judgment, of which all the members are denied. If a proposition *A* is *B* can be developed into the disjunction: If *A* is *B*, either *C* or *D* or *E* is true; and if it can then be shown that neither *C* nor *D* nor *E* is true, then the presupposition, the proposition *A* is *B*, is itself denied; if the demonstrandum before us is—

A is *B*,

But *B* is either *C* or *D*,

And *A* is neither *C* nor *D*,

then the proposition *A* is *B* is refuted (cf. § 57, 4, § 58, 3. I., pp. 370, 372).

9. Methods of proving negative propositions, or the falsity of positive statements, first become important when they are the means, not merely of guarding against the error which attempts an untenable affirmation, but of grounding a positive statement. The importance of the disjunctive judgment lies in the fact that it makes it possible to pass from a negation to an affirmation. It is in this way that we get the forms of proof known as indirect, in opposition to the simple derivation of a proposition from principles by means of categorical or hypothetical syllogisms.

10. Proof by exclusion starts from a disjunctive judgment, and demonstrates one member of the disjunction by the denial of all the others; this denial again may be demonstrated in the different forms, leading to negative judgments, *modo ponente* or *modo tollente*.

The proof which alone is generally called indirect or apagogic (ἡ ἐκ ἀδύνατον ἀπαγωγή) is only a special form of this proof by exclusion.

As ordinarily represented, its essence consists in demonstrating a proposition by the refutation of its contradictory opposite, and in achieving this refutation by showing that the opposite, when developed into its consequences, leads to the impossible. It proceeds, therefore, as follows to prove *A* is *B*:—

If *A* is not *B*, then by the introduction of other premises it can be shown to follow that *C* is *D*; but the proposition *C* is *D* is false, therefore it is false that *A* is not *B*, therefore true that *A* is *B*.

In this purely formal representation it might easily seem as if the indirect proof needed no help but that of the principle of the excluded middle in order to reach its goal. But if we look more closely, we shall see that in a case where there is really nothing else introduced the whole circumlocution is unnecessary and the direct proof possible.

If, that is, it is true that—

If *A* is not *B*, *C* is *D*, then it is also true that
If *C* is not *D*, *A* is *B*.

But the proposition *C* is not *D* is known to be true; thus the same connection, according to which the false proposition *C* is *D* follows from *A* is not *B*, enables us to demonstrate directly from the true proposition *C* is not *D* the proposition *A* is *B*.¹

If an indirect proof is to lead to anything which cannot be reached directly, its essence must consist in attaining to a sphere of action in

¹ It is the same in the categorical form. If the proposition to be proved is all *A*'s are *B*, the contradictory would be some *A* is not *B*.

From this we may proceed to infer, either in the second figure :

Every *C* is *B* ·
—————
Therefore some *A* is not *C*;

or in the third :

All *A*'s are *C*
—————
Therefore some *C* is not *B*.

If the first conclusion is false, then it is true that all *A*'s are *C*; if the second is false, then it is true that all *C*'s are *B*. Hence it is true both that all *A*'s are *C*, and that all *C*'s are *B*; and from that it follows directly that all *A*'s are *B* (cf. § 25, 8. I., p. 155).

which it can move independently. This happens when it succeeds in gaining by the denial of the demonstrandum positive propositions which can be developed into their consequences; and this is possible only by means of a disjunctive judgment, which exhausts the circle of possibilities. The others all proving by their consequences to be false, only the original proposition remains, and as the only one possible it is necessary.

It is for this reason that the indirect proof so often runs in several parallel series; when several possibilities present themselves, each one must be refuted before the exclusion of all other members of the disjunction can establish the one which is left.

Disjunctive judgments are therefore the true foundations for indirect proofs; they proceed in one series where the disjunction has two members, in several series where it has more than two.

11. When we see that the genuine indirect proof is based upon a disjunctive judgment which contains more than the mere antiphrasis of an affirmation and a negation, the objections often raised against it to the effect that its power of conviction is less than that of the direct proof, and that it masters truth only by a stratagem, disappear. The disjunction upon which it stands must proceed from and develop the nature of things; all our concepts gain whatever they may have of determination from distinction and opposition, and the proof which starts from such oppositions employs ultimately only what is contained in the nature of the concepts or their relations. It shows no imperfection to prove the equality of two lines by the fact that one can be neither longer nor shorter than the other, for the concept of equality gets its determination from just this negation of all difference, and its very nature is to present a limiting case to which we approximate by diminishing the differences. When Euclid proves indirectly that two lines which, meeting in the same point with a third, make angles of which the sum = $2R$, form one straight line, the proof constrains one line to be the continuation of the other by the exclusion of every other direction, and follows as much from the nature of the figure as any direct proof.

If we are dealing with the proof of negative propositions, then the indirect proof may be said to be the normal method, inasmuch as it obtains from the affirmation itself the negation of its possibility by showing the self-destructiveness of its consequences. Thus Spinoza is fully justified in proving a number of negative theorems apagogically (e.g. *Eth.*, 1, 5. 6.

Coroll., 12, 13; II. 10, etc.), showing that the opposite alternatives are by their positive determinations impossible.

12. The problem of proof is solved when the proposition in question can be exhibited as a syllogistic conclusion from premises which are without exception true and valid, which therefore rest finally upon immediately certain propositions. Moreover the truth of a proposition which is not self-evident cannot possibly be grounded in any other way than through such a proof; only when a proposition is the necessary consequence of necessary propositions is it itself necessarily valid; and in order to show it to be so, we must make use of the syllogistic forms in which alone that necessity is to be found. Proof, therefore, in the strict sense, is possible only in so far as there are definitions and axioms.¹

III. REDUCTION.

§ 82.

The framing of possible premises for given propositions, or the construction of a syllogism when the conclusion and one premise is given, is called REDUCTION. It leads first to hypotheses, but when the propositions obtained by reduction prove to be immediately necessary, it is a means of finding the first principles of deduction.

INDUCTION, which takes place by means of comparison and abstraction of general concepts, is a special form of reduction.

1. From proof we pass first to the explanation of a proposition which is already known to be true, but which is also shown to be the necessary consequence of general grounds and thus takes its place in a connected deduction from first principles.

If I know that the sum of the digits of all compounds of 9 from twice to ten times 9, is 9; or *vice versâ* that all numbers between 10 and 90 of

¹ I can only briefly refer to errors of proof, which are partly of a formal nature, based either upon insufficient determination of concepts and terms or upon ignorance of the syllogistic rules; and partly such as violate the conditions of proof because mingling with their premises propositions which do not possess unconditional validity, thus assuming a ground of proof too hastily (*alreivθau τὸ ἐν ἀρχῇ, petitio principii*), or because something other follows from the deduction than that which was to be proved (*ἐρεποσύνησις*). The latter error does not, of course, take place, when instead of the proposition required a more general one is found in which it is contained. The rule "*qui nimium probat nihil probat*" does not apply to the proof which yields more than is needed, but only to that which yields too much, *i.e.* something notoriously false, in addition to the proposition to be proved, and thus betrays either an error of inference or a false premise.

which the sum of the digits is 9, can be divided by 9; I have here a proposition obtained from immediate observation of the numbers 18, 27, 36, etc., an actual coincidence without exception, within these limits, of 9 as the sum of the digits, and divisibility by 9. The question is whether we can find any necessity in this coincidence; whether it can be represented as the consequence of other known numerical laws; and to decide this question we must proceed just as if we were dealing with a proposition which had still to be proved. By analysing the denary expression of a number, upon which is based the statement of the sum of the digits; by substituting $10-1$ for 9 or $9+1$ for 10, which follows simply from the definitions of 9 and 10; and by introducing propositions about multiplication and division, we find the middle terms which make it possible to prove that a number of the form $m \cdot 10 + n$ is divisible by 9 when $m+n$ is divisible by 9, and that every compound of 9 must give a number of which the sum of the digits is 9 or can be divided by 9; and it is then evident that the proof can be applied, not only to numbers with two digits, but to all numbers whatever which have for the sum of their digits 9, or a compound of 9.

2. In this case the propositions from which the proof is derived, and the explanation given, are known beforehand. But the same process may also serve to discover *principia demonstrandi*, which have not yet been consciously put forward as propositions.

It is here that we come upon a new phase in the construction of judgments which follows a direction the reverse of deduction; it is frequently called analysis, but as this term is ambiguous reduction is a better name.

To start with the simplest cases: if two propositions all *A*'s are *B*'s, and all *A*'s are *C*'s, are given, then if we employ them as premises of a syllogism, it follows only in the third figure that some *C*'s are *B*'s, or some *B*'s are *C*'s; in other words, that *C* and *B* are compatible, or do not necessarily exclude each other. But these same propositions admit of another interpretation; they may be taken as conclusion and minor premise of a syllogism having for its major premise "all *B*'s are *C*'s," or "all *C*'s are *B*'s," so that the syllogisms in which they would naturally be placed would be either

All *B*'s are *C*'s

All *A*'s are *B*'s

All *A*'s are *C*'s;

or :

$$\begin{array}{l} \text{All } C\text{'s are } B\text{'s} \\ \text{All } A\text{'s are } C\text{'s} \\ \hline \text{All } A\text{'s are } B\text{'s.} \end{array}$$

In the first case A is B is the minor proposition and A is C the conclusion ; in the second case *vice versâ*. According to the major premise introduced they would enter into the one or the other necessary relation ; a ground could be given for the meeting of the two predicates in the subject in so far as predicate B necessarily involved predicate C or *vice versâ*.

If we have two negative propositions : no A is B , no A is C , nothing can be obtained from them so long as we regard them as premises. But if in the natural order of a comprehensive thought they should become the premise and conclusion of a syllogism, this would run as follows :

$$\begin{array}{ll} \text{All } C\text{'s are } B\text{'s} & \text{or} & \text{All } B\text{'s are } C\text{'s} \\ \text{No } A \text{ is } B & & \text{No } A \text{ is } C \\ \hline \text{No } A \text{ is } C & & \text{No } A \text{ is } B \end{array}$$

in the second figure. Or if we convert the propositions and take them as major premises,—

$$\begin{array}{ll} \text{No } B \text{ is } A & \text{No } C \text{ is } A \\ \text{All } C\text{'s are } B & \text{All } B\text{'s are } C \\ \hline \text{No } C \text{ is } A & \text{No } B \text{ is } A \end{array}$$

in the first figure, the assumed premise being now the minor premise. Here again the connection between B and C is the ground for both being denied of A .

Finally, suppose an affirmative proposition all A 's are B 's, and a negative proposition no A is C , to be given ; if we take them as premises, it is only in the third figure that an inference can be made :

$$\begin{array}{l} \text{All } A\text{'s are } B\text{'s} \\ \text{No } A \text{ is } C \\ \hline \text{Some } B\text{'s are not } C, \end{array}$$

where the conclusion merely denies necessary connection between C and B . But if it were true that no B is C or no C is B , then we should get :

No B is C
 All A 's are B

No A is C
 in the first figure

No C is B
 All A 's are B

No A is C
 in the second figure.

Instead of the particular negation following from the two propositions, we might presuppose a universal negation as the ground of the relation between them.

Or in the more general formulæ of the hypothetical judgment: two propositions—

If A is true, B is true,
 If A is true, C is true,

are necessarily connected when B is a ground of C , or C is a ground of B ; for then the syllogism is possible:

If B is true, C is true.
 If A is true, B is true.

 If A is true, C is true.

or: If C is true, B is true.
 If A is true, C is true.

 If A is true, B is true.

In the same way, given two propositions:

If A is true, B is true,
 If A is true, C is not true,

the second follows from the first when we have the true proposition: If B is true, C is not true; and the first follows from the second, when, If B is not true, C is true. For when we add, If A is true, C is not true, it follows *modo tollente*, If A is true, B is true.

3. This framing of possible premises for given propositions is, of course, as much bound by the syllogistic rules as deduction; it is similar to those processes of calculation which proceed backwards, inasmuch as it proceeds from the result to the elements which give rise to it; it is, as it were, a division of the product by one factor in order to find the other. We do not, however, get an unambiguous result, but generally—as in the extraction of roots—a double possibility with reference to the supposed

major proposition according as one of the propositions to be reduced is taken as minor premise or as conclusion ; even apart from this difficulty, the result to which this process leads is at first only a possibility, and the discovered premise only a hypothesis, of which we cannot say that, because when taken together with a true proposition it yields a true conclusion, it is therefore itself true (cf. § 59, 3. I., p. 374).

4. The same process of reference to a possible major proposition may appear when a predicate C belongs to a subject A , and the question arises as to which of the determinations of A has C for its consequence ; it may, for instance, depend upon one alone of its attributes, B , and then we should have, All B is C , A is B , therefore A is C . In our former example, the compounds of 9 from twice to ten times have 9 for the sum of their digits, and then the question arises whether this attribute depends only upon their being compounds of 9, or also upon their having two digits ; whether, that is, they follow necessarily from the whole subject as the complex of all its determinations, or from only one of its determinations.

This question would be still more pressing if several subjects, A^1 , A^2 , A^3 , having the predicate C , should all agree in falling under the general concept A . For then the particular propositions A^1 is C , A^2 is C , A^3 is C , appear as common consequences of the same major premise all A 's are C , in which all alike find their explanation ; the reduction brought about in this way is ordinarily called induction.

Here again, however, no necessity is shown why the predicate C should be the consequence of the common A : iron, gold, and silver are heavier than water ; iron, gold, and silver fall under the concept of metal, but it does not follow that the fact of being metal is by itself the ground of greater specific weight.

Only if A^1 , A^2 , A^3 should constitute the whole logical extension of A , would it be possible to infer the validity of the proposition, All A 's are C . We should have :

A is either A^1 , or A^2 , or A^3 ,
 A^1 , and A^2 , and A^3 are all C ,

Therefore all A 's are C (according to § 57, I., p. 368).

This proposition may then be taken as the major proposition by which to explain the particular propositions A^1 is C , etc.

5. We shall have to show in a later section what part is taken in empirical inquiry and the inductions grounded upon it by this process of

assuming major propositions from which given propositions follow as conclusions. Here the process of reduction comes into consideration only so far as it bears upon the problems of deduction.

In the first place, it is clear that reduction is one of the ways in which hypotheses arise of which the validity can be decided by a process of proof; it is therefore of use as a heuristic process leading to the construction of questions. It is, however, chiefly important as a process for discovering ultimate first *principia demonstrandi*; when, that is, a proposition found by reduction proves to be self-evident, an axiom. The most universal presuppositions of thought do not generally appear in consciousness in their most abstract form; their force makes itself felt as the ground of certainty and self-evidence in special and concrete cases. Just as our ideas are not at first consciously formed from their simple elements, which are interwoven without explicit consciousness and need therefore to be reconstructed by logical reflection, so also our convictions are not constructed from first principles by means of explicitly stated syllogisms; what appears in consciousness is the general principle in a particular application, from which it must be disengaged and raised into clear consciousness in its pure form. In the practical sphere, also, we are always coming upon convictions as to the right or the expedient which have not been constructed at every step with conscious reflection, but of which the certainty is nevertheless maintained by the inner necessity of universal rules.

This is especially true of logical axioms themselves. Locke is perfectly right in saying that many men are all their lives unconscious of the principle of contradiction, although in the concrete case they maintain the incompatibility of affirmation and negation with the fullest conviction. The principle of contradiction, like all other logical principles, is found only by reduction; the question being raised why A is B and A is not B cannot both be true at the same time, it becomes apparent that the impossibility does not attach to this particular A or B , that it is not merely impossible for B to be affirmed and denied at the same time of A , but that the impossibility is grounded in the relation between affirmation and negation themselves. Thus, what is needed is an analysis which will set free the concept "to affirm and deny B of A " from the special case, and disengage from the particular instance the general concept "to affirm and deny something of something"; the proposition "it is impossible at the same time to affirm and deny something of something" now becomes the

major proposition to the particular case, which derives from it alone its absolute certainty.

This analysis of the general from the special is aided by comparison, which shows that it is also impossible to say both C is D and C is not D , x is y and x is not y ; the abstraction of the general, which is necessary to the attainment of the general proposition, takes place more easily when the general appears as the common element in different cases.

This is what Aristotle means when he teaches that first principles are obtained by induction,¹ by a process which mounts from the particular to the general. He did not mean that the general actually derived its validity from the particular, but that the recognition of general principles has its origin in the particular case; they rest their claim to be principles upon the fact that they bear their own necessity within them and are immediately true, a *πρότερον φύσει* over against the concrete example which is the *πρότερον καθ' ἡμᾶς*. This disengaging of principles is akin to the process of induction in the proper sense, which endeavours to derive the truth of a general proposition from the truth of a number of particular instances, only in so far as both are grounded upon abstraction of the general concept from particular examples.

What is true of logical axioms can be said also of mathematical axioms. The fundamental laws of space intuition do not at first come into consciousness by themselves, but attached to certain particular figures; but as we come to see that the predicate is not grounded upon that which constitutes the speciality of the particular case, the general proposition disengages itself as the condition of the necessity. That two straight lines cannot enclose a space becomes evident first from experiments made with two particular straight lines; but when we find that the predicate does not depend upon the position of these, or upon the angle made by them, but that we can disregard these details, then the necessity of the general proposition becomes manifest.

In the logic of jurisprudence this process of disengaging the general legal principle from some special determination of which it is the ground is known as analogy.

Here again it is true that the validity of some special proposition attaching a judicial consequence F to a condition ABC , or the validity of several propositions attaching the consequence F to the conditions ABC , ADE , cannot at first give more than the possibility that the

¹ *Analyt. post.*, II. 19, 100b, 4.

consequence F depends upon the general element A , and not upon the modifying determinations B or C , D or E ; the certainty of the proposition that F depends upon A must be evident in itself or upon other grounds if the analogy is to lead to an unquestionably valid proposition.

6. Sometimes the reduction may indeed be transformed by an indirect process into a valid proof. If the proposition were established that the condition ABC has the consequence F , and if it could be shown that the consequence F cannot be grounded either in the presence of determination B , or in the presence of determination C , or of both together, then by a process of exclusion A would remain as the only possible ground of F .

IV. THE DISCOVERY OF HYPOTHESES.

§ 83.

The fundamental heuristic methods, by means of which hypotheses are framed, are the CONVERSION of given propositions, INDUCTION and ANALOGY.

1. We have until now postponed the question as to how the problems arise which become the object of proof; we have assumed that there is a question before us to be answered by Yes or No, a hypothesis connecting a definite predicate with a definite subject.

How do such hypotheses arise, and what methodical process can lead to their construction? The method which plays at combining every possible predicate with every possible subject has indeed been recommended as *ars inventiva*; but the aimlessness and arbitrariness of such combinations have always condemned this art to futility. A question can only be reasonably asked where there is some occasion to expect a predicate; and our methods of framing questions must take into consideration the motives which will suffice to induce us to undertake the trouble of seeking a proof.

The combinations which may lead to a surmise are in themselves of many kinds; they may however be reduced to a few fundamental forms.

2. The first to suggest itself when some universal proposition is given is the attempt to convert it; in other words, the inquiry as to whether the predicate of such a judgment belongs exclusively to the subject, and is therefore a characteristic attribute, and whether the consequent of a hypothetical judgment has only one ground and may therefore in its turn

serve as ground for the antecedent. When we know that in the isosceles triangle the angles at the base are equal, the question suggests itself whether all triangles having equal angles at one side are isosceles; or to express it differently, whether equality of two sides is the exclusive ground of equality of two angles, so that when a triangle is not isosceles it has not two equal angles.

Since the conversion of the universal affirmative judgment gives only a particular proposition, and the validity of the ground is not proved by the validity of the consequence, it follows that a special proof is always necessary for the converse of a universal affirmative proposition, and for the conversion of a hypothetical judgment.¹

¹ It has been stated by F. C. Hauber, and quoted by Drobisch in the Appendix (p. 234 of ed. 5), that the conversion of a universal affirmative hypothetical judgment needs no special proof, if we allow that

S is either a or b or c
 Σ is either α or β or γ ,
 and that if $S \dots a$ then $\Sigma \dots \alpha$
 if $S \dots b$ then $\Sigma \dots \beta$
 if $S \dots c$ then $\Sigma \dots \gamma$

Since it follows, without further proof, that

if $\Sigma \dots \alpha$ then $S \dots a$
 if $\Sigma \dots \beta$ then $S \dots b$
 if $\Sigma \dots \gamma$ then $S \dots c$

In this general form the proposition, together with its proof, is false.

The proof begins: Assuming that if $\Sigma \dots \alpha$, then S is not a , then since it is presupposed that the disjunction is complete, either $S \dots b$ or $S \dots c$.

But this assumes without proof that if $\Sigma \dots \alpha$ then there is an S to which a or b or c belongs; the possibility that S is neither a nor b nor c , hence that it does not exist at all, is left out of consideration. The disjunctive judgment S is either a or b or c tells us that if S is then it must be a or b or c ; but if it is false that $S \dots a$, this may be either because it is b or c , or because it does not exist at all, because the ground of the disjunction is removed.

E.g. If a quadrangle is a parallelogram, it is either equilateral or unequilateral.

If the diagonals are drawn within a quadrangle, they will intersect either at right angles or at oblique angles.

If a quadrangle is an equilateral parallelogram, the diagonals will intersect at right angles.

If a quadrangle is an unequilateral parallelogram, the diagonals will intersect at oblique angles.

Does it follow from this that if the diagonals of a quadrangle intersect at right angles, it is an equilateral parallelogram; and if at oblique angles, it is an unequilateral parallelogram?

Or: A triangle is either acute-angled or right-angled or obtuse-angled.

Of three given angles either the sum of each two is greater than the third, or the sum of the smallest is equal to the third, or smaller than the third.

If a triangle is acute-angled, the sum of each two of its angles is greater than the third.

3. This is one way in which a question may arise, and it leads of itself to another. Reliable conversion of the universal proposition all A 's are B gives us at first only some B 's are A ; but the question is raised whether all B 's are A , and this attempt to convert universally may be represented as an attempt to raise a particular into a universal judgment, from the fact that P belongs to one or some S 's to suggest that it belongs to all S 's; *i.e.*, that it is the determinations thought in the concept S , and not some peculiarity of the particular S of which P is true, which make this predicate necessary.

When, *e.g.*, we find that 3^2-1 is divisible by 2, 4^2-1 by 3, 5^2-1 by 4, 6^2-1 by 5, etc., then the question arises whether this property of the squares of the numbers minus 1 to be divisible by the original number minus 1 depends upon the peculiar properties of these particular numbers, or whether it is universally true that n^2-1 is divisible by $n-1$, or in a still more general form, that n^m-1 is divisible by $n-1$.

4. The way in which we arrive at the hypothetical statement of such a general proposition may be called, on the one hand, *reduction*, as the addition of a universal major proposition to two or more definite judgments; on the other hand *induction*, in so far as we infer from a number of instances to the possibility of the general rule; the process of reduction thus appears as the heuristic method of discovering general propositions, for which the deductive proof is to be discovered subsequently. The history of mathematics shows that even in the sphere of strict deduction a great number of propositions have been originally found by way of induction.¹ We cannot, however, speak of inductive inference, if the term

If a triangle is right-angled, the sum of the two smallest angles is equal to the third.

If a triangle is obtuse-angled, the sum of the two smallest angles is smaller than the third.

Here again it does not follow that if three angles are given of which each two together are greater than the third, they are the angles of an acute-angled triangle; it would first have to be shown that wherever three angles are given, they are the angles of a triangle.

Hence we cannot dispense with the proof that the proposition: if S is a or b or c , then Σ is α or β or γ , must be convertible into the proposition if Σ is S is; it is only on this condition that we can be sure of the convertibility of the propositions combining the particular terms of the disjunction.

¹ In the disputed question whether the fundamental propositions of mathematics are obtained by induction or in some other way, it is necessary to make a distinction, which is sometimes overlooked in the meaning of the word induction. It cannot, of course, be questioned that we first become aware of the simplest relations of numbers by means of intuition, and that the geometrical axioms also, such as that two straight lines

inference is to be used in its strict sense; the most comprehensive comparison of instances cannot lead to the certainty of the universal proposition unless based upon a logical division, unless, that is, it becomes a divisive inference (§ 57).

5. Co-ordinate with induction as a heuristic process stands analogy, which is also important only as a means of framing hypotheses. In its simplest form it may be represented as follows: if a judgment A is B is given, and a subject A^1 possesses some attributes in common with A , it may be supposed that A^1 will also possess the predicate B .

6. Analogy takes a more definite form when a comparison of given cases not merely suggests the connection of a general A with a general B ,

cannot include a space, and that equal straight lines can be made to coincide, depend upon intuition which is concerned at first with particular straight lines. That two and two are four we first learn—assuming that we know the meaning of the numerical terms—by a particular attempt to add together two pairs of things. But when we notice what it is we do, we see at once the necessity that it should be so, that from the nature of our counting, whatever it may be applied to, it is impossible ever to arrive at any other result; we become aware in the particular concrete case of the necessity which dominates it. It is the same with geometrical axioms. It is not our experience of different instances which first shows us that our intuition of space is constant; if it were, this constancy would be only empirical, and not to be relied upon unconditionally; we know that it cannot be otherwise. The proposition that two equal straight lines coincide when laid one upon the other, shows itself as quite evident so soon as we have realised by an example what it means; and this is possible just because the concept of the straight line is not empirically abstracted from sense objects, but is a constituent element in our idea of space. Wundt (*Logik*, ii. 108) says quite rightly that the really mathematical elements are those which are subjective and belong to our activities of thought; that what remains when we have abstracted from the particular objects of sense is nothing else than the activity of thought which is operative in the formation of mathematical ideas. This is why it is possible in particular instances to become conscious of this activity and of the laws to which it conforms. If this recognition of the general and necessary in the particular instance is to be called induction, then the term has an essentially different meaning from that which it has where the general proposition is only hypothetically, and without any certainty of its necessity, applied to a number of instances including no exception. That all heavy bodies fall, is a proposition which is accepted in consequence of invariable perception, but of which we merely assume and do not see the necessity; for a body to fly towards the sky would contradict no law of our own mental activity, as would be done if twice two were five. We can only speak of induction in the ordinary and strict sense when the proposition is believed merely because the particular instances of which it is an enlargement are given as facts; as soon as they are known to be strictly necessary, though the proposition may have been originally found by way of induction, it is no longer believed because of this induction, but on the ground of immediate self-evidence or of a deductive proof. Hence the fact that in mathematics also we start from particular instances does not affect its deductive character; the process resembling real induction is only an auxiliary operation, and attains its end only when we can see that the propositions obtained by it are strictly necessary.

but from the modification which B undergoes when A is varied leads us to expect a definite modification of B for every variation in A . It is an essential condition here that the variations both of A and of B should fall into a series which can be recognised to be governed by a rule; this will be the case when they are of a quantitative nature, and manifest therefore increase or decrease, either in general or according to a definite formula. In this case the construction of a general formula according to which B varies with changes in A is a reductive process, and the suggestion that a further modification of A will be in accordance with the same rule is framed by analogy (in the narrower sense).

We may find a simple example of this in the binomial co-efficients. The calculation of $(a+b)^2$, $(a+b)^3$, $(a+b)^4$ shows the co-efficients

$$\begin{array}{c} 1, 2, 1 \\ 1, 3, 3, 1 \\ 1, 4, 6, 4, 1; \end{array}$$

from these instances may be discovered the rule according to which each subsequent series is derived from the one before, and by analogy it is assumed that the following powers will also follow the same rule.¹

In the same way every process of interpolation is a process of this kind of analogy.

V. DETERMINING QUESTIONS.

§ 84.

Every question which cannot be answered by Yes or No, but aims at determining an element of the proposition (subject, predicate, or part of the predicate) presupposes as valid a more general proposition, and endeavours merely to determine what is thought in this in indefinite generality. If this general element can be developed into a disjunctive judgment, the question presents itself in a more definite form as disjunctive.

The answer to such questions is obtained by the same methods which lead to the proof by which determinate questions are decided.

1. In addition to the questions of which the origin is shown in § 83, we have others which cannot be answered by Yes or No, but which try to determine some part of a judgment. Their general character

¹ The close connection between induction and analogy will be discussed again later on.

always consists in the attempt to find for a general concept the special determination demanded by the other elements of the judgment. For every question of this kind presupposes a general concept before it can be possible, and the interrogating word itself contains the concept of this general element. When I ask: what is A ? it is assumed that A has some predicate X by which it is distinguished; when I ask: how large is A ? I assume that it is a quantity which can be determined by some measure; what still remains to be achieved is the more exact determination of the general predicate, which is needed by the nature of the subject.

2. If the general concept which is presupposed in every question can be developed into a disjunction having a finite number of terms, we may in this way obtain from the one original question a number of determinate and mutually exclusive questions, and thus proceed directly to hypotheses for which a proof may be sought. The question: What is the relation between two manuscripts, A and B , of the same work? leads at once to the disjunction that either A is derived from B , or B from A , or that both are derived independently from the same, or from different, older manuscripts.

3. The process of answering determining questions does not differ essentially from the process of deciding determinate questions by Yes or No; here also the determinate predicate must be introduced by means of a deduction, and the problem is to find the premises for this deduction.

In the first place, we can proceed only by starting from the given elements of the judgment, and developing these by means of known premises introduce the missing element. If I ask, what is the length of the perpendicular of an equilateral triangle of which the side = 1? a consideration of the figure shows first that the relation of this perpendicular to the known parts is that of one side of a right-angled triangle of which the hypotenuse = 1 and the other side = $\frac{1}{2}$, which therefore are already known. By the Pythagorean theorem it follows that this side must be $\sqrt{1^2 - (\frac{1}{2})^2}$, and this gives us the answer to the question. The same operation which would prove the proposition that the perpendicular is $\sqrt{\frac{3}{4}}$ of the side, would also answer the question.

4. Here again we may employ that expedient of analysis in dealing with convertible propositions which we described above as a means of discovering proofs. We assumed there the truth of the demonstrandum and developed it into its consequences; here the missing determination is assumed to be given and its relations to the known determinations developed deductively—the general mode of procedure in algebraical

analysis, of which the chief expedient is to denote the missing element in such a manner that it may be handled as if it were known. If I want to know what number is greater by 5 than 7, I may either proceed to subsume the predicate under a general concept by arguing that a number which is greater than another may be got from it by addition, and that therefore the desired number must be the sum of 7 and 5 ; or I may proceed analytically by reasoning that the desired number x must be such that $x-5=7$, and developing this equation deductively, I find by the addition of 5 to both sides that $x=5+7$.

5. It need not be pointed out that what are generally called problems in mathematics are logically identical with such questions as these ; the problem of constructing a cube which shall be double a given cube coincides completely with the question as to the length of the side of a cube which is twice the size of a given cube. Thus every solution of a problem is also the proof of a proposition.

VI. THE CALCULATION OF PROBABILITIES.

§ 85.

Where, though it is not possible to decide a question, the desired answer can be at any rate limited to a finite number of possibilities by means of a disjunctive judgment of which the terms are so far equivalent that for our knowledge they represent equal specializations of a general concept, or equal parts of its total extension, then we may proceed to an estimation of the PROBABILITIES of the particular possibilities ; its measure is a fraction of unity of which the denominator is the number of the equivalent terms in the disjunction.

To this is added a further process of deduction, of which the form consists originally of inferences from a combination of disjunctive judgments, and which is based upon the framing of exhaustive disjunctive judgments. In both processes the application of mathematical methods, and more especially of the theory of combinations, is necessary.

That which we can calculate in this way is merely the amount of subjective expectation which is justified by the relation between knowledge and ignorance upon which the disjunctive judgment is based. Inferences as to the relative frequency of the particular possibilities according to the measure of their probability, are reliable only when they are supported by certain presuppositions as to the conditions of this occurrence.

Corresponding to deduction in the sphere of probability is a reduction which finds its special application when, from the numerical relations of actual instances, that disjunction of possibilities is affirmed upon which they are most probably based. The results thus obtained also have no more than subjective probability.¹

1. When the development of a determining question leads only to a disjunctive judgment (§ 84, 2) without any apparent possibility of proceeding to a decision, the investigation is brought to a halt before a question which is insoluble. Deduction in the proper sense ceases, and thought can merely survey the different possibilities, uncertain as to which of them is valid.

Even in this case, however, a further deductive process presents itself when different disjunctions can be combined. The inferences from disjunctive judgments were examined in § 58 only in so far as they led from the affirmation of one term of the disjunction to the negation of the others, or from the negation of one term to the limitation of the disjunction to a smaller number of possibilities, or from the negation of all the terms but one to the affirmation of this one. But the disjunctive judgment also permits of forms of inference which can be developed only in disjunctive judgments, and which depend upon a combination of disjunctions.

2. *a.* Suppose we have two independent disjunctive judgments,

A is either b or c

A is either m or n

the relation in which b and c stand to m and n being neither that of necessary connection, nor yet that of exclusion; by combining them we should get:

A is either bm or bn , or cm or cn

(A parallelogram is either equilateral or not equilateral,

A parallelogram is either rectangular or oblique-angled; hence

A parallelogram is either equilateral and rectangular, or unequilateral and rectangular, or equilateral and oblique-angled, or unequilateral and oblique-angled).

¹ Cf. with this section J. v. Kries, *Die Principien der Wahrscheinlichkeitsrechnung. Eine logische Untersuchung*, 1886, and my notice of this work *Vierteljahrsschr. für wiss. Phil.*, xiv. p. 90 sq.; also an article by Stumpf in the *Sitzungsb. der Münchner Academie*, 1892: *Ueber den Begriff der mathematischen Wahrscheinlichkeit*. I have benefited by what is contained in the first of these writings so far as to give a more accurate form to some sentences.

Again, suppose three judgments to be given under the same conditions :

A is either b or c
 either m or n
 either x or y

we should obtain :

A is either bmx or bmy
 or bnx or bny
 or cmx or cmy
 or cnx or cny .

b. If, again, two judgments are given :

If A is, then either m is or n is,
 If B is, then either x or y or z is,

it follows :

If both A and B are, then either mx or my or mz , or nx or ny or nz is.

It needs little consideration to show that the disjunctions thus obtained contain all the combinations of the particular terms of one series with each term of the other series, and that they must therefore have a number of terms equal to the product of the number of terms of the particular disjunctions from which they were derived.

3. We get another form of derivation in the development of the particular terms of the disjunction by disjunctive judgments. Let these be given :

1. If A is, it is either b or c .
2. If A is b , it is either p or q .
3. If A is c , it is either x or y or z ,

then by substitution of disjunctions 2 and 3 in disjunction 1 (assuming that p , q , x , y , z exclude each other) it follows :

If A is, it is either p or q or x or y or z .

A quadrangle is either a parallelogram or a trapezium or a trapezoid.

If it is a parallelogram, it is either (*a*) a square, or (*b*) an oblong, or (*c*) a rhombus, or (*d*) a rhomboid.

If it is a trapezium, then it has those sides which are not parallel either (*e*) equal or (*f*) unequal.

If it is a trapezoid, it either (*g*) has a re-entrant angle, or (*h*) has not.

A quadrangle is either *a*, or *b*, or *c*, or *d*, or *e*, or *f*, or *g*, or *h*. The number of terms in a disjunction obtained in this way is the *sum* of the number of terms of the particular disjunctions from which it is derived.

4. Should the development of a disjunction proceed by simple hypothetical judgments, then what would take place would be a simple substitution of consequences, assuming these to be mutually exclusive; the derived judgment would then have the same number of terms as the original judgment.

If A is either b or c or d , and it is true that
 If A is b , then it is x
 If A is c , then it is y
 If A is d , then it is z ; then it is also true, assuming that x, y, z exclude each other, that
 A is either x or y or z .

If, however, the consequences of b and c (e.g.) should not exclude each other, but should combine in some common characteristic, then there would be a decrease in the number of terms,—

A is either b or c or d .
 If A is b , it is x and also w .
 If A is c , it is y and also w .
 If A is d , it is z , then it would follow that
 A is either w or z .

If, finally, all three terms of the disjunction should have the same consequence w , the conclusion would be A is w , according to the form of inference in § 57.

5. It is clear that these forms of inference by the combination of disjunctive judgments hold good both for those disjunctions which, being based upon the division of a concept, enumerate the more special determinations of a general subject, and which, therefore, enable us to find the number of kinds resulting from combined divisions: and for those hypothetical disjunctive judgments which state how many possible cases may occur on the assumption of some condition or set of conditions.

The judgments: When a coin A is thrown it will fall either heads up (Ah) or tails up (At);

When a coin B is thrown it will fall either heads up (Bh) or tails up (Bt), combine according to $2b$ to form the proposition:

When A and B are thrown, there will follow either $Ah Bh$, or $Ah Bt$, or $At Bh$, or $At Bt$.

6. The theory of this branch of inference from disjunctive propositions

has not, however, been developed in a form corresponding as this does to the ordinary logical mode of representation; it has taken shape as the mathematical theory of probabilities, which is shorter and more elegant, at any rate in more complicated cases, and which is fundamentally nothing but a mathematically formulated chapter from logic.¹

The possibility of a mathematical treatment lies primarily in the fact that in the disjunctive judgment the number of terms in the disjunction plays a decisive part. Inasmuch as a limited number of mutually exclusive possibilities is presented, of which one alone is actual, the element of number forms an essential part of our knowledge; our ignorance differs in character and in significance according as the uncertainty extends over 2, 3, or 4 possibilities.

But before we can pass from the number of terms in the disjunction to any definite measure of the confidence with which we may expect the truth of a particular term, we need another presupposition. Our knowledge must enable us to assume that the particular terms of the disjunction are so far equivalent that they express an equal degree of specialization of a general concept, or that they cover equal parts of the whole extension of the concept; or—in hypothetical disjunctive judgments—that the alternative consequences are equally possible, *i.e.*, that nothing is forthcoming to favour one consequence rather than another; or, again, that the extensions of the specializations of the ground from which the consequences proceed are equivalent. This equivalence is most intuitable where we are dealing with equal parts of a spatial area, or equal parts of a period of time; that a grain of corn dropped upon a chess board must fall either upon a black or upon a white square, that an event of which nothing is known as to the time of its occurrence must take place upon the first day of the week, or the second, etc., these are instances of such disjunctions. But even where this obvious equality is not forthcoming we may ground our expectations upon a hypothetical equivalence, where we see no reason for considering the extent of one possibility to be greater than that of the others.

The disjunction: A triangle is either right-angled or acute-angled or obtuse-angled, is a correct one, but its terms are not specializations of equal value; the expressions obtuse-angled and acute-angled cover an endless number of variations in the size of the angle, of which each one, considered

¹ F. A. Lange rightly saw that the theory of probability is based upon the disjunctive judgment, and is in this way connected with logic.

as the specialization of the general concept of the angle of a triangle, is equivalent to the one instance of the right angle, which is only one case amongst indefinitely many. In the same way, if we know that there are one white and ten black balls in an urn, the disjunctive judgment that a ball drawn from amongst them must be either black or white is indisputably correct, but its terms are not equivalent because the predicate black represents an element common to ten variations of the event, and the predicate white only one variation. If a coin is thrown up twice successively, it is quite true that it will fall twice heads or twice tails, or once heads and once tails; but the last case represents two possibilities, heads first and tails second, and *vice versa*.

To speak of an equivalence in the terms of the disjunction or of any relation between their extensions is impossible when a disjunction merely opposes affirmation and negation; for the bare negation is quite indefinite. I can say of every A that it is either b or not b , of every body that it is either green or not green, of every expected or possible event that it will either occur or not occur; but the negative term of the disjunction is quite indeterminate, and cannot be regarded as any determinable part of a total extension.¹

Assuming that the terms of a disjunction are equivalent, their number gives us a measure for the probability of each one amongst them. The confidence with which we are justified in expecting the truth of a particular one is evidently greater when the choice lies between two only than when we have the choice between three, four or more possibilities; it diminishes in proportion to the number of the terms, and may, therefore, be expressed as the reciprocal value of this number.

On the other hand, the proposition, "the probability that given condition A the proposition B_1 is true is $\frac{1}{n}$," merely states that a disjunctive proposition having n equivalent terms, of which one is B , exists in the form :

If A is, then B_1 or $B_2 \dots$ or B_n is.

¹ There is, *e.g.*, no meaning in saying that the probability that there is iron upon Sirius is equal to the probability that there is none; there is no universal common to both. If we assume that there are only the n elements known to us, we might determine the probability that a certain chemical simple body is iron as $\frac{1}{n}$, for now we have a universal given which specializes into a given number of positive members of disjunction; to calculate the probability of a composite body containing iron we should need a disjunction of all the chemical combinations and all their possible conglomerations—an impossible task.

7. Hence follow the simple fundamental rules of the theory of probabilities :

1. The probability which is to be ascribed under condition A to one amongst various equally possible cases is measured by a fraction of which the numerator is 1, and the denominator is the number of mutually exclusive possibilities stated in the disjunctive judgment.
2. If an instance x has by itself the probability $\frac{1}{m}$, and an instance y the probability $\frac{1}{n}$, then the coincidence of the two xy will have the probability $\frac{1}{m} \times \frac{1}{n}$ (see above 2 a and b).
3. If amongst various possible instances of which the probabilities are known, several have the same consequence, then the probability of this consequence will be equal to the sum of the probabilities of the particular instances in which it occurs (4).

These principles are generally illustrated by the familiar examples of one or more urns, in which there are different coloured balls, or of one and several dice.

Take, for instance, a die : I have here the disjunctive judgment that when thrown it will show on the upper surface either 1 or 2 or 3 or 4 or 5 or 6 ; the probability of each of these numbers is $\frac{1}{6}$.

If two dice are thrown together, the case falls under 2 b ; with each term of the one disjunction there may be combined one term of the other, and I get a combined disjunction of 36 terms ; the probability that 1 and 1, 6 and 6 will be thrown is $\frac{1}{36}$.

If it is not taken into consideration whether the one number of a combination (e.g. 2 in the combination 2 and 5) appears upon the first die, the other number, 5, upon the second, or, *vice versa*, 5 upon the first and 2 upon the second, then the probability that 5 and 2 will be thrown together in some way will be $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$, for amongst the combined disjunctive judgments we shall get both 2 and 5 and 5 and 2.

If we merely look to the sum of the numbers thrown, then in the complete disjunction

12 = 6 + 6	will appear	once,
11 = 5 + 6 (I 5, II 6 ; I 6, II 5)		twice,
10 = 5 + 5 = 4 + 6		three times,
9 = 4 + 5 = 3 + 6		four times,
8 = 4 + 4 = 3 + 5 = 2 + 6		five times,
7 = 3 + 4 = 2 + 5 = 1 + 6		six times,

$6 = 3 + 3 = 2 + 4 = 1 + 5$	five times,
$5 = 2 + 3 = 1 + 4$	four times,
$4 = 2 + 2 = 3 + 1$	three times,
$3 = 2 + 1$	twice,
$2 = 1 + 1$	once.

Hence the probability of 12 = $\frac{1}{36}$.

" " of 11 = $\frac{2}{36}$.

" " of 10 = $\frac{3}{36}$.

" " of 9 = $\frac{4}{36}$.

" " of 8 = $\frac{5}{36}$.

" " of 7 = $\frac{6}{36}$, etc.

The sum of all the probabilities is $\frac{36}{36} = 1$, that is, some one of these combinations must be thrown.

Exactly the same probabilities result from two successive throws with the same die; we get a combination of the same disjunctive judgments for the first and the second throw.

If I know that an urn contains only white and black balls, without knowing either their number or proportion, then I have the disjunction:

Every ball drawn out will be either white or black, and the probability of drawing a white one is $\frac{1}{2}$.¹

¹ v. Kries has disputed this proposition on the ground that we are only justified in a numerical statement of probability when we have certain knowledge of the objective equality between the areas belonging to the particular possibilities as parts of the whole area. I think I have (*l.c.*) shown in answer to this that we can never have such absolutely exact knowledge even in what seem to be the most favourable instances; that we must confine ourselves to assuming that the possibilities are equal if our knowledge affords us no ground for expecting any one more than the others. In the example before us, our knowledge that only black and white balls are present justifies us in stating an exhaustive disjunction as follows: Let the unknown number of balls be n , then the urn contains

either 1 black and $n-1$ white balls,

or 2 black and $n-2$ white,

or $n-1$ black and 1 white ball.

The terms of this disjunction are absolutely equivalent. In the first case, the probability of drawing a black ball is $\frac{1}{n}$, in the second $\frac{2}{n}$, and so on until $\frac{n-1}{n}$; the probability of drawing a white ball is in the first case $\frac{n-1}{n}$, in the second $\frac{n-2}{n}$, and so on to $\frac{1}{n}$; the sums of these particular probabilities for black and white are equal, therefore the statement $\frac{1}{2}$ is justified for both the black and the white balls. All that we can yield is that our expectations are more uncertain in proportion as our ignorance is greater; but it cannot be disputed that in the case before us they are equal for black and white.

But if I know that there are 6 black and 4 white balls in the urn, the disjunction is one of 10 terms; the probability of drawing any given one is $\frac{1}{10}$, but 6 of these give the same result black, and if I look to this only, the probability of a black one is $\frac{6}{10}$, the probability of a white one $\frac{4}{10}$.

This last example is well adapted to show the purely subjective character of the foundations of the theory of probabilities. It measures nothing but the degree of expectation, justified upon the ground of a disjunctive judgment stating the number of mutually exclusive disjunctive propositions which are present for my state of knowledge; a judgment which may change as soon as my knowledge becomes more exact.

8. This purely subjective character of the theory of probabilities is often concealed by the fact that the illustrations chosen contain a further knowledge, which, in itself, is not contained in the statement made by the disjunctive judgment. In the case of the die, for instance, we know with absolute certainty upon geometrical grounds that when it lies upon a plane surface, one of the six sides will lie uppermost, and hence follows the disjunction from which we estimate the probability of each throw at $\frac{1}{6}$. But we know further from considering the way in which the die is handled, and from comparing our observations of a series of throws, that the different throws do actually succeed each other with great irregularity and will therefore continue to do so in future; and this irregularity is of such a kind that in a long series of throws the particular throws will appear an approximately equal number of times, so that we know that the real causes which bring about the particular throw will vary in such a way as to realize the various possibilities, without giving definite preference to any one throw over the others. We have not merely no ground for assuming that the possibilities are unequal; we have ground for assuming that objectively they are at least approximately equal. Since, however, we are not in a position to determine the degree of approximation, perfect equivalence is hypothetically assumed, as it was when our starting point was merely the simple disjunctive judgment that every throw must yield either 1 or 2 or 3 or 4 or 5 or 6.

When the probability of any particular throw is to be calculated, there is complete ignorance of the special conditions actually existing, complete ignorance as to the degree of strength in the throw or the number of times the die will turn, etc., complete ignorance as to how the preceding throw may perhaps influence the next one, and we must, therefore, in our calculation

treat every throw as independent of the others, though we must suppose that as a matter of fact it is not independent. As we do not know how the position, say, of the die, in which it is taken up from the preceding throw, will coincide with other variations in the conditions of the movement, there remains, so far as our knowledge extends, the equal possibility of all six numbers for each throw; and accordingly in each particular case we determine the probability of each number as $\frac{1}{6}$.

Those assumptions which we are justified in making as to a variation in causes which passes gradually through all values, first become important when we are no longer determining the probability of a single throw, but are directing our conjectures to what will happen in a long series of throws; we then infer that the causes vary in such a way that in a long series each number will occur the same number of times, that the numbers of the actual events will, therefore, be in proportion to the fractions of the probability. If this were not so, the assumption would fall to the ground; if, in a large number of throws, 6 should occur considerably more often than $\frac{1}{6}$ of the whole number, we should be justified in assuming that there was something in the construction of the die which favoured the throw 6, and we should be obliged, even so far as concerned one particular throw, to drop the assumption that all the terms of the disjunction were equal.

But we are justified in applying the methods of the theory of probabilities not only in those cases where we are able to make and believe assumptions as to a uniform variability of causes such that, given a large number of instances, all the terms of the disjunction will be realized in equal proportion; the methods apply wherever a disjunction with equivalent terms can be established, and we have no ground to hold one to be real rather than the others; they apply also when some of the terms of the disjunction are actually impossible, or where only one would be possible in reality—as is indeed always the case when we confine ourselves to a single instance. The assumptions, if justified, give a more definite meaning to our estimates of probabilities, in so far as they are supported by at least an approximate knowledge of the causes which are likely to bring about the particular terms of the disjunction; but they cannot be made the ground of an essential distinction. The fraction representing the probability in the strict sense can state only the amount of subjective expectation which is justified, which the amount of our knowledge permits us to entertain; it always depends upon assumptions which

we cannot strictly verify, and for this reason it cannot in any way prescribe to reality the course which it must take.

Thus even the less probable still remains possible, and a high degree of probability is yet far from certainty; so much only is certain, that one of the disjunct cases will occur; which of them it will be depends upon just those circumstances which we do not know and which no logical considerations will prevent from declaring themselves in favour of the less probable case. The man who buys 500 out of 1,000 chances in a lottery will count upon winning with greater probability than the man who buys one; but should the latter win, there is no difficulty to be overcome because of the greater probability of another issue to the event, one of the 1,000 equally possible cases has occurred regardless of the expectations which the players were justified in entertaining. The possessor of 500 chances could not hope to win with more than one amongst them, and it is just as improbable that any given one amongst them should win as that the single one belonging to the other man should win.

9. It follows from the whole character of the calculation of chances that it is a matter of indifference so far as concerns the principles of the process whether the probability to be determined is that of a future event, or of a past event, or of a proposition of permanent validity. In the games of chance which generally serve for illustrations the expectation of a future event no doubt predominates; but the characteristic feature is that we are limited to a disjunction of possibilities, concerning the reality or necessity of which we know nothing. We may expect in just the same way confirmation of our conjectures as to a past event; our knowledge and ignorance may be related to the past or present exactly as it is to the future. So long as I have not seen the list of winners my expectation that I have won with a number which was drawn yesterday has exactly the same value logically as my expectation that I shall win in a drawing to take place to-morrow.

Inasmuch as the whole process is based upon the fact that a general concept is established while its determinations are uncertain, every statement of probability concerning the particular depends ultimately upon a hypothetical judgment of universal validity. The probability of throwing 6 with a given die is $\frac{1}{6}$, only because we know that any body of the shape of a die will afford equal expectation for each of its six sides so long as we know nothing of the conditions of its position. Every statement of probability for a particular case can be represented as the consequence of a

general proposition ; for it is only upon the ground of general propositions that we can obtain the fundamental disjunction. Here again we see the deductive character of the whole process.

10. The art of correctly manipulating the deductions belonging to the calculation of chances depends less upon the simple application of the principles according to which the disjunctive judgments are reduced to numbers than upon the establishment of correct disjunctive judgments from the conditions known to us,—judgments, that is, in which the number of disjunctive terms corresponds to the number of originally different and equally possible cases. When 5 coins are thrown together, a superficial observation seems to show only 6 different cases as possible, viz., 5 heads, 4 heads 1 tail, 3 heads 2 tails, 2 heads 3 tails, 1 head 4 tails, 5 tails. But when, constructing the original disjunctions, we find that the coin *A* may give either *Ah* or *At*, *B* may give either *Bh* or *Bt*, etc., we learn by actually combining these five disjunctions that the second case (4 heads 1 tail), as well as the fifth, is the result common to 5 originally different cases (for the one coin which shows tails may be either *A* or *B* or *C* or *D* or *E*), while the third and fourth are the result of 10 cases. In this way the probabilities for the 6 cases take the value, not of $\frac{1}{6}$ each, but of $\frac{1}{32}$, $\frac{5}{32}$, $\frac{10}{32}$, $\frac{10}{32}$, $\frac{5}{32}$, $\frac{1}{32}$ (*i.e.* they proceed as the binomial co-efficients for the fifth power).

It is chiefly for the correct statement of these disjunctions in more complicated cases, and for the calculation of the number of combinations of the same elements, which appear in different order, that the help of mathematical methods is necessary. Their fundamental principles follow from the results obtained by the combination of disjunctive judgments, and it is no part of our task to carry them out in detail ; it is enough to have explained the relation of this branch of deductive method to the deductive process in general.

11. To this form of deduction also there is a corresponding form of the reduction of given propositions to the premises from which they may have proceeded.

In the first place, every disjunctive judgment of the form *A* is either *ax* or *bx* or *ay* or *by*, or of the form *AB* is either *ax* or *bx* or *ay* or *by*, refers us back to the components from the combination of which it may have proceeded ; the only question to be asked is whether the terms which are given together represent a complete combination of simple elements.

In the first case this reference gives at once A either a or b , either x or y : in the second case we have the double possibility

A is either a or b , B is either x or y ;
or : B is either a or b , A is either x or y .

Given one of these premises, the other would be found by a sort of algebraical division.

But it is not in such cases as these that reduction is generally employed in connection with the calculation of chances ; it is employed when the numerical relations, in which the disjunct possibilities of a given presupposition actually occur, give rise to the problem of constructing the fundamental general disjunctive judgment of which they may be regarded as instances—in other words, when we are passing from an *empirical* to a *logical* division.

When a black ball is drawn from an urn at the first time of drawing, absolutely nothing can be inferred from the fact but that a black ball was there ; whether the drawing of the black ball was the only possibility, or whether other possibilities may be assumed, we have no grounds whatever for conjecturing.

But if, the second time, a white ball is drawn, we should already be justified in forming as a conjecture the disjunctive judgment that either a black or a white ball will be drawn ; but whether the disjunctive judgment is correct in the sense that it enumerates all possibilities, that the urn therefore contains balls of no other colour, and whether it is an accurate statement of different *equivalent* cases remains uncertain.

If repeated trials should fail to give a ball of other colour than black or white, the conjecture that either a black or white ball must be drawn because none of a third colour was there would gain very rapidly in certainty ; and if a large number of trials should give an equal number of black and white balls, we should at once conjecture that there was an equal number of the two colours in the urn, although the same result would not in itself be absolutely impossible if there should be only one black and one hundred white.

The principle upon which these two conjectures are based may be formulated as follows : that assumption is always the most probable upon which the result which is actually there most probably follows.

If, for example (referring to our first conjecture), there were in the urn one black, one white, and one red ball, then the probability that in six trials only black and white balls would be drawn is only $\frac{9}{125}$; but if there

are in the urn only a black and a white ball, then it is necessary that only black and white should appear. Thus the latter assumption is incomparably more probable than the former.

If, again, there should be one white and three black balls in the urn, then a simple calculation will show that, given four trials, the probabilities of the only possible cases are as follows :

4 white, $\frac{1}{256}$; 3 white, 1 black, $\frac{3}{256}$; 2 white, 2 black, $\frac{6}{256}$; 1 white, 3 black, $\frac{10}{256}$; 4 black, $\frac{8}{256}$.

If, on the other hand, there were two white and two black balls, then we should get for—

4 white, $\frac{1}{16}$; 3 white, 1 black, $\frac{4}{16}$; 2 white, 2 black, $\frac{6}{16}$; 1 white, 3 black, $\frac{4}{16}$; 4 black, $\frac{1}{16}$.

In the second case, therefore, the probability that an equal number of black and white balls will be drawn is $\frac{6}{16}$ or $\frac{3}{8}$, which is considerably greater—nearly twice as great—as in the first case, where it is $\frac{6}{256}$; the second assumption is proportionally more probable than the first, and it is in a still higher degree preferable to one assuming a still more unequal relation, which, when deductively developed, would give a still smaller probability of equality.¹

It is sufficient here to indicate the main features of this process of reduction; the theory would proceed to show that the greater the number of trials resulting in only black and white balls, the more rapidly the probability would increase that there were only black and white balls in the urn, and that the two colours were there in the same or nearly the same proportion as they appeared when drawn, and that a very great number of trials would be a ground for very probable conjectures concerning the fundamental assumptions to be made.

The application of this reduction to problems of inductive investigation will occupy us later on; here we have merely to show that the probability calculated in this way of disjunctive hypotheses has the same character of subjective probability as that derived immediately from dis-

¹ It is due to the same process of reduction that we refuse to believe that all is straightforward if, e.g., each of the players in a game of whist holds cards all of one colour. Apart from the fact that, in consequence of the way in which the cards are shuffled and dealt, this result is certainly still more improbable than would follow from the mere calculation of possible combinations, our suspicion rests upon a comparison of two assumptions, from one of which the result would be very unlikely to follow, while from the other it would be certain. The second assumption is that of intentional arrangement of the cards. According to the general principles of reduction, this hypothesis is by far the more probable of the two.

junctions, and that this purely subjective character is modified only so far as we know from other sources of laws affecting the question—such as laws concerning the uniform change of the conditions giving rise to the different cases.¹

In § 48 it was shown that immediately certain judgments which are not statements of self-consciousness can only refer to our ideas as such, not to anything having particular existence; thus the application of strict deduction is limited to those judgments in which we deal only with our own activities, which are governed by inner necessity, and it extends so far as we become conscious of this necessity in axioms. It is limited, therefore, to logic, arithmetic, geometry, chronometry (if the term may be allowed in this sense), and the purely constructive mathematical doctrine of motion; but as soon as our statements refer to anything existent which lies beyond our own self-consciousness and the sphere of the constructive formation of concepts, our certain consciousness of necessity disappears, and the methods which determine the construction of our judgments are altered. For every judgment that something is, is valid only when we have general rules according to which a subjective something indicates an existing something; the question is, how to find these rules, and in what way to become certain of them.

The sphere of ends, and the judgments referring to them that something ought to be, seem, on the other hand, to admit of a purely deductive development. Here the validity of the propositions that something ought to be is grounded ultimately in ourselves, in the consciousness of our volition and of its necessity; and there may be ultimate ethical axioms which, like logical axioms, are accompanied by the feeling of evident cer-

¹ The attempt made by John Venn, in his otherwise instructive and clever book, *The Logic of Chance* (London, 1876), to find a purely empirical basis for the principle of the theory of probability, makes it especially important to point out that, according to the principles of the theory itself, it is only by chance, even with large numbers, that we can clearly know the original disjunctive judgments and the probabilities to which they lead. If, for instance, we calculate the probability that in 200 throws a coin will fall exactly 100 times heads and 100 times tails, it is somewhat smaller than the probability of the proximate cases taken together; hence an inference from actual experiment would give in the majority of cases, not $\frac{1}{2}$ for heads and $\frac{1}{2}$ for tails, but only fractions which approximate to $\frac{1}{2}$. Thus the fundamental relation can never be deduced with certainty directly from the observed numbers, but only by way of the reduction we have described, which is based upon the subjective view of probability.

tainty. The principle, *e.g.*, which is parallel to the logical principle of contradiction, that we ought not to will what is contradictory, *i.e.*, that all the ends of our will must harmonize, is undoubtedly such an axiom. Nevertheless, ethics is not adapted to purely deductive science ; its minor propositions cannot be obtained either from mere concepts or from subjective necessity. In so far as it is to be a practical science, and its Ought is to pass into Is, the spheres within which it is applied are of a real nature, and the validity of the minor propositions, by means of which the deduction advances, is of the same kind as the validity of propositions concerning the existent. Theoretical and practical science, which refer as knowing and willing to the existent, are so far different that in the former the conclusions are given to which the premises are to be sought, while in the latter the major premises are established and the certainty of the minor propositions still remains to be found ; they are so far alike that neither admits of a purely deductive treatment by means of axioms and definitions. Here again we see the middle position occupied by final concepts between constructive and empirically abstracted concepts.

Because our judgments that something is always start originally from the immediate self-consciousness in which some affection is affirmed which we refer to reality, and because the existent which thus becomes the object of our belief is always some definite particular, therefore our investigation of the methods by which we arrive at knowledge of the particular must start from these fundamental acts, *i.e.*, from judgments of perception.

One general consideration remains. If we assume that the most valuable judgments are those which attribute to a subject a predicate of the greatest possible specialization, which do not merely subordinate it to a wide general concept, but also state that special determination of the concept which belongs to the subject ; if we further assume them to be those which do not merely state the general kind of consequence which follows from certain conditions, but include also its specialization within the narrowest possible limits (*cf.* § 55, 1, I., p. 357) ; then it follows that the most valuable major premises for deductions are those which enable us to find for every variation in the subjects or assumptions falling under them an appropriate variation in the predicates or consequences resulting from the major premise.

The proposition that in a right-angled triangle the larger side is opposite the larger angle is obviously less valuable as a major premise than the proposition that every cathetus is the sine of the opposite angle multiplied

into the hypotenuse. In place of a proposition which contains both in the condition and its consequence the indefinite term "greater," including indifferently an indefinite number of more special determinations, we get another which accurately determines for every variation in the magnitude of the angle the appropriate variation in the magnitude of the side.

Such propositions are possible where its specializations follow of themselves from the general concept as its natural development, and where one and the same relation is possible between the terms of the two series in which the connected concepts are unfolded; they are possible, therefore, wherever we are dealing with numbers or with relations of measurement between magnitudes of any kind.

The expression "formula" is generally used in mathematics to denote those hypothetical major premises which state for every value of the specializations falling under the first general concept the appropriate value of the specialization falling under the other. Their simple expression is made possible by the general symbols which express the whole series of successive values of a number, and what they generally affirm is the equality of the value of each term in a series to another value which stands in a definite arithmetical relation to the appropriate value of the other series. The general concept upon which the formula is grounded is not the concept of equality, but that of this constant relation; this relation is the general expression of the connection to which all particular instances are subject, or, to use the traditional expression, it is the concept of the given function which one variable is of the other.

The simplest relation between variations in magnitude belonging to the special determinations of a concept *A*, and variations in magnitude belonging to those of another concept *B*, is when to equal variations in *A* there correspond equal variations in the appropriate *B*; that is, the relation of proportionality. The constant element in all the various instances, the concept connecting them, is here the constant quotient between the numbers which measure the one and the other magnitude.

When both magnitudes are measured by the same measure, as in the case of the periphery of the circle and its diameter, then this quotient contains also the relation of the differences by which the one magnitude increases to the differences by which the other increases; the formula for the circumference $p = d\pi$ or $\frac{p}{d} = \pi$ contains in the product, or in the quotient, the general concept of proportionality, but in the constant π it

contains the fixed relation between the increase of the one magnitude and the increase of the other.

When two magnitudes are measured by a different measure as space and time are in motion, then the constant states the number of units by which the one magnitude increases when the other is increased by unity ; in the formula for a uniform motion $s=vt$ the constant v contains the number of spatial units which are traversed in one unit of time.

The importance of such major propositions, which enable us to develop for every variation in the subject the appropriate variation in the predicate, and for every variation in the condition the appropriate variation in its consequence, will be especially evident when our object is to derive from perceived phenomena universal judgments expressing the laws of these phenomena. For the name of law in its strictest and fullest sense is generally given only to those universal judgments which exclude every element of indefiniteness from the predicate or consequence, and which therefore possess the logical character of a mathematical formula, applying with equal strictness to every value of the variables, even though we should imagine these to vary, as in the differential calculus, by the smallest possible increase.

CHAPTER IV.

THE METHODICAL PRINCIPLES OF CONSTRUCTING JUDGMENTS OF PERCEPTION.

§ 86.

THE ideal at which thought aims demands that our judgments of perception shall be so constructed as to ensure objective validity and exhaustive accuracy, and this both with reference to the description of particular things and events, and with reference to their determination in time and space.

The methodical construction of judgments of perception, which has its first starting-point in the unpremeditated reference of our sensations to particular things in space, completes itself in the objectively valid determination of single subjects, to which sensuous qualities and spatial and temporal relations are attributed as predicates.

1. The general characteristic of our JUDGMENTS OF PERCEPTION is, that by them the subjective affection given in sensation, which in the first instance is the object of immediate self-consciousness, is referred to an existing object. We have shown in § 47 that for this reference to have objective validity we need general laws according to which it may be carried out, according to which more especially we may assume definite objects corresponding to the sensations, and may ascribe to these objects their place in time and position in space; only upon this condition can the judgment of perception, which always starts from a purely subjective fact, claim to have objective validity. In methodology we have therefore to inquire how we may find such universal principles, and what logical character they possess.

But there are still further conditions to be satisfied by our judgments of perception. If the universe which we are endeavouring to know manifests itself only through perception, then in the interests of knowledge it is essential that the judgments in which we give utterance to our perceptions should express the content of sensation as completely and accurately as

possible, and that nothing which is contained in our subjective consciousness should be lost in formulating the judgment. Every difference of sensation must find expression in the conceptual predicate, which must be free from every kind of indefiniteness. We are not here concerned with the subsumption of a given fact under a general concept, but on the contrary with the specialization of the predicate to such a degree that there is no longer any difference between it and its subject. Ancient and modern logic differ perhaps in no point more clearly than in the fact that logic of old was satisfied with the subsumption of every Given under a general concept in which many small differences were disregarded; while, on the other hand, logic as actually practised in modern times, though not yet completely confirmed in theory, insists upon this general concept being specialized with complete definiteness, so as to be the full expression of individual difference. Our investigations in chapter I. showed that this complete accuracy of predication is possible where the predicates are reduced by their conceptual construction to a continuum which can be mathematically represented, and can therefore be expressed by measurement; and that this possibility extends as far as our power of perceiving small differences and subjecting them to measurement.

2. The ends at which we aim in thought demand not only that our expression of particular perceptions should be exhaustively accurate, but also that we should endeavour to make these perceptions extensively complete. This means that from the beginning the particular should be regarded as a part of the whole and in its perceptible relations to the whole. Hence follows also the necessity of determining for every judgment of perception which is to be complete the position occupied by the perceived in time as well as in space; and the question arises as to the possibility of objectively valid time and space-determinations. According to § 46, 5 (I., p. 305) even subjective statements about the facts of sensation as given in self-consciousness are not complete until the time-determination involved in them has been objectively fixed; a further question then presents itself as to the relation of the time-determinations of objects and their changes to the time of subjective sensation. Space-determination, on the other hand, demands that every object of perception shall be ranged in the one space which embraces all the existent, its position being thus determined with reference to all other objects.

3. Here again in perfecting our method we must start from activities which are being carried on without premeditation, and we cannot at the

commencement free ourselves from assumptions which have their origin according to psychological laws. The conscious construction of judgments of perception is always preceded by the habit of referring sensations to particular things which are in definite places and which persist in time ; it first appears as the description of particular things as given in perception, when for the indefinite judgment of denomination (expressed in one general predicate) is substituted the analysis of the object into its particular simple attributes, activities and relations, leading to the reconstruction of the idea from elements which have been apprehended with the greatest possible definiteness.

When the description starts from unities which, owing to some limitation, offer themselves as subjects for judgments of perception, then further reflection constrains us to define more accurately the concept of *thing* itself, and to inquire about the methods which will enable us to find an accurate determination for the subjects of which we predicate sensuous qualities and spatial and temporal relations.

4. The questions of method which arise here remain unaffected by our final conclusions as to whether what we perceive are ultimately mere phenomena, or real things existing independently of our thought. Even in reference to phenomena there can only be objective and universally valid propositions if there are definite laws according to which the affection of the individual can be transformed into a judgment which is objective and valid for all. We in no way, therefore, prejudice the question as to realism or idealism when we start from the familiar assumption that real things exist in an objective space and in objective time ; and all we are concerned with is, how we may construct from the manner in which they appear to the individual a judgment concerning their objective nature and their real relations, which shall be valid for all.

§ 87.

It is necessary, in order that even our purely subjective statements about what is contained in our consciousness may be fully determined, that our own subjective time should be referred to a TIME-SYSTEM which is common to all, and so far objective.

Because the correspondence of one individual consciousness with another is only possible by means of external sensation, this must depend upon external perceptions which are shared by all, and which occur

simultaneously for all. Recurrence of these forms the ground for a division of time into periods which are the same for every one.

The complete determination of our statements of time necessitates a STANDARD OF TIME. Owing to the uncertainty of the subjective estimation of time this can never be reached directly, but only by the aid of hypothetical assumptions concerning the laws of motion.

The use of these standards of time for the temporal determination of subjective events, and still more for the determination of the objective events of which they are perceptions, necessitates the determination of the relation between the time of the event and the time of the perception, and the reduction of the subjective series of perceptions to the objective series of changes in things.

This reduction can only be carried out by assuming causal laws, in which the sensation is regarded as a temporally determined effect of objective change; but to establish such laws is possible again only by assuming that in the case of continuous events the succession of sensations corresponds to the succession of the events.

From this point of view Kant's doctrine that an objective time-determination is only possible by means of a principle of causality, according to which objective events prescribe to themselves their place in time, is, as stated in his formula, inaccurate.

1. If we inquire first about the time-determination of our narrative judgments, in so far as they are the purely subjective statements of immediate self-consciousness (§ 46, 5. I., p. 305), we find that they presuppose the reference of our subjective discrimination of definite points of time and their intervals to a time which is uniformly valid for all individuals; they necessitate therefore the reduction of the time of the individual to a time which is the possession of all, and the elimination, therefore, of subjective differences in the estimation of time.

Though the idea of the chains of inner events and their continuous succession in time, by virtue of which they form a series in which every conscious act has its place between others, is familiar to every individual consciousness; and though we easily distinguish on the whole longer and shorter durations—at any rate in the immediate proximity of our Now—yet this idea, in its immediate connection with our momentary consciousness, cannot be raised into one which is objective in all its parts; that is, in the first place, valid for all conscious and thinking subjects in the same way, because necessary. We assume, indeed, that there is one time for

all, in which we and every one are and live, and in which, as in a frame, the conscious moments of all are ranged in a certain order of simultaneity and sequence, one time which is reflected in the time-consciousness of each person ; but how shall we bring about a relation and comparison between the time idea of one and that of another ?

2. For our purely subjective time-consciousness the starting point is always the immediate Now, with which memory connects the nearest Past with tolerable confidence as to the sequence and duration of its particular moments, and the more remote with diminishing certainty with reference both to the Before and After, and to the particular intervals. But this Now is not a fixed point ; it advances incessantly in time, and as the head of a comet is followed by a constantly changing tail, so this moving Now is followed by a chain of memories which is always changing its form.

If we are to bring about a community of the time-consciousness, we must begin by establishing fixed points which are common to all, by reducing the Now of one man to comparison and coincidence with the Now of others, and by determining a common measure for the intervals between the past and the common present.

3. Such points of coincidence can only be found where we are certain that different people are simultaneously conscious of the same fact, and where, therefore, the content of the one consciousness can be compared as to time with the content of the other ; and since the reference of the conscious content of one person to that of another is only possible through the external world, this must be where a phenomenon which is external for both is simultaneously perceived. But a single coincidence would leave the series Before and After without points of comparison ; a progressive comparison can only be carried out by the repetition of coincidences which enable us to continue to compare the subjective time-idea of the one with that of the other.

Thus we see that periodically recurring external phenomena, which are shared in by all alike, form the first basis of a common time consciousness, and most important amongst these are the alternation of day and night, the rising and setting of the sun. From them we get a division of time into enumerable sections, in which the points of division are the same for every one ; and the most obvious and natural application of this division would be, reckoning backwards from the to-day of every one, to determine the days and nights in which other remembered experiences (whether of the individual or of the community) must be located, which

will serve as further points of comparison. The endless succession of such marks both backwards and forwards prevents there being any natural starting point except the present. But to-day is always passing away, and is, therefore, ill adapted to be the starting point for a fixed and uniform reckoning; for this reason, in reckoning forwards we start from some arbitrarily selected point which is determined by some memorable event. The primary division of time, the day, is next followed by the month; the reappearance of the new moon enables us to mark off a definite (though varying) number of days into a period; while the year, owing to the gradual change of the seasons, forms a less easily recognised period, and was late in becoming an accurately determined measure of time.

4. This division of time according to the popular calendar is sufficient to bring about uniformity amongst the time-ideas of individuals, at any rate with reference to the limits of considerable periods; and it needs no further assumption than that the rising and setting of the sun are perceived simultaneously, and that the days are counted in the same way by all. But because there is no certain and natural division of the day it is not sufficient to the further task of affording a fixed and universally valid measure of time, which would enable us on the one hand to compare the durations of particular intervals according to one standard, on the other hand to determine particular points of time at any intervals, however small, as parts of the day; and it is here that the difficulties of chronometry begin.

5. Daily experience teaches us that the purely subjective measurement of time is varying and deceptive; we are unable to form any judgment which is certain and coincides with that of others as to the equality of time-periods if they pass beyond a very small limit in size and are not given in immediate repetition. Nevertheless, if we are to determine objectively the relations of duration between different states of consciousness, and to determine every moment in one and the same time, we must have an absolutely fixed measure and an absolutely fixed point from which to measure; and these are not given in the merely popular division of time into days. The equality of days cannot be in any way certainly and immediately recognised—indeed, the solar days are actually unequal—nor would equality in the days give us that possibility of division which is needed for an available measure.

A real standard of time could only be given by the perfectly uniform motion of a perceptible body, which would enable us to measure times

which could not be directly compared by means of the spaces traversed, which spaces we could bring into direct coincidence by a measure. But whether a motion is uniform or not can never be empirically known with certainty, because there is no immediate and reliable certainty in our equation of subjectively estimated successive time-magnitudes.

If we define a uniform motion as one in which equal spaces are traversed in equal times, then in order to know it as such we must suppose that we are able to measure directly the equality of two periods of time.¹ Thus all the means which we can employ for measuring time depend upon assumptions which can never be strictly proved; they depend ultimately upon the assumption that the rotation of the earth is constant in velocity, and that the time between one culmination of a fixed star and another is, therefore, always the same; and, again, upon the assumption that the oscillations of the pendulums regulating our clocks are isochronous. It may be urged that it is strictly proved that oscillations of the same pendulum within fixed limits are isochronous, but this proof depends partly upon the Galilean law of inertia, which, again, is in no way empirically demonstrable if only because it contains the concept of uniform motion. Our conviction that in our clocks and our astronomical observations we really measure equal times depends ultimately upon nothing more than the agreement of those motions which various physical principles lead us to expect with those motions which we actually observe.

It is instructive to follow the course actually taken by science in this connection. External perception first leads us to think of uniform motion, when, *e.g.*, the motion of the stars appears to our rude, subjective estimation as uniform; this thought is elaborated to mathematical accuracy, and the assumption is made that the stars in their daily revolution move with absolute uniformity. But in proportion as the standard based upon this assumption is generally applied, and laws concerning the times of motions are established, the realization of which is estimated by this standard, there appear slight divergences; and at the present time we actually do not know whether the rotation of the earth is constant in velocity or not, and whether there is any real motion to be found corresponding to the mathematical ideal.

Thus upon closer consideration of even the first problem, that of

¹ Cf. C. Neumann, *über die Principien der Galilei-Newton'schen Theorie*, 1869, p. 14. Liebmann, *zur Analysis der Wirklichkeit*, ed. 2, p. 86 sq.

establishing an objectively valid temporal determination of judgments concerning our immediate inner experiences, we find that at best it can only be approximately solved. We start from the thought of one time in which everything happens, and our ideal is the division of this time into absolutely equal intervals; but we can only succeed in marking off intervals of this time in an objectively valid way by means of external perception of periodical changes or of successive positions of a body in motion, and our conviction that these are equal depends upon a number of assumptions and presuppositions with reference to physical laws of motion for the validity of which no strict proof is possible, and which we only assume because the facts of our sensation can be explained by them with the highest degree of probability.

How to construct the best clocks is a technical matter, and the methods of regulating them also aim only at determining with the highest degree of probability the relation of their time and the absolute time of our assumption. But the fact that an objectively valid measurement of time is possible only by means of external marks, and at best approximately, opens up new questions for our consideration.

6. In the first place the use of these expedients in the measurement of time assumes that the objective event which marks any point of time is in fact perceived absolutely simultaneously by every one, and that there is no difference between the intervals of subjective perception of periodical events and the events themselves. Then the use of these external expedients reduces the ranging of any conscious moment in the objective time series to the problem of framing with objective validity a judgment concerning the simultaneity of two events, or of determining whether some affection of the Self appeared before or after the perception of an objective event—the peal of a bell, or the movement of the second-hand from one mark to another.

7. Now the succession of our conscious moments as such certainly seems to be given with such immediate self-evidence, that we are immediately certain about the Before and After, about the unmistakable succession of an inner event *B* to an inner event *A*; and we seem forced to accept our perception as laid down in the judgment that *for us B* follows *A*, as the simple expression of an actual subjective fact. No one doubts the validity of his own judgment that he saw the flash of a cannon at a distance before he heard the report, or mistrusts a similar statement on the part of another; no one thinks himself mistaken in hearing the

first words of a sentence before the last ; the affirmation of this succession is vouched for by the immediate truth of those statements of self-consciousness which refer to actual existence. And yet the events of which we have to determine the succession need only be crowded close together to make our judgment uncertain. The activities belonging to the successive sense perceptions, and to their comparison in judgment, which must take place together, disturb and confuse each other ; especially when the perceptions concerning the simultaneity or succession of which we have to judge belong to the provinces of different senses, or lie within different series.

If that which is the object of consciousness consisted of a simple series of subjective events, and if our apprehension of them, by which we comprehend them into one temporal whole, went on apart from and without obstructing them, like the activity of a spectator who watches a play without having any effect upon it, then the immediate apprehension of every inner event would also assign to it its place in time. But the problem is more complicated than this. Our inner life does not progress in this simple line ; several distinguishable series proceed simultaneously ; we do not cease to see while we hear, and hearing continues while we consciously endeavour to compare the seen and the heard. The uncertainty which we sometimes feel gives rise to the question whether our judgment in comparison is capable of recognising unmistakably even the simultaneity of every moment in one series with every moment in the other, and of referring both in the correct order to the one line of time. This is especially the case when the moments to be compared are themselves composite acts, as, *e.g.*, the perception of the coincidence of two spatial objects, of the second hand of a clock with a given mark, of the image of the star in the telescope with the thread.

We will pass over the psychological questions arising out of the relation between an intermittent conscious comparison, which, it may be, can only grasp events after they have already happened, and the subjective object of comparison (the various perceptions) ; questions which, upon closer investigation, threaten to limit even the proposition as to the immediate certainty of our statements about ourselves. For our purpose they are fruitless. If, in fact, no certain judgment were forthcoming as to whether *A* preceded *B* or were simultaneous with it, then this uncertainty itself would be the ultimate psychological fact upon which we could build nothing further ; if our judgment decides that *A* preceded *B*, it will help

us nothing to call its correctness into question hereafter, we must accept it as the momentary fact, and assume that *A* really came into consciousness before *B*, although we may have expected something else.

This difficulty is primarily of interest not because we are investigating the relation between our conscious comparison and our purely inner states, but because we are trying to obtain from the simultaneity or succession of subjective events a knowledge of the simultaneity or succession of an objective occurrence; and the whole of this difficult question would hardly have arisen for psychology unless there had appeared in the estimation of the temporal relations of objective events discrepancies between different observers, and discrepancies between results obtained by them and conclusions which we are forced to accept upon other grounds. The so-called personal equation of astronomers has called attention to this point; it happened that the statements of different observers as to the time in which a star crossed the intersecting thread in the telescope were at variance, closer investigation showed that variation occurred even with one and the same observer, which proves that there is no fixed relation between the subjectively apprehended succession of perceptions and the succession of perceived objective events.

8. This leads first to the more general question as to how we can pass from the subjective succession of our sensations of external events to objectively valid judgments concerning the succession of these events themselves.

The assumption with which our unschooled perception starts in referring sensations to external things is, that something external is and happens in the same moment in which we perceive it; hence that the succession of events coincides point for point with the succession of our perceptions. In the moment in which we see it, what we see is such as we see it, and is in the place in which we see it; that which we hear sounds at the moment in which we hear it, and what we feel touches us at the moment in which we feel it.

But the observation of daily life is itself sufficient to break down this assumption by demonstrating its contradictions. If we stand beside a cannon, we see the flash and hear the sound in immediate succession; if we stand some way off, the sight and the sound are separated for us by an interval. Or other results are contradicted: the eclipse of the satellites of Jupiter appears sometimes before, sometimes after it is due, according to the calculation of their motion from general conditions.

Recognising that sound and light are gradually propagated, we are thus forced to make a corresponding deduction in respect to what we immediately perceive, and when we further take into consideration our observations as to the velocity of transference by the nerves we find that at all points there must be a discrepancy between the times of perception and the times of the perceived event, and that it is impossible, without further expedients, to bring into a time series with objective validity, not merely the subjective sensations, but also the events to which they correspond, and especially to determine their relation to the events by which we measure time.

9. * If we consider the way in which these subjective discrepancies are compensated, and a reduction of the time of perception to the time of occurrence accomplished, we might at first sight find in it a confirmation of the Kantian proposition that an objective time-determination is possible only through the concept of causality, and its connected principle that everything which happens presupposes something upon which it follows according to rule,¹ the mere order of perceptions having only a subjective significance and warranting no objective judgment that *B* succeeds *A* in time.

Every reduction employed in compensating subjective discrepancies in time-results aims first at establishing a causal relation between the objective event and the subjective sensation, the former being regarded as the cause of the latter ; an accurate determination of the way in which the objective cause brings about the subjective sensation includes the difference in time which intervenes—owing to the transmission of light and sound through media, and of the peripheral impression through the nerves—between the event and conscious sensation, and it shows how the varying magnitude of this discrepancy depends upon varying distance and upon individual temperament. It is due to these causal relations that we are able, when the distance is known, to calculate from the point of time in which a shot is heard back to the point of time in which it was fired ; and it is due also to these causal relations that we are able, from the interval between the perception of the flash and the hearing of the report, to determine, with at any rate approximate accuracy, both the distance of the cannon and the true time of the event. And if we ask, further, how the personal equation of astronomers is compensated and made capable of

¹ As stated in the first edition ; the second differs by formulating the principle : all changes occur according to the law of connection between cause and effect.

reduction, we find that it is done by mechanical contrivances which produce a report at the moment in which a visible mark reaches a given point, and so enable us to compare the subjective apprehension with this objective fact, warranted by mechanical laws. The causal connections in the mechanism of apparatus give us exact time-relations for the visible and audible event; the latter *cannot* precede the former, because its production depends upon the motion of the indicator to a given point. We may illustrate the same fact by a more familiar instance: when a spectator watches a battalion exercising from a distance he sees the men suddenly moving in concert before he hears the word of command or bugle call, but from his knowledge of causal connections he is aware that the movements are the result of the command, hence that objectively the latter must have preceded the former, and he is at once conscious of the illusion involved in the reversion of the temporal series in his perceptions.

It would seem, then, that an objectively valid temporal determination is only possible where there is a relation, by virtue of which one event as cause determines for another as effect its position in time, by introducing it of necessity and according to rule in temporal sequence; only in so far as I know that *A* is the cause of *B* can I say with certainty that *A* precedes *B*, whether in my subjective sensation *A* precedes *B* or is simultaneous with, or succeeds it. Only because I know that my sensation is the effect—mediated by various intervening facts—of a change in the object, must the lightning flash from cloud to cloud before I see it, and the bell be struck before I hear it.

10. But as against this view it is just as obvious that I can only discover concrete causal connections and causal laws by perceiving that *A* regularly precedes *B*; all our knowledge of causal relations in the real world may be ultimately reduced to the perception of temporally connected changes, in which the activity of the cause precedes the change effected in the object upon which it acts. No one would think of connecting lightning and thunder as cause and effect unless it were a regular experience for lightning to precede thunder; our belief in the simplest as in the most complicated causal connections depends ultimately upon the fact that an objective succession of *A* and *B* was established *before* we could gain the conviction that *A* is the cause of *B*. Thus we are confronted by an antinomy: before we can establish a causal connection, a rule according to which *B* follows *A*, we must be able to affirm with objective validity that

B has followed *A* ; but before we can affirm this with objective validity we must have recognised a causal connection between *A* and *B*.

11. How can we escape from the contradiction ? Where shall we find certain principles to guide us in establishing objective relations ? Perhaps a more careful examination of the Kantian view may throw light upon the question.

In the first place, Kant evidently goes too far in affirming that the perception of a manifold is always successive, and that accordingly, in so far as it is a question merely of subjective apprehension, there is no distinction between the perception of the co-existent parts of an object and the perception of successive events. It is not correct to say that the perception of the parts of a house is successive in the same sense as the perception of the different positions of a boat gliding down the stream. No spatial apprehension of an extended object would be conceivable at all unless a manifold of distinguishable parts were seen in the strictest sense *together* ; even if we should limit this seeing to the place of clearest sight, to the centre of the field of vision, even here a *surface* is seen, and therefore a plurality of the distinguishable parts of this surface perceived together. Moreover, as even in the less central parts of the field of vision movements of objects easily catch the attention, we may, when we glance over the particular parts of a house, fixing one window after another, be quite certain in our judgment that the whole surface is at rest ; those parts which, by the movement of the eye, fall to the side of the field of vision, do not vanish from perception, and succession is really only to be found in the differences of clear and less clear vision, which we are led by the motor sensation of our eye to refer to a movement on our part and not to a change of the object ; more accurately, it is to be found in the particular acts in which we make successive judgments about parts of the house which are seen simultaneously, recognising them as window, door, etc. Thus when we regard an object at rest we perceive the persisting unchanging position of its parts, and our subjective consciousness of the passage of time finds its only expression in the judgment that the object of our perception is unchanging and persistent, hence that it affords no occasion for distinguishing *in it* a Before or After, because there is no spatial difference between the later and earlier content of perception. But it is different when a change in the position of the parts of an object takes place. When—to take Kant's example—we see a boat gliding down the stream, our image is made up of an environment at rest and an object

moving within it ; we have immediate consciousness of the continuous succession of varying positions of the boat in the total image, of its increasing distance from one object and increasing approximation to others. The subsequent perception does not merely differ from that which precedes, as would be the case if the perception were of another object, but it is partly the same, partly different ; and for this reason the succession is now apprehended as one going on in the object of perception. Kant's proposition, " Every apprehension of an occurrence is a perception which follows upon another," is incorrect, because it does not state that where we perceive an occurrence, an event, a change, the content of the subsequent perception differs from that of the preceding in the spatial configuration or in the quality and intensity of the sensation of the *same objects* ; when I turn myself round 180° in a landscape, then the perception of the west horizon follows that of the east, but I apprehend no event because the second perception is entirely new and shows no connection with the first. Kant should have said : Every apprehension of an event is a perception which follows upon another partly different from, partly identical with it, and which is accompanied by the consciousness both of the difference and of the identity.

Now when we begin to refer our perceptions in general to an externally existing object, it is necessary also to refer the succession of our different perceptions to a successive variation in the object ; and where the variation is quite continuous, as when one and the same body moves through the field of vision, then we are also perfectly certain of the objective succession in the sense that what was last perceived happened last, that there was the same sequence of different states in the object as in our perception. When I see the boat glide down the stream, then it is necessarily involved in the general reference of my perception to objects that I should assume the sequence of perceived positions to be the sequence of actual positions, and not that the boat should in reality have gone up-stream, I having perceived its first position last and its last first. All localization in objective time-relations, and ultimately in space-relations also, depends upon this assumption ; if we need an explicit confirmation, we shall find it in the harmony between the succession of perceived movement in our limbs and the succession of the subjective acts of will, which set them in motion.

Absolute necessity, indeed, cannot be demonstrated for this assumption, any more than absolute necessity can be shown for referring our perceptions to an existent. The certainty with which we regard perceived

motions as objectively happening in the same sequence depends ultimately upon nothing but the agreement of all those time-determinations,—which we gain, *e.g.*, through the different senses—amongst themselves and with the purely subjective apprehensions of time; the motion of my arm as seen, as felt, and as willed, all agree.

12. If we develop those assumptions by help of the view that our subjective sensations are the effects of objects, we shall find that the assumption as to their manner of acting is so defined that the later state produces its subjective effect later than the earlier state; and inasmuch as the concept of efficient action involves in its genesis temporal continuity between the activity of the cause and the production of the effect, this is also necessarily assumed in employing the concept of efficient action.

Inasmuch as the action is not necessarily immediate, but takes place through media and transmission by the nerves, we get the possibility of non-coincidence between the particular points of time in the objective motion and the particular points of time in the perceived motion; but when we have to do with a single continuously changing object the sequence of successions must at least be the same.

From this point of view it is manifest that an objectively valid determination of the temporal sequence of the stages *a b c d* in the continuous change of an object is possible if we assume that the order of my perceptions corresponds according to rule with the order of changes in the object; that a series of perceptions *a b c d* which appears to me in this order cannot, if the conditions are exactly the same, appear to some one else in the order *d b c a* or any other. But these rules themselves can only be established upon our original assumption, that we are in a position to begin by perceiving successions with objective validity from which we may derive the rules; the movement of my hand before my eye, and my seeing it, follow each other immediately, and thus the presence of the hand appears as the cause of the seeing.

When, by comparison, we have found such rules according to which the time of the perception is determined by the time of the event—*e.g.* the laws of the velocity of transmission of sound and light—then non-continuous changes may also be brought into an objectively valid order, the time-relation between a visible phenomenon and the sound of a bell may be determined, and all perceived events brought into one time-sequence.

In establishing, for example, the velocity of the transmission of sound we start first from the fact that the hearing of the sound is regarded as the

effect of the stroke upon the bell, and this because of the immediate and invariably recurring continuity of the events when I observe them from a short distance. I am at once certain of the immediate succession between the visible perception of the hammer which strikes the bell and the sound, and it is upon this basis only that I can establish any causal connection at all; but the fact that the sound is always heard at a greater interval after the movement is seen as I move farther away, enables me to discover that the action upon my ear depends upon the distance in a definite relation, while I never come across a similar delay in perceptions of sight; and thus a means is given by which we can interpolate the blows as heard into the series of sight perceptions with objective validity.

13. Thus it is not in that perceptible fact which precedes an external event that we must find, as Kant does, "the condition for a rule according to which this event follows always and of necessity"; it is in the preceding of one event by another that the condition lies for the rule that the apprehension of the second follows the first, or, to put it more generally and more accurately, the temporal relation of the apprehensions of two events must follow from the temporal relation between the two events and from the law according to which the moment of their perception is determined by the moment of their occurrence.

The corrections by means of which we reduce the subjective time of perception to the objective time of the event, and allow for discrepancies between individuals in the time of perceptions, do not therefore directly touch the causal relations of perceived events amongst themselves, but refer to the laws according to which our perceptions, as they proceed from the object, are determined as the same for all men, or as differing for different men. If one man is placed near a clock and another near a cannon 3,000 feet away, and the cannon is fired in objective simultaneity with the first stroke of the clock, then the first man will hear the report when the clock strikes for the fourth time, but the second man will not hear the first stroke of the clock until three seconds after the report; neither of them will be justified in regarding the sequence of his perceptions as the sequence in the object until he knows the rule according to which sounds from different distances succeed each other in perception. This rule as to the sequence of the event and its perception is necessary to enable us to reinstate the objective event from its subjective discrepancies, but it is absolutely unnecessary that there should be any rule according to which the report succeeds the stroke of the clock, or *vice versa*; otherwise

we could never affirm the fortuitous succession in time of two independent events.

In the same way, if one astronomical observer places the passage of a star through the indicator at one second later than another, the discrepancy is overcome by establishing the law according to which the sequence of objective events is reflected in the consciousness of each ; there is, however, no causal connection between the passage of the star and the striking of the second-clock.

We cannot, then, believe it to be "an indispensable law of the empirical idea of the time-series that the phenomena of past time determine every existence in subsequent time." We maintain, on the contrary, that the empirical idea of the time-series is given and absolutely certain in our immediate consciousness of inner events, that the sequence of our conscious states, which are also phenomena of the inner sense, is that which originally occupies time, and with reference to this we can no more speak of a distinction between a merely subjective and an objective sequence than we can of a rule according to which each moment of consciousness determines the following moment. From this basis we will proceed to ask what objective order in the causes of our inner events is necessitated by the laws according to which these inner events are produced by the objects of external intuition.

14. This immediate certainty could only be confirmed if I could succeed in comprehending into one causal chain all the events in the universe, in regarding every moment as the necessary consequence of the preceding moment ; if, that is, I were not limited to merely tracing out different parallel series of independent causal connections, such as the movement of a clock and the course of a fire which goes on quite independently of the clock, but could see a complete linking together of all things by virtue of which the present total state of the universe proceeds from that immediately preceding by a reciprocal determination of all its parts. I should then also perceive the necessity of the simultaneity of different events which at present I can only recognise as matter of fact. But before we can attain to such knowledge we must have a knowledge of the sequence of isolated concurrent series, and this is based upon the assumption that what takes place objectively in continuous succession is also perceived successively. Indeed, Kant himself points out that we merely assume that every event has its place in time assigned to it ; the assumption in no way helps us to *know* its place in time.

What must be admitted in Kant's view is this: If an objective time-determination is possible only under assumptions which represent the sequence of subjective perception as depending according to law upon the sequence of events, then these assumptions must themselves harmonize with those causal laws of phenomena which are grounded upon them, and must rank merely as a part of the causal laws by which we interpret the course of events and endeavour to know them as necessary. For in so far as our own perception must be regarded as merely a part of the general causal connection and as manifoldly conditioned by it, the aim of our investigation is to represent that which enters into our perceptions as a result of a system of laws (uniformity) connecting all events; and there is no doubt that in detail the particular values of time-determinations which we ground upon the subjective perception of inner events are subject to subsequent corrections, although the basis upon which we rely must have some claim to objective validity before we can even try to establish universal causal laws about phenomena.

We must allow, also, that so far as we fail to establish absolutely fixed laws as to the time-relations between perceptions and events—and small subjective deviations take place with the most practised observers—an absolutely certain measure is impossible.

It is for this reason that the difficulties which confront us in subjectively estimating with any confidence the simultaneity of perceptions lying in different series (*e.g.* the visible perception of a signal and the audible perception of a sound, etc.) lead to the construction of self-registering apparatus, in which the two series of events leave visible marks on a uniformly moved surface. We have more confidence in the temporal uniformity of the causal connections in the apparatus than in the uniformity of the effects of external objects upon our nervous system and upon our consciousness. Instead of having to compare fleeting moments of time we get for observation stationary spatial objects; and the task of determining the position of the moment at which one perception occurs between the moments at which other perceptions occur, is reduced to one which is much easier and can be carried out with the help of external aids—that of establishing the position of one spatial point between others. Similar expedients serve to determine time-differences which lie beyond the limits of our powers of discrimination.

But our confidence in the reliability of self-registering apparatus depends ultimately upon our confidence in the objective validity of mechanical

laws ; these are themselves obtained by means of induction from perception, and thus each particular time-determination indicates a background of premises which are quite general in their application, and of which the logical character must be more completely shown in the theory of induction. These premises are in the last instance hypotheses, from which our perceptions follow with uniform necessity.

15. To sum up the result of this investigation : all time-determination starts first from the assumption that the objective sequence of changes in objects corresponds to the subjective sequence of those changes as perceived. This assumption continues so long as it does not lead to contradictions ; where it leads to contradictions it is not immediately abandoned, but further assumptions are based upon it concerning the time-relations between event and perception by which these are connected with varying conditions, and the assumptions are continually modified until all the data agree. Thus the whole process is hypothetical ; we modify our original and unpremeditated assumption, and find a guarantee for the correctness of the proceeding neither in immediate empirical observation, nor in an *a priori* principle (for which, in the generality in which Kant states it, we should have no rules of application), but in the agreement of all the consequences which follow from our hypotheses.

§ 88.

The problem of passing from the subjective localization of our impressions of sight and touch to objectively valid judgments about the POSITION, FORM and MAGNITUDE of the objects found in space, presupposes first an idea of space which is the same for every one, and a uniform principle by which we may determine the relative positions of the perceived limits of objects.

Uniformity in the idea of space is guaranteed by the nature of geometrical definitions and axioms. The principle of localization is expressed for the sense of touch in the proposition that a sensation of resistance is the sign of contact between two bodies, of the partial coincidence of their limits ; and for the sense of sight in the proposition that points which cover each other lie in the same straight line. Both propositions rest upon assumptions concerning causal relations between the objects and our sensation.

The exact statement of relative positions presupposes a uniform measurement, and this presupposes as standards of measurement bodies which are not subject to variation. But because the assumption that there are

unvarying bodies is refuted by its consequences, actual measurement gives us only the relations between variable things, and objective determinations of magnitude in space can only be attained with approximate probability on the ground of assumed laws as to the changes of things.

The predication of movement as change of place in an objectively valid sense presupposes an absolutely fixed space to which changes of relative positions can be consistently referred. This absolute space is not an object of perception, but can only be inferred from causal laws concerning the action of motive forces.

These causal laws, again, depend in their turn upon the perception of relative motion, together with the assumption that motive forces exert the same action whether the bodies are at rest or in motion.

Thus every judgment concerning the position or magnitude of an object is obtained by deduction from hypotheses, and these can only be presented as probable because of the agreement of their consequences with the propositions of geometry and with the subjective sense-impressions; they can never be strictly proved.

1. The problem of describing perceptible objects according to their spatial relations in objectively valid judgments, and of stating their form, magnitude, and position, starts from similar data, and finds a similar solution, as the problem of time-determination.

The localization of our visual images and impressions of touch, which proceeds at first naturally and without conscious rules, leads to the idea of the co-existence of corporeal objects of different form and magnitude in the space around us. The spatial image which every one gets in this way in immediate sense-conditioned intuition is referred to a single centre, the position of his own body, the spatial image of which determines both the localization of tactual sensations and of visual impressions. It may be represented as a system of polar co-ordinates having its origin in the body; the revolutions of the body and of the eye measure the angles made by the various lines of direction, and the length of the rays up to the object is measured for the sense of touch by motor-sensations of the hand in grasping, for the eye by a series of indirect means. To this extent every one has his own spatial world of perception, and the first question is, how this subjective idea of space can be elevated to one which is common to all and valid for all, in such a way that spatial apprehensions may agree, and there may arise as the basis of our judgments a space which is common to every one.

This community in our idea of space results partly from the uniformity of the geometrical laws according to which localization is carried out, and from the allied uniformity of the total space-idea ; partly from the power of movement of the particular individual which enables every one to put himself in another person's place ; and partly from the fact that every one is perceptible for others at a given position in space, and that, therefore, his space intuition as it is from his point of view can be constructed by others from their knowledge of how their own intuition would be altered if they were in his place.

This explains why the space-idea of a particular person disengages itself so easily from the place where he may happen to stand, and his originally merely perspective view of objects in space reduces itself to an idea of the distribution of objects in space which is independent of his particular position—at any rate so far as it is possible for him by his own motion to combine the different perspective views into a complete image, which, though never as such the object of sense-intuition, is yet due to a construction which combines different intuitions, just as the completed form of a building is obtained from ground-plan and elevation.

2. But that of which we get this uniform idea is after all only space itself as continuous extension of three dimensions, and the laws prescribed by it to the boundaries of the bodies which fill it continuously ; whether the form, magnitude and position of each particular perceptible object are determined in exactly the same way by every one, we cannot tell from mere measurement with the eye, however small the differences which it can perceive may be under favourable circumstances. We must have measurement by a standard which is the same for every one.

The problems of measurement are in all essential points identical for the determination of the form and magnitude of particular bodies and for the determination of the position in which they stand to each other ; in all cases what we have to do is to express the magnitudes of straight lines by their relation to a fixed standard, and the magnitude of angles by their relation to the right angle. There is no more essential difference between measuring the edge of a cube and determining the distance between its corner and that of another cube, than there is between measuring the angle of a triangular surface of one and the same body and measuring the angle made by the lines drawn from the corner of one body to the corners of two others.

3. The fundamental assumptions upon which all measurement depends

have reference, first, to the question how the position of any body may be determined beyond doubt; and next to the question how serviceable standards may be constructed.

As to the first point, all localization presupposes ultimately that *felt contact*, the impression of resistance, which we feel by direct contact of a solid with our organs of touch or by means of another solid, is a proof of the spatial contiguity of two solids, a proof of the coincidence of their boundaries in space at the point of contact. It is obvious how closely this assumption is connected with the principle that two bodies cannot occupy the same position in space; it is a simple consequence of this principle of impenetrability. Here the sense of touch takes precedence over that of sight. Thus all measurement depends ultimately upon the application of measures to the object to be measured and to each other; and it is the sense of touch which primarily decides whether an application—*i.e.*, spatial contiguity—has actually taken place.

4. But it is also clear that only relative positions can be determined in this way; the momentary coincidence of the boundaries of one body with that of another at first merely establishes a relation between the two. If this determination is to lead to an absolute statement valid for all space, then we must have bodies in absolute rest, situated at a fixed distance and in a fixed position, to which we may refer the permanent or changing positions of all other bodies, and which will serve as the starting point for all our measurement.

It does not at first sight seem difficult to satisfy this condition. The motionless masses of the surface of the earth upon which we move afford a fixed system of points in the mutual position of which we perceive no change. To these we first refer in all our determinations of position in the sky and upon the earth; the fixed axes of space we find in above and below, west and east, north and south, and so long as any one system of points is agreed upon as the starting point it seems possible to have absolute determination of position by stating the position of any point with reference to the axes determined by the system.

5. It seems easy also to provide standards which can be used. Solid bodies give us invariable form and magnitude; and if we assume that points which, for our eye, cover each other are in a straight line—an assumption by which localization is guided from the very first—we can get straight edges for our measures, as well as straight lines to be measured by them. All we need is an arbitrarily chosen normal standard by the application

of which we may measure any distance, and then we can proceed by the help of trigonometry to measure all our angles. It is from this fundamental measurement of the straight line that all others are derived, *e.g.*, the measurement of angles by the parts of the periphery of a circle, the equality of which is determined by the equality of their chords. However great the technical difficulties may be in dividing a measure into smaller intervals, the periphery of a circle into equal arcs, they cannot be of first-rate importance so long as our assumptions hold good.

To the direct measurements of distance we must finally add—again assuming that we can know when a line is straight—those of trigonometry, which are based upon the same geometrical propositions by which we are guided in all localization.

6. If we disregard for the present the uncertainty in accurate measurement of lines and angles due to the limits of our power of sense-discrimination, and ask merely whether in this way we get objectively valid statements about the dimension and position of objects, then it is obvious that if the results of our measurement are to be objectively valid, the presuppositions referred to must be strictly true.

The first presupposition, that sensation of resistance signifies contact, cannot be shaken without putting an end to all possibility of referring our sensations to real objects; nevertheless, reflection shows us that we are here dealing, not with a self-evident axiom, but with an assumption concerning a causal relation between object and sensation, and concerning the particular mode of action of the cause which excites the sensation. This assumption is hypothetical, for physics has gone so far as to dispute the possibility of actual contact; and its hypothesis that one body is prevented from penetrating into the sphere of another by the mere force of repulsion without contact has necessitated the distinction between a merely phenomenal occupation of space by perceptible matter and the quite different real places of efficient subjects, although, indeed, these real places can only be determined on the ground of the spatial determinations of phenomenal matter. With this distinction, then, our first principle can be maintained as a presupposition of measurement; not, however, as an indisputable axiom concerning that which really is, but as an inevitable assumption, by which we must be guided, at any rate at first, in referring our sensations to things, because it is the first which offers itself.

7. The other presuppositions, on the contrary, have proved not merely to be not necessarily valid, but because of the contradictions to which

they led amongst themselves or with geometrical propositions, to be also untenable.

In the first place, there is no possibility of obtaining a material measure which shall be strictly invariable. In a greater or less degree change of temperature affects all bodies and all objects of measurement ; and if we try to nullify this change by reduction to a normal temperature, we are confronted by similar uncertainties in determining the normal temperature ; strictly speaking, we have always to deal with relations between magnitudes which vary in different degree according to different laws. It is only by presupposing definite laws, inductively obtained, which again contain the assumption that under the same circumstances the same bodies are absolutely the same, that we can get any basis from which to work ; and this basis itself ultimately presupposes the assumption that the movement of objects in space involves no change in their form and magnitude, an assumption which is called into question by recent speculations concerning the nature of space, in order to illustrate the proposition that we have no knowledge with reference to the phenomenal, but mere conjectures, in which we endeavour to guess how things are most probably constituted.

Moreover, the presupposition upon which is grounded the judgment concerning the position of different points in a straight line, has proved to be incorrect ; the phenomena of refraction affect all measurements at any great distance, and necessitate corrections which rest upon inductively gained laws concerning the motion of rays of light. Here again, then, the objective validity of space determination depends upon the validity of inductively obtained laws ; only geometrical propositions remain unshaken, the statement that three perceived points form a triangle having sides and angles of a certain magnitude can never be proved with geometrical strictness, but only upon the ground of hypotheses.

8. Finally, our whole spatial image of the real world is shaken when we find that the fixed axes of space, to which all positions are to be referred, are nowhere to be discovered. For we find no objects at rest to serve as a frame within which to insert all others. If, indeed, all we wanted was to determine the position at any given moment of particular points with respect to each other, this would be possible if we knew even one body of invariable dimensions, no matter whether this moved with respect to the others or not. But it is a part of the problem of localization to construct the orbits of moving things, and here the mere relativity

of empirical space determination overwhelms us with perplexities when we have to make an objectively valid statement as to the orbit and direction in which a body is moving.

The mathematical conception of motion presupposes a space at rest and absolutely determined by a number of fixed points, within which one point alters its position continuously with respect to other points. If this concept of motion is to be applied to a perceived body, then we need in the same way a system of perceptible points which are at rest, and the motion of the body will be described by determining the successive points of its orbit by means of their position with respect to the fixed points. It is thus that we determine the line in which bodies fall as perpendicular to the earth; and it is in this sense that a locomotive running on straight rails moves in a straight line in relation to its surroundings, which are thought of as at rest.

But when the points which we took to be at rest are regarded as themselves moving with reference to others now taken as fixed, then the predicates of the motion are at once changed by the reference to another background. When we take the sun as a fixed point, and think of the rotatory movement of the earth with reference to it, then the path of the falling body changes, and becomes a curve; if we think of the earth as moving forward in space, we get yet another orbit; if we think of the sun as moving amongst fixed constellations, then the position of the points traversed by the falling body changes again with reference to the space of the fixed stars. But the system of fixed stars may itself move, and thus there really remains nothing perceptible which we can regard as at rest, and the absolutely fixed space which we seek is represented by no visible object; no relation can be discovered between it and the spaces of perceived things. Who shall say whether the heaven and the earth are at rest, or whether they move, whether their motion in space is towards north or south, east or west? It might seem an idle question, since it must be ultimately a matter of indifference in what direction everything moves. But it is not so; an absolute, fixed space is the one condition which can give objective validity to a judgment concerning the path, direction, and velocity of the motion of a body, in the sense that in it motion is ascribed to the body as its predicate, and not merely as a phenomenon relatively valid for me under subjective assumptions; it is, moreover, the only presupposition which makes it possible to establish universal laws of motion, for such universal laws cannot possibly take into consideration the

constantly changing relative positions of all bodies in space to which merely relative motion must be referred.

Any perceived motion, therefore, authorizes only the judgment that certain points change their position with reference to each other, not the judgment that one of them is moving in a given direction; and in the apparently simplest task of a judgment of perception, that of stating the magnitude of an object, its position in space and the direction of its motion, we are met by the impossibility of obtaining an objectively valid result by a directly empirical method. In measurement we must start with standards of varying magnitude, and we can only approximately determine ideal constant magnitudes from those which are variable by means of general laws concerning change of volume. Only upon the ground of general assumptions, such as Galileo's principle of inertia, can we come to any conclusion in our spatial determination as to the distribution of relative motion amongst the bodies moving towards or away from each other; and from amongst a confusion of perceived relative motions we must obtain the fixed axes to which we may ultimately refer actual and absolute motion.¹ The one thing which gives us anchorage in the dizzy

¹ A simple example may make this clear. When a stone falls to the earth, the principle of the relativity of all motion tells us that it is a matter of indifference whether we think of the earth as at rest and the stone as in motion, or of the stone as at rest and the earth in motion; the result, the meeting of the two, is the same in either case. But Newton's *Law of Gravity* rejects this arbitrary way of looking at the matter. We must not think of the earth as traversing the whole distance to the stone, but of both as moving towards their common centre of gravity, which in this instance lies at an immeasurably small distance from the centre of gravity of the earth; so that the whole way, except an immeasurably small portion, must be travelled by the stone, to which, therefore, the motion must be ascribed as predicate. And this relation remains the same even if I think of both as moving; the motion otherwise belonging to the stone must always change in proportion to its distance from the earth, while the earth itself experiences only a minimum change. It is the same in the motions of the planetary system. From the point of view of relative motion I can think of the centre of the sun as at rest, and determine the motions of the planets with reference to it, or I can think of the earth as at rest, and make the sun and planets describe their orbits (which will now indeed be more complicated) with reference to it. The mere perception of the motions does not decide between the Copernican and Ptolemaic systems of the universe; but Newton's principle teaches me that the latter is absolutely impossible, and that even the former—the assumption that the sun stands still—is not exact, in so far as mutual attraction involves a motion of the sun about the common centre of gravity of the system, though this never falls outside the sun; and this view remains consistent when we think again of the whole system as moving within the universe of the fixed stars. The relative motion of each body in our solar system with respect to the fixed stars must be analysed into that which is due to forces acting within the solar system, and that which is due to other causes. Our whole understanding of the laws of motion, the only possibility of getting

chaos of endlessly involved relative motions, and enables us to carry out a relative localization, is the assumption that moving forces in their effect upon material masses are independent of the motion in which these are already involved, and that the change of place demanded by them takes place in direction and amount indifferently, whether the object affected is at rest or not; in the latter case, what takes place is the combination of motions called for by the parallelogram of the forces. But this assumption also is no self-evident axiom, and has lately been seriously attacked; if the attack were justified, we should only feel more pressingly the difficulty of saying how a body moves in absolute space, in such a way as to be a ground upon which to formulate the causal laws.

9. It is not a part of our work to describe all the technical expedients by which the difficulties of exact measurement and localization are diminished, and which depend upon accurate knowledge of causal laws; all we have to do here is to point out the methodological principles involved in transforming judgments of perception, concerning the position of visible and tangible points, into objectively valid judgments. Upon examination it appears that the processes by means of which each particular measurement is determined are always in their nature deductive.

The problem is to localize particular perceptions in the spatial system already geometrically established, and the localization takes place by the help of general assumptions as to the laws of motion and change in perceptible objects—assumptions which do not possess the character of absolute axiomatic necessity, but are guaranteed only by the fact that they render possible a localization in accord with geometrical truths and with itself.

On the other hand, the establishment of the causal laws themselves depends, as in temporal sequence, upon the preceding localization as it occurs before reflection; this is not overthrown, but remains as the first approximation, and calls for corrections only where its own consequences would force us into contradictions. When, therefore, we try to grasp the conditions of a perfectly valid time and space determination, we find that each particular case must be woven out of a whole network of deductions from general propositions, the results of which ultimately agree with the

any insight into the confusion of relative motions, depends upon the assumption of an absolute space; even though we should never succeed in disentangling the empirical data in such a way as to know what it is which we must regard as absolute motion. Cf. C. Neumann, p. 15, and O. Liebmann, p. 96 sq., of the writings quoted on p. 240.

immediately given, with the simple sensation of touch or sight, in so far as it is apprehended as a purely subjective phenomenon, and has as such immediate validity.

§ 89.

Because our power of discrimination by the senses is limited, direct measurement leads only to the judgment that two magnitudes are for us indistinguishable, not that they are equal: in consequence it leads always to rational numbers.

For the same reason it becomes possible that repeated measurements of the same object will yield different results, and the problem arises of deducing from these the true magnitude.

This problem can be solved only with probable approximation by making assumptions about the mode of action of the causes giving rise to the errors, by developing these assumptions deductively, and by determining from them what assumptions as to the true magnitudes will lead with the greatest probability to the values found.

Thus the method of least squares, and the rule of taking the arithmetical mean of the values found, as well as the determination of the probable error in the numbers thus obtained, all depend upon deduction from hypotheses.

1. In connection with the limits of discrimination allowed by our sensations there arises a series of questions.

All measurement in time and space comes ultimately to this: that we have to decide as to the temporal coincidence of sensations or as to the spatial equality of objects, *i.e.* as to the coincidence of their limits; but sooner or later all the differences of magnitude reach a limit, beyond which they make no further impression upon us. And this difficulty is seriously increased where the limits of the object to be measured are themselves affected by this imperceptibility, for it holds also for the differences of intensities and qualities by which we recognise the limits of objects. The boundaries of the zodiacal light, the boundaries of the spectrum, the boundaries of a half-shade, cannot be determined with certainty, because the differences of illumination between the outermost parts of the object and its surroundings are too weak to be perceptible, and are caused by the varying disposition of the eye to appear wavering. But even in the most favourable instance, where we have to do with the measurement of a sharply defined object, it is impossible to establish absolute equality

because we cannot distinguish between it and a minimum difference in magnitude.

Here, again, we abstain from speaking of the technical expedients by means of which the relations most favourable for perception may be brought about, the inaccuracy of observation reduced to a minimum, and differences which are imperceptible directly made perceptible by skilful multiplications ;¹ they all depend in part upon mathematical propositions, in part upon causal laws. We are concerned only with the general methods by which we attain to judgments concerning relations of magnitude, and with the logical character of these. And here it is at once obvious that directly we cannot get beyond the statement that two magnitudes are undistinguishable for us we cannot say that they are equal ; and if we wished to be accurate, we should have to confine them within limits, between which they would lie, and beyond which the one magnitude could be definitely known as larger and the other as smaller.

2. It follows, in the first place, that in measuring we always obtain rational numbers, although actually magnitudes would be incomparably oftener incommensurable with our units of measurement than commensurable.

3. The second consequence is, that different measurements of the same object by the same observer, or by different observers, may give different values ; and the question now arises as to what is the true magnitude of the object, what is the amount of the error which has occurred in the different measurements. Here we stand before a problem which is, strictly speaking, insoluble, unless some definite ground can be found which will explain the contradiction between the results ; such is the case, for example, when we recognise the variation of the standard with the temperature as the ground of different results, and are able by consideration of such circumstances to reduce the result of one measurement to that of another and to reconcile them. In addition to these differences which admit of definite correction, there are others which are purely errors of observation, and which indicate partly a varying relation between the object and our physical functions, partly unknown sources of deviation in the instruments used for measurement, and the way in which they are used, or errors in the assumptions which we make in all indirect measurements when it is not the object itself, but a magnitude varying in

¹ Cf. with this Jevons, *Principles of Science*, I, p. 313 sq.

a definite relation to it, which is measured. If we can discover no ground which will explain the discrepancy in the values found by measurement, then all we know is either that one value is correct and all the others false, or that all are without exception false; but which value is the correct one, or where between all the incorrect values the undiscovered true value lies, seems to be a question to which there can be no answer.

4. Suppose the simplest instance, that two measurements of the same straight line have given two different values—say 100 and 101—and that we are not in a position to discover a ground for the discrepancy; so far, nothing is proved but that our measurement is unreliable. Both values cannot hold good; and when we enumerate the possibilities all we can say is: either both are too small, or both are too large, or the true value lies between 100 and 101 (thus including as limiting case the case in which it coincides with one of the two numbers), but for none of these possibilities is there any preponderating ground. The statement that we must at once conclude that the desired value lies between 100 and 101 is completely without ground; on the contrary, if we make no further assumptions, it is just as probable that there is some tendency in the relations between the objects and our senses and in our methods of measurement to magnify the result, and that this tendency has merely acted more powerfully in the second case (101) than in the first, or that there is a tendency to diminish, which has reduced an objectively greater amount in the one case to 101, in the other to 100. The assumption that the truth lies between the two is only preferable when we can argue from the way in which the measurement itself has been carried out that some compensation takes place; as, for example, when an angle has been measured first upon one side of a moving and divided circle, and then upon the opposite side, in which case we assume that the point round which the circle revolves does not lie exactly in the centre; or when a weight has been tested first upon one side of the balance and then upon the other, in which case the discrepancy is explained by an inequality in the beam. If no such explanations present themselves, then there is so far no ground for estimating the possibility that both values are too large, or both too small, as less than the possibility that they represent errors in opposite directions.

The same thing recurs when we have a large number of measurements; there are plenty of instances in which it has finally appeared that all were incorrect in one or the other direction, and we shall find a ground for

supposing this to be the case, more especially when constantly repeated measurements yield a series of values all varying in one direction.¹

On the other hand, even where one method of measurement results in considerably larger, and another in considerably smaller values, nothing is established; the difference may just as well be due to the fact that the one is more accurate than the other, although the easiest assumption may be that the larger values indicate a too much and the smaller a too little, the true value lying within the limits of those obtained.

General considerations of this kind therefore merely show the necessity of investigating whether any inferences may be drawn from the grouping of the numbers. Only when it is presupposed that the same methods are employed and that numbers obtained by them vary unaccountably, or when there is absolutely no ground for regarding the results of one method as more accurate than those of others, are we justified in making certain assumptions as the basis for the calculation of the probability of a given magnitude as the true value.

5. The first of these assumptions is, that it is in itself equally possible for a measurement to yield too small a value, as for it to yield too large a value; hence that in any particular case the probability of a positive and that of a negative error are equally great. (The probability of an absolutely accurate measurement, which represents only one case amongst infinitely many, is so small that it may be left out of consideration.)

This assumption places us in the same position in measurement as in the throwing of a coin, where we are equally likely to get heads or tails. If we consider what we may expect from two measurements which we are going to make, then in—

(1) the value found will be too large or too small, and in (2) also the value found will be too large or too small. Because the two instances are independent of each other, the combination of these disjunctions will give a four-fold disjunction: either the first value is too large and the second too large, or the first is too large and the second too small; or the first is too small and the second too large, or the first is too small and the second too small.

The probability that both will be found too large or both too small is therefore never more than $\frac{1}{4}$; while the probability that the one will be too large and the other too small is $\frac{1}{2}$.

In the same way if we make three observations, the probability that the

¹ This is the case, *e.g.*, in determining the diameter of the fixed stars.

values will be all too large or all too small will never be more than $\frac{1}{8}$; on the other hand, the probability that the true value lies between the extremes of the observations will be $\frac{3}{8}$. With the number of measurements, therefore, the probability increases that they do not all err on the same side.

6. Where we get more than two results another point has to be taken into consideration. In estimating the probability of three measurements in accordance with our first presupposition, we find that it is $\frac{3}{8}$ for the case that two measurements are too small and one too great, and $\frac{3}{8}$ for the case that two are too great and one too small. But if we suppose the numbers obtained to be 115, 111, 110, we shall not estimate the probability that two are too large and one too small (hence that the real value lies between 111 and 110) as equal to the probability that two are too small and one too large (hence that the value falls between 111 and 115). Our ground for this will be a second assumption, the assumption that smaller errors are more probable than larger, and that conversely that magnitude will most probably be the true one which presupposes the smallest error.

If, for the present, we leave the attempt to determine the degree according to which the probability of an error diminishes as it increases in magnitude, then we shall find that the general assumption that larger errors have less probability leads first to the following results.—

Where a measurement yields two values, a and b , a being greater than b , all that we can infer from this is, that the value sought for, x , will be between a and b . For if it lies between a and b , then whatever we may assume it to be, the sum of the errors is $(a - x) + (x - b) = a - b$; but if it were larger than a by n , or smaller than b by n , the sum of the errors would be $2n + (a - b)$; and this, according to our assumption, is less probable. On the other hand, it remains from this point of view absolutely undetermined whether $x = a$, or $x = b$, or whether x lies somewhere between a and b ; for the greater probability of the smaller error on the one side will always be compensated by the smaller probability of the greater error on the other side. There is nothing to determine which is the more probable, the assumption that the two errors are equal, hence that x lies midway between a and b , each error being $\frac{1}{2}(a - b)$, or the assumption that $x = a$, where, though the larger error in b is less probable, the disappearance of error from a is proportionately more probable.

If we had obtained three results, a , b , c , then, according to our assump-

tions, it would be most probable that x corresponded exactly to the middle value b , wherever it might lie between a and c ; for wherever we place it between a and c , the sum of the deviations from a and c will remain the same, while, by assuming x to coincide with b , we reduce the third error $x-b$ to 0.

In the same way, where we have four observations, we must assume the desired value to lie between the two middle values, although we shall not be able to assign any definite position as the most probable; with five observations we must equate it to the middle value.¹

7. But since both universal practice and theory have decided that in such cases it is the arithmetical mean of all the observations which is the most probable measure of the magnitude, there must be further considerations determining the relation between the probability of larger errors and the probability of smaller errors in such a way that the former diminishes much more rapidly than the numbers expressing the errors increase. For without some such assumption, if we have three values 15, 11, 10, that which yields the smallest sum of errors will be 11. For :

$$15-11=4, 11-11=0, 11-10=1,$$

where the sum of the errors = 5; while, if we take the arithmetical mean $\frac{15+11+10}{3}=12$, then

$$15-12=3, 12-11=1, 12-10=2,$$

where the sum = 6.

In the same way, with the 5 values 15, 11, 9, 8, 7, the middle number 9 would give the sum of the errors 11, while the arithmetical mean $\frac{15+11+9+8+7}{5}=10$, would give 12.

The rule that we must take the arithmetical mean cannot therefore be derived simply from the desire to select that value which involves the smallest sum of inevitable errors in the observations; there must be further considerations which make a few large errors less probable than a larger number of small errors.

¹ If we have 4 values a, b, c, d of decreasing magnitude, then, wherever we may insert x between a and d , the sum of the errors $a-x, x-d$ is the same; but if we insert x between a and b , or between c and d , say by n larger than b or by n smaller than c , then the sum of the errors $x-b$ and $x-c$, or of $b-x$ and $c-x$, will be equal to $2n+b-c$, that is, greater by $2n$ than if it lay between b and c , and had for the sum of its errors only $b-c$. Where we have 5 values a, b, c, d, e , we get in the same way the minimum error $a-e+b-a$ by taking the middle value c .

8. These considerations are derived in the first place from the assumption that every given deviation is the result of an indefinite number of particular causes, of which each one may in itself be the ground of a deviation in either direction ; from this it follows that the probability of all factors working together in the same direction, and thus yielding the maximum of error, is very small in comparison with the probability of combinations in which the tendencies to error compensate each other in part or entirely.

We may take as the simplest case two causes, A and B , which are as likely to give rise to a deviation in one direction as in the other, and which take effect in every measurement,¹ causing the value found to be always one too small or too large. Here four cases will be possible :

$A + 1, B + 1$, error $+ 2$; $A + 1, B - 1$, error 0 ; $A - 1, B + 1$, error 0 ; $A - 1, B - 1$, error $- 2$.

Thus the probability that the errors will compensate each other is $\frac{1}{2}$, the probability that the value found will be too large or too small $\frac{1}{4}$. With repeated measurements, therefore, we may expect that half the results will be correct, one-fourth of them too large, and one-fourth too small ; then the arithmetical mean of all the observations will also give the true value.

If we assume 6 causes, each of which takes effect in every measurement, and is as likely to cause the found value to be one unit too small, as too large, then all 6 causes may take effect in the same direction, either positive or negative, and thus give rise to the errors $+ 6$ or $- 6$. Or they may act 5 as positive, 1 as negative, or *vice versa*, giving rise to the errors $+ 4$ and $- 4$; or 4 positive, 2 negative, or the reverse, the errors being $+ 2$ or $- 2$; or finally, there may be 3 positive, 3 negative, where the errors compensate each other, and we get the right result.

If we develop the number of combinations in which these cases occur, we get :

¹ The result is the same if we modify the assumption, and assume two causes A and B such that A increases the result by 1, and B diminishes it by 1, assuming also that A and B are as likely to act as not ; then we have

A either increases the result, or does not act ;

B either diminishes the result, or does not act.

From this we get the 4 instances :

A increases the result, B does not act, error $+ 1$.

A increases the result, B diminishes it, error 0 .

A does not act, B does not act, error 0 .

A does not act, B diminishes the result, error $- 1$.

The difference lies merely in the arbitrarily assumed amount of the error.

1 combination yielding the error +6; 6 combinations yielding the error +4; 15 combinations yielding the error +2; 20 combinations yielding the error 0; 15 combinations yielding the error -2; 6 combinations yielding the error -4; 1 combination yielding the error -6.

Thus in every 64 observations we may expect to find 20 correct, 15 which give a result too large by 2, 15 in which it is too small by 2, 6 in which it is too large by 4, 6 in which it is too small by 4, finally, only one in which it is too large by 6 and one in which it is too small by 6. Here again the arithmetical mean of these 64 cases would give exactly the right value.

In proportion as we assume the causes to be more and the amounts of their action smaller, it becomes obvious that comparatively small deviations are very probable, and if we add together positive and negative deviations, even more probable than the right result (for in the example given above 30 cases give a result which differs by 2 from the true value, and only 20 give the right result); extreme cases, on the other hand, diminish in number very rapidly, and the probability that all 6 cases will take effect in the same direction is only $\frac{1}{32}$.

When we examine the number of combinations for the different errors which result from assuming different numbers of causes, we see at once that they are expressed by the binomial co-efficients of the powers, of which the exponent is the number of causes.

9. These considerations are enough to show why, if we take first two observations, the arithmetical mean is the most probable value of the magnitudes measured. Let there be two observations, of which the difference is 4. Other things being equal, both may yield a value too large or both a value too small; one may be right, and only the other false; finally, the one value may be too large, the other too small.

Assuming as above 6 sources of error, each of which falsifies the result by 1, it is possible that :

- (1) One observation represents the error +6, the other the error +2.
- (2) One observation represents the error -2, the other the error -6.
- (3) One observation represents the error +4, the other the error 0.
- (4) One observation represents the error 0, the other the error -4.
- (5) One observation represents the error +2, the other the error -2.

If we calculate the probabilities on the basis of the above assumptions, we obtain first the total probability that a difference of 4 will occur in two

observations to be $\frac{0000}{4000}$; ¹ this will be distributed in such a way that the comparative probabilities of the 5 previous cases will be expressed by the following numbers :—

1. Case (+ 6 and + 2) by the fraction $\frac{30}{64^2}$.
2. „ (- 2 and - 6) „ „ $\frac{30}{64^2}$.
3. „ (+ 4 and 0) „ „ $\frac{240}{64^2}$.
4. „ (0 and - 4) „ „ $\frac{240}{64^2}$.
5. „ (+ 2 and - 2) „ „ $\frac{450}{64^2}$.

The probability that the true value is represented by the arithmetical mean of the two observations is therefore greater than that of any given one of the other values ; although it follows from the same numbers that if this mean is accepted as the true value, the probability is in favour of its being erroneous, since the other possibilities taken all together have a probability of $\frac{840}{64^2}$, as against $\frac{450}{64^2}$ for the arithmetical mean.

This explains the further necessity of stating the probable error of such a mean ; it is done by determining the limits within which the value in question will lie with a probability of at least $\frac{1}{2}$. From the previous examples—which must deal with whole numbers—it follows that there is a preponderating probability that the true value will not differ from that determined by the mean by more than 4, that it will therefore lie between the mean ± 4 ; for the probability that it lies beyond these limits is

¹ The proportional probability that two observations will give the various possible differences, 12, 10, 8, 6, 4, 2, 0, is represented by the following numbers (the numerators of fractions, of which the common denominator is 64^2) :

For the difference	12	2
„	10	24
„	8	132
„	6	440
„	4	990
„	2	1584
„	0	924

This is another proof that not only the smallest possible difference of 2 units, but even the next greatest of 4 units, would be more probable to occur than would the agreement of both measurements in any result, whether correct or incorrect ; while the probability that both measurements will agree in the right result is represented only by the fraction

$$\frac{400}{64^2}.$$

expressed by the fraction $\frac{6}{900}$, the probability that it does not lie beyond them by the fraction $\frac{4+0+2+0+2+0}{900}$ or $\frac{8}{900}$. If we assumed a larger number of causes to calculate from, we could determine still more exactly how great the deviation from the found mean is, for which the probability that the true value falls within its limits equals the probability that it falls outside.

Similar considerations would show that for a larger number of observations also it is the arithmetical mean which has the greatest probability.¹

10. In the mathematical treatment of this subject the arbitrary assumption of a given number of causes affecting the measurement to a given amount is avoided by basing the calculation upon an infinite number of causes, which take effect in infinitely small amounts; in this way errors of any value whatever can be explained, while on the previous assumptions there would be intervals in their values. In this way we get a proportion of the probable frequency of errors of different degrees of magnitude, to which the proportion of their frequency (on the assumption of a finite number of sources of error) which is expressed by the binomial co-efficients approximates as the number taken increases.² At the same time a

¹ For 3 observations of which the differences are 2 and 2 we get, under the above conditions, as numbers for the ratio of probability, that they represent :

1. The errors +6, +4, +2	1
2. The errors +4, +2, 0	20
3. The errors +2, 0, -2	50
4. The errors 0, -2, -4	20
5. The errors -2, -4, -6	1

Thus the arithmetical mean of 3 numbers has already a much greater preponderance of probability than that of 2; and in the same proportion this probability increases with the increase in the number of observations.

² This law of errors, as it is somewhat hyperbolically called, may be represented to intuition by a curve. First the increasing magnitudes of the errors are brought to view by marking on a horizontal line, right and left of a point corresponding to error 0, equal intervals, which represent equal amounts of increase in positive and negative errors; by taking this line as the axis of the abscissæ and erecting at the beginnings and ends of the intervals ordinates of which the magnitude measures the probability of the errors represented by the abscissæ, and connecting the ends of these ordinates, we get a curve of the figure of a bell; its highest point lies upon the ordinate which stands on the error 0; thence it falls, at first slowly, then more rapidly, so as to approximate to the axis of the abscissæ at the point where the greater and less probable errors begin, finally following the axis with reversed curvature as its asymptote. This is involved in the mathematical formula, which will not regard any error, however great, as absolutely impossible, and cannot therefore allow the ordinates as the expression of the probability of the errors to fall to 0.

This is enough to show that we ought not to speak of a law, but of a mathematical construction, embodying conditions which cannot be found in reality, but which form an

general proposition has been demonstrated which applies also to those cases in which the errors do not simply consist in a too much or too little, and are therefore not related to each other as the distance of different points in a line from one given point, but deviate from the true value in different directions (as shots upon a target, *e.g.*, deviate from the centre); the proposition, namely, that the most probable value is that for which the sum of the squares of its differences from the observed values, hence the sum of the squares of the assumed errors, is a minimum. In assuming the arithmetical mean where the differences lie in one direction only, we simply apply this more general proposition; and we get similar general rules for the determination of the probable error of a result thus obtained. We cannot follow out the mathematical treatment further; here again we find that the method in which a result is obtained, even though it be only a probable result, is in its nature deductive and determined by assumptions concerning causal relations between objects and our sensations.

11. As we have seen in § 70, all determinations of intensities and qualities of sensation, in so far as they can be conceptually defined, lead to the problem of the measurement of spatial magnitude. With regard to them also it is true that neither the absolute equality of two sensations, nor the absolute equality of two spatial magnitudes, is the object of perception; but that the predicates which are most probably objectively valid must be inferred from presuppositions which cannot themselves be actually proved.

§ 90.

Inasmuch as direct measurement can only determine discrete points in objects at rest, and can only be intermittent where the objects are in motion, it cannot exhaust the spatial continuum of a given figure, or the temporal continuum of a perceived motion or change.

Where no mechanical expedients are available we have to proceed by inference, seeking geometrical concepts of configuration, or functions between spatial or qualitative changes and time, from which the data given by observation may be deduced as consequences.

So far as this process fails, it is supplemented by models or drawings of the figures, and the graphical representation of changes.

approximate, and because of its generality an easily applicable, expression for that which really occurs in different ways. Where more specific data are wanting it is necessary to work from such a hypothesis, and as a matter of fact it has been confirmed in particular cases in an extraordinary way; *e.g.*, in the series of numbers in numerous astronomical measurements of the same magnitude.—Cf. Jevons, *Principles of Science*, i. 445.

1. Really direct measurement, by which we endeavour to determine the form and magnitude of an object, can merely give us the distance from each other of certain discrete points; for this reason it can never exhaustively express the configuration of bodies, since their continuous limits can never be absolutely determined by a number of points, however great. No trigonometrical measurement, however extensive, can determine the shape of the earth in all parts of its continuous surface in such a way that the position of any particular point in it would be certainly given; no system of measurements of a human skull shows its complete configuration, but merely a system of points between which there might be various curvatures of surface. Only when the bodies measured are bounded by plane surfaces, or exhibit simple geometrical forms—such as the sphere, the cylinder, the screw—so that the coincidence of their form with one which is geometrically determinable can be demonstrated by mechanical motion, can we ascertain, with as much certainty as measurement is capable of, the coincidence of all points in their surface with the measure prescribed by the geometrical concept; the continuity of motion then steps into the gap which cannot be filled by measurement. In all other cases we can only supplement the description expressed by definite predicates and statements of size by an image, which may be plastic, or projected upon a plane surface, or stereoscopic, and which in all complicated forms tells us incomparably more than could be done by the most accurate enumeration of magnitudes. The value which the art of the draughtsman, the painter, the modeller, and the skill of the photographer have for science, marks also the limits set by the continuum to the fulfilment of the claims of logic.

2. But the image can represent only objects at rest. The difficulties of exact judgments of perception increase in proportion as we have to do with objects which change, and are called upon to represent exactly as they are perceived a movement, the change of a quality, the gradual transformation of one shape into another, in judgments with conceptually determined predicates. We referred in § 69 to the uncertain and wavering nature of our perceptions of movements and changes, which makes them unfit to be the bases of fixed concepts; now we get the reverse difficulty of ascertaining the coincidence of the mathematically formulated concepts of motion with the actual perception, or of developing that concept which coincides with the perception.

The intuitive perception of a motion or change can indeed be assumed to be continuous, and allows, or rather constrains, us to apprehend the

event as a continuum ; but all measurement of place and time, even if we assume it to be perfectly accurate, is intermittent. When a body sinks in front of a scale, even though it falls with the diminished velocity of Atwood's machine, or upon an inclined plane, we can only determine accurately the position in which it is at any given time, and carry out the comparison of time and space, at definite intervals of time—say every second ; that which intervenes escapes our observation. In other cases of very slow motion the spaces traversed are too small to be perceived, and not until they have attained perceptible magnitude is it possible to get a measurement of the distance traversed, in which the difference in the spatial determinations can no longer be attributed merely to errors of observation ; we find such slowness as this in the motion of the fixed stars. Other conditions give rise to intermittence in other ways ; the constellations, for instance, can only be observed at night ; and even if they were uninterruptedly visible, we should be prevented from determining their position from moment to moment by the nature of our instruments of measurement.

3. How shall we proceed to an objectively valid judgment concerning the motion of an object ; a judgment, moreover, not merely about its motion in general, but about an absolutely determinate species of motion ? In most cases, indeed, continuous perception is sufficient to assure us that the same object makes a continuous passage from one place to another ; for though measurement is intermittent, yet the intuitable image is of such a kind that we are certain of having perceived no sudden leap sideways or backwards. But what we have to do is to determine the motion conceptually, to express the form of its path and the relation of the space traversed to time for every moment.

4. It is not necessary to prove that we must obtain by inference what cannot be immediately observed. Our first clue in this process is the assumption, arising from intuition itself, of continuous change of place ; and in the case of non-uniform velocity, of continuous variation in velocity without sudden leaps. We might say that this presupposition was inductively obtained, inasmuch as it has undoubtedly arisen from actually intuited motions, if it were not that the apprehension of motion itself displays those *a priori* activities by which we combine and connect particular moments of time. All that we really do is to bring into consciousness a law of that activity of intuition through which the idea of the continuous first arises, and from this point of view Kant was fully justified in declaring the principle of the continuity of all change to be *a priori*, because it contains

the condition under which alone the apprehension of that which happens in time is possible. We may supplement the principle by saying that it would not even be possible to perceive interrupted motion strictly as such.

5. But this general presupposition does not tell us what particular motion takes place in the particular case. The problem here is to deduce from our observations the geometrical form of the orbit and the numerical expressions for the relation of space and time which will exactly represent the motion of the observed body.

If a series of observations in the times t_1, t_2, t_3 , gives us the positions of a body in its orbit, such that a straight line may be drawn through them, and gives their distances from a fixed point s_1, s_2, s_3 , etc., such that the quotients $\frac{s_1}{t_1}, \frac{s_2}{t_2}, \frac{s_3}{t_3}$ are equal, then we affirm that the body moves in a straight line with a uniform velocity. But our inference presupposes the concept of the straight line and the concept of uniform motion; and what we do is to show that the given positions and times are related just as they would be if the body had moved in a straight line with uniform velocity. We compare the given observations with the ascertained and familiar consequences of the concept of rectilinear uniform motion, and from their agreement with those consequences we infer that the whole orbit was traversed according to the law which was satisfied by the observed positions.

If observation of a body falling upon an inclined plane showed that at the end of the first second one foot was traversed, at the end of the second 4 feet, and at the end of the third 9 feet, we should have the figures 1, 2, 3 for the time, 1, 4, 9 for the space. Our problem is to find the formula which connects these numbers in such a way that the connected times and spaces will appear as the expression of one and the same relation, the formula $s = t^2$; here, again, we can only say that the observed positions are related as they would be if the body had traversed its path according to the law $s = t^2$. In this case it is easy to find the relation which combines all the numbers because the series of the squares is familiar to us; it would have been more difficult if the numbers had been greater. It is clear, however, that, strictly speaking, there is no kind of necessity for assuming this simple relation, since other formulæ are conceivable which would give the same values. No doubt repeated observations which conform to the same assumption strengthen it with increasing probability, but they do not make it necessary.

6. If three points of an orbit were determined as not lying in a straight

line, the possibilities would be incalculably many; all possible curves may pass through three points. The greater the number of points, the more curves are excluded, but no curve is necessarily determined by a finite number of points, and we are always brought to the problem of formulating a hypothesis, of deducing the positions from it, and of comparing them with those which are given.

We must leave it to mathematics to develop the methods by which from a number of connected values of x and y to find the form of the function which obtains between them, and so to determine, on the one hand, the equation of the curve which passes through the observed positions, on the other hand the law of the acceleration. It is always essentially a process of reduction, the discovery of a universal major premise for given minor premises and conclusions; more exactly, the discovery of the formula or of the general concept according to which the various minor premises, $t_1=1, t_2=2, t_3=3$, have for their necessary consequences the corresponding conclusions $s_1=1, s_2=4, s_3=9$ (p. 232).

7. We find the most famous example of such a process in the discovery of Kepler's laws. Given the positions of Mars according to a series of observations, the problem was to find the motion of the planet, both the form of its orbit and the law determining its velocity in that orbit. It is well known how long Kepler toiled at this problem because he started from the assumption that the planets must move, according to the dogmas of the natural philosophy of the ancients, in circles and with a uniform velocity; no matter how he selected his circles, his observations never agreed with the consequences of that assumption. Then he tried the ellipse, which was suggested by the eccentric circles of the Copernican system, and he then found that the lines drawn from the sun in accordance with the observed positions of Mars followed the same law of increase and decrease as the radii vectores which are drawn from the focus of an ellipse to its periphery. It needs no great mathematical knowledge to see that not even the most marvellous acuteness could have succeeded in constructing from observations alone the ellipse as a continuous line governed by a simple law; if Kepler had not known the geometry of conic sections, and had not had in his mind the attributes of the ellipse as proceeding from purely geometrical considerations, to serve as major premises for his calculations, he would never have discovered his first law. Nor was it observation which led him to compare the areas described by the radius vector with the time; geometrical knowledge alone enabled him to calcu-

late these areas, and so to discover that they are equal for equal times. It is the same with the third law; given the mean distances from the sun, to find the one principle from which their successive values may be ascertained as consequences. He tried all possible major premises, starting with the regular bodies of the *mysterium cosmographicum*; finally, he discovered the fixed relation which holds between the mean distances and the times of revolution. That relation of the cube to the square is the general concept, of which the relations of the mean distances to the times of revolution represent particular instances.

There is no example which shows so clearly as this that it is a deductive process with which we are concerned; a number of major premises are tried and found useless, they have to be developed into their consequences, but are refuted by the discrepancy between these consequences and the results of observation. Finally, the agreement between the consequences of the last hypothesis and the observation justifies the conclusion that the ellipse and the law of the areas contain the desired major premise, and that those positions and parts of the orbit which have not been observed, and the velocity in those parts, will also agree with this presupposition.

It is also because Kepler's laws are, strictly speaking, false and do not agree exactly with the results of observation, that their history and the history of the further development of astronomy is so instructive as to the actual methods of scientific investigation. They represent an ideal case only, not an actual state of things, and nevertheless Newton's theory was developed out of these laws, by the same method of reduction, into a universal major premise, of which the consequences (*i.e.* disturbances) explained the discrepancy between the actual motions as accurately observed and the strictly elliptical motion first assumed.

8. Where a comprehensive formula cannot be found because its mathematical expression is too complicated, or because the assumed regularity is wanting altogether, nothing remains but to have recourse to the *image*, and to represent graphically the continuum which we cannot get from actual observations because of their discontinuity. If, for instance, we mark off distances on a line which are proportional to times, and erect perpendiculars from the points of division to represent spaces, and connect the upper ends of these perpendiculars by a continuous line, then this line will give us the representation of continuous motion, and by its convex or concave curvature we can at once recognise gradual decrease or increase of velocity.

The logical process in the construction of such a curve is essentially the same; the only point is that the assumption which we make with reference to continuous changes now appears in the line of the curve, which is drawn so as to show continual curvature. But it still remains clear that we make an assumption which does not seem to follow necessarily from the given data, and that at best we can only hope to get an approximation; it is clear also that we infer from the fact that the curve as drawn connects all points without difficulty that it expresses the course (which cannot be represented in a formula) of the actual motion, and that those parts of it which lie between the marked points also correspond to the intervals of motion which lie between the points of observation.

9. The uncertainty of all such inferences which depend upon the mere assumption of a law has naturally made the need felt of avoiding intermittence in observation, and where it is practicable of causing motions to register themselves by means of a mechanical apparatus which follows the motion exactly; the curve corresponding to the motion is marked upon a uniformly moving piece of paper. It is in this way that by means of a lever we obtain a curve for the gradual rise and fall of the pulse, and it is in this way that the rising and falling of the barometer registers itself; an image of the motion is presented for spatial observation and measurement, and thus all the difficulties which occur in the logical process are eliminated, while there are substituted for errors of observation the more easily recognised defects in the working of the apparatus. The construction of the apparatus itself, and knowledge of the relation between the curve and the actual motion, depend upon simple geometrical and mechanical laws.

10. Exactly the same methods must be applied when, instead of motion in space, we are dealing with changes of any kind, changes of qualities and intensities. So far as qualities and intensities are measurable at all, they may be reduced to numbers and proportional spatial magnitudes; and we proceed in the same way to construct from the temporally separated observations the continuous course of the change, either by means of a formula or by the graphical method. It is thus that we construct the curves of temperature for the daily rise and fall of the atmospherical temperature, or the curve of the temperature of a patient in a fever.

In many cases it is, of course, impossible to show the course of a change from moment to moment; when, for instance, it is too rapid to admit of several observations, or too complicated for us to be able to

follow the simultaneous change of a number of parts. In rapid chemical action our observation is limited to comparing the beginning and final point of the change, and to ascertaining the difference between the second state and the first ; we determine the changed state and not the process of change.

§ 91.

Since the reference of sensibly perceived qualities, relations of magnitude and changes, to individual things can only take place in the syntheses described in § 72, perception can lead directly only to the description of PHENOMENA, and we must distinguish between this and the reference of the phenomena to their REAL SUBJECTS.

In so far as spatially limited figures are given to perception, these are the subjects of judgments of perception.

Where spatial limits are lost to observation, the reference of predicates to their subjects is guided by the principle of the permanence of matter, and determined by measurement of weight.

Neither the principle of the permanence of matter, nor the principle that the weight is the measure of the quantity of matter, is *a priori*.

1. In our preceding considerations we have dealt with the means of determining in an objectively valid manner the predicates belonging to the things given to our perception. We begin by assuming these to be simply given, as they appear to the ordinary sense-apprehension ; they form the starting points to which, by analysis of their particular aspects, judgments are attached which express in fully determined predicate-concepts their perceptible magnitudes, their attributes, and their changes. But our investigations have shown in § 47, 9 (I. p. 312) and § 72 (II. 78 sq.) that the idea of individual things as subjects of our judgments of perception is brought about by a synthesis of those extended and successive qualities of sensation which alone are immediately given, and that this synthesis is not determined in one invariable way by any necessity, but is originally governed only by the negative principle that two things cannot occupy the same position in space ; finally, in § 78 we found different forms of unity for these syntheses, according to which the meaning in which we speak of a single thing is differently determined. There still remains for our investigation the question : how is it possible for our judgments of perception to be objectively valid in the sense of

referring their predicates unerringly to those subjects to which they belong necessarily and with universal validity, and which are connected with the predicates by every one in the same way ?

2. When we learn that what is immediately given consists only of our sensations, and that these are themselves only the effects of presupposed real things upon consciousness through the medium of our sense organs, then we are first led to distinguish between the mere phenomenon, the sensibly perceived appearance, and the thing itself which gives rise to the appearance. We then find ourselves called upon to free ourselves in all observation from habits of referring perceptions to things, from every interpretation of the seen and heard, and to describe the phenomenon as such ; so that the subject of our judgment shall be in the strictest sense only that which is subjectively and directly perceived, and we shall therefore be entirely in the position in which the so-called impersonal judgments arise, in so far as what they express is merely the perceived phenomenon without reference to a definite thing (cf. § 11, I. p. 59 sq.). When we see a rainbow, we can only say that we see colours which are arranged in a certain way and describe an arc ; as to what is coloured we shall prefer to withhold our judgment, since we have no perceptions of touch or any kind of analogy to guide us. When we see lightning, we can speak only of the transitory flash of light ; or when we see a dark round spot upon the disc of the sun, of this optical impression. It is left uncertain what it is which flashes, and whether the dark spot is a planet passing before the sun or something else. But the question what it is which flashes and is coloured means only : in what way would the assumed object, which takes effect as light upon our eyes, affect our eyes and our other senses under other conditions ; in what way would it, under given conditions, change itself and other things ; in other words, what complex of spatial and temporally successive phenomena would it present by itself or in connection with others, for it is only as the one ground of perceptions and change of perceptions that it is ever thought of.

3. Our attitude towards the rainbow or flash of lightning is fundamentally the same as that in which we originally approach all perceptible objects ; hence the direct and immediate expression of our perception can be merely that we perceive at a certain place such and such colours, such and such qualities of touch, etc. Here we see again how the concept of the one thing must originally be determined by the bounded

form, and how the individual subject, to which the predicates of the description apply, is first given by the permanent form, which even in motion remains the same or changes continuously. In every description of plants and animals the subject of the judgments is only the form which appears ; the predicates are attributed to this spatial whole. The subjects thus gained are finally established by the fact that every one agrees in the apprehension of their forms and of their spatial boundaries ; their identity attaches to the constancy of the image and to the uninterrupted continuity of perception, and so long as our statements merely refer to the permanent or changing qualities of sense within certain spatial limits the objective validity of the statement depends only upon similarity in our organizations and consequent sameness in sensations, and upon coincidence in measurement.

4. But as soon as we substitute for the form, as the element determining unity, the actual individual subject which appears in this form, the meaning of the statement changes, and it passes beyond the merely phenomenal to an interpretation of the phenomenon, to a process of inference which no longer merely gives immediate utterance to the perception, but introduces general propositions by which the interpretation is guided. While what I say is merely that what appears at a given place within perceptible limits is yellow, shining, smooth, hard, etc., I describe the phenomenon ; as soon as I say that it is gold I interpret the phenomenon by a general concept, and carry out an inference of subsumption, of which the major premise is the definition of the concept of gold (§ 56, I. p. 367) ; this statement therefore presupposes the classificatory formation of concepts. So long as we are dealing merely with objects at rest and of fixed limits the concepts of particular kinds of matter (for instance) may be regarded as simple constant complexes of perceptible qualities (as Locke represents them), and the concept gold might then be taken merely as an abbreviation, as a simple substitute for a plurality of perceptible qualities ; it was no doubt in this way that the concepts of particular kinds of matter originally arose, and from such concepts all description of things by definite words must start.

But, as we have shown in § 47, 9 (I. p. 312) and § 72, 14 sq. (II. 86 sq.), the fact of change in what is perceived, while we still need to apprehend that which changes continuously as one thing, forces us beyond this simple way of forming concepts, and makes us uncertain as to our affirmation of the identity of the substance which appears in different forms and trans-

formations, if it is to be grounded upon the mere perception of spatial continuity. Where, as in the mixture of fluids or the solution of a solid body into a fluid, the boundaries of two things originally distinct in space disappear; where chemical combination does not merely obliterate spatial boundaries, but withdraws the original qualities of the combined materials from the senses and replaces them by the entirely new qualities of the compound; where, as in evaporation, parts become invisible which were previously visible, or as in the precipitation of gases and fluids, visible drops and solid bodies arise where there was before nothing to be seen, in such cases as these immediate perception leaves us uncertain how we are to maintain the unity of the bodies which were separated, and how the later subject of perception is related to that which was seen earlier. There is no *a priori* principle which forbids us to assume that substances disappear; on the contrary, the assumption is suggested by the simplest interpretation of perceptions. It is only on the one hand our logical needs (§ 72, p. 84), on the other the inference from analogy which passes from the plurality of bodies which we see persisting to the others, which favours the assumption that even substances which have become invisible still exist somewhere, an assumption which finds its confirmation in our constantly increasing knowledge of the possibility of reproducing elements from compounds, or of precipitating vapours. The proposition that matter persists has become certain by way of induction; but the history of the concept of matter shows clearly how slow the early general assumption of a permanent substratum for all changes has been to take definite form, how long the Aristotelian theory held its ground that this substratum is completely without quality and equally capable of all forms and definite attributes, how the concept of the chemical element first vindicated this predicate of the permanent for determinate kinds of substances, and the development of chemistry afterwards yielded a comprehensive inductive proof for the general presuppositions, thus establishing definite concepts concerning the material subjects to which our predicates refer. But it still remained a doubtful question how the constancy of subjects could be known in the particular case by way of perception, and how it could be determined whether successive states were really referable to one and the same thing, until the principles that weight is the measure of substance, and that the same material body must invariably have the same weight, gave us the clue by which to establish with certainty the identity of substances as subjects of our judgments of perception, and to form inferences from perceived

motions as to the identity or non-identity of the things to which successive predicates belong.¹

It is this which gives its importance to the balance as a supplementary help in all our description of external things and events; it does not directly determine any predicate of sense-perception, as the measure determines visible magnitude, but by measuring through a perceptible motion the quantity of matter which is contained within definite spatial limits, it not only adds an important relation to the immediately apparent qualities, but serves principally to determine whether that which confronts us in successive perceptions is *the same* subject or not. Thus it may really be called a metaphysical instrument which, by the information it gives, guides us in our reference of sense-given to the permanent ground which we are constrained to add in thought, and raises bridges for the understanding, by which it may pass from the world of sensations to the non-sensuous world of the subjects which we assume as the cause of the sensations, and which can be apprehended only in the concept. It is true that it does not remove the impossibility of finding absolutely individual subjects, such as would satisfy the claims of logic, in the province of intuition where we can know only the extended, that is the divisible; but it does enable us to control their numerical relations, and to determine their increase or decrease.

Thus the validity also of our statements as to the sense in which the predicates of our judgments of perception belong to their subjects depends upon a comprehensive frame-work of hypotheses, which have arisen in part from our logical needs, and have in part been gained by way of induction. In reference to the limits of accuracy in weighing, and in establishing the most probable results of the process, the same methods are, of course, applicable as in other measurements.

§ 92.

Our desire to attain extensive completeness of perception in space and time would be realized by a DESCRIPTION of the universe containing all particulars of perception in their spatial and temporal order, which would take the form of a catalogue of all particular objects and their changes.

In so far as this exhaustive completeness of description is not possible,

¹ Cf. above, p. 169, and v. Kries, "Ueber Messung intensiver Grössen," *Vierteljahrsschr. f. wiss. Phil.*, vi. 260 sq.

a substitute is found in the STATISTICAL NUMERATION of like things and events, which presupposes a given classification of objects.

Within narrower limits this numeration serves partly for the description of the particular in its course in time, and partly for the description of collective wholes as to their composition and change; beyond those limits its task is partly to present for our survey the spatial and temporal distribution of like things and events, partly to bring before intuition the numerical relations in which the various specific differences of a general concept are realized.

1. Perception, and the description expressing it, begins within an incalculable plurality of things, and of events taking place in and between these things, as this or that happens to present itself or is selected by subjective interest. But even the localization of them in space and in the time common to all forces us to extend perception until it embraces all space and all time, and so to establish a spatial order for the plurality of perceived things and a temporal order for their movements and changes, within which every particular object appears as part of a spatial whole, and every period of time occupied by a plurality of contemporaneous events as part of a temporal whole. An extensively complete perception would become a cosmography, in which for every point of time there would be assigned to everything its position in space and its state; within this ideal frame-work are ranged those fragments of the representation of larger spatial wholes and historical events which we are able to establish by exact perception. And the basis of this all-embracing image of the universe is the determination of the spatial distribution of objects which are—at any rate for us—invariable in time, of the fixed masses of the earth and constellations, geography and uranography; when this ground-plan is once given for any point of time, then movements of the objects to and fro are introduced, and can be known by means of comparison.

2. Within the whole, which comprehends a plurality of perceptible things, every part which forms a unity is determined as a particular thing; it is also determined as single, and distinct from all which are to be found near it in space. The principle that the same thing cannot at the same time occupy different positions in space secures numerical distinctness between objects and the determination of their number at any given time. The principle that different states of the same thing can only be successive and not simultaneous secures difference in point of time.

3. Astronomy, favoured by the fact that the greater part of its objects are easily surveyed and constant, and by the wonderful development of its instruments and methods of measurement, has realized as fully as possible the idea of an accurate representation of the spatial distribution of its objects. By undertaking in its charts and catalogues to determine the position at any given time of every permanently visible object in the heavens, and by tracing their motions, it approximates to the ideal of a cosmography embracing the whole realm of the perceptible; and the completeness with which it recognises and distinguishes the particular as such finds expression in the fact that it denotes every object as particular by its proper name or the number equivalent to it.

4. In constructing a world-catalogue to contain all perceptible objects and to present them in a spatial system as particular with name or number, we are opposed by the number, the transitoriness and the irregular motion of most earthly objects, as well as by the absence of spatial separation in the masses of the earth's surface. Nevertheless geography, no doubt chiefly with a view to the results of human activity, has to a large extent set before itself the same task of determining the particular as such in its spatial distribution and number, and of cataloguing, of giving proper names to the mountains, rivers, etc. But where this way of making the particular as such in its completeness the object of knowledge is impracticable, an approximating process offers itself in the counting of particular things of the same kind.

5. So far as the classification of substances and of forms according to perceptible characteristics has been carried out, every description of a particular object serves either for its subsumption under existing concepts, or to show us that they are insufficient and that the classification needs to be extended or corrected. If we leave aside for the present the latter case, then the first consequence of this subsumption is that the subjects of our judgments of perception can be named by a general term expressing a concept. As perception then takes the extensive direction towards completeness of knowledge of the particular, and observes the plurality of the particular as such, it leads to the numerical statement of those objects which fall under one and the same concept, and determines the number of things of the same kind which it comes across. Here the particular place or particular time occupied by each particular thing is no longer regarded, and any chance differences in individuals by which each one is distinguished from all others, and which must be noticed in a description,

also disappear; but the particular as such still maintains its own in so far as it is not merely taken as the indifferent representative of a general concept, but retains its value as an existence numerically distinct from every other. Quite apart from the fact that for the real relation of particular things amongst themselves and to others it is not a matter of indifference in what numbers they are forthcoming, these numerical statements have an interest in themselves with reference to the ideal of an extensive completeness in our knowledge of the particular.

6. There is no familiar name for this kind of classified cataloguing of the numbers of particular objects which fall under a general concept, except the terms statistical statements and tables; the peculiarity of all statistical surveys consists just in the fact that they do not numerate and catalogue particular objects and instances as such, but that they give us the total numbers of homogeneous objects and phenomena, and so merely summarize particular perceptions according to certain categories. To this extent statistical numeration belongs to the province of the observation and description of the particular, but the inferences which can be drawn from such numbers must be reserved for a later section.

7. The object with which the numeration of particular objects is undertaken is not always the same.

In the first place, the numerical statement is subservient to the description of collective wholes, the constituent unities of which are either all of the same kind or fall under a limited number of different concepts. In the exact description of a group of stars such as the Pleiades, we must have before everything a statement of the *number* of the stars forming it; in the exact description of a flock of sheep, a statement of the number of sheep; in the description of a family or people, a statement of the number of members; in the description of a caravan, a statement of the number of camels, horses, and men composing it. If it is taken into consideration that the unities forming a collective whole manifest in addition to the fact that they fall under the same generic concept differences, either of quality or of mere quantity, then the description will be more complete if the total number of the individuals which fall under the generic concept is broken up into the numbers of individuals corresponding to specific concepts or varieties. The description of the Pleiades states how many stars there are of the 4th, 5th, 6th magnitude, etc.; the description of a flock of sheep, how many rams, ewes, and lambs; the description of a people, how many males and females, how many of different ages, how many of different

callings, etc. ; the conceptual order appears as a point of view by which to guide the numeration within the limits given by the collective unity.

Such a description of different wholes composed of analogous elements leads us further to institute a comparison between them, and to recognise differences in their composition.

Where the province within which a census is being taken is not given as a definitely limited collective unity, the problem would at first be merely to determine the total number of objects falling under a concept,—to count how many men, etc., there are. But a twofold interest connects itself with this process ; on the one hand, to determine how the total number of the individuals of a genus is distributed in space ; on the other hand, to see how the total number of the individuals of a genus is distributed amongst its species or its quantitative differences, and thus to obtain a basis for comparison, for the establishment of numerical relations, which will be analogous to measurement.

In the first proceeding we may, at any rate to some extent, find a substitute for that ideal completeness of perception of the particular in which we were called upon to assign to every particular thing its position in space ; and even where this may have been done, as in the lists of stars, the statement of the numbers of stars in the particular parts of the heavens enables us to make a comparative survey of their distribution. In statistics of population we count the total number of human beings inhabiting the earth. We do not, however, stop here, but by dividing the earth into spatial parts and stating the population of the particular continents, countries, and provinces, we make a comparative survey ; we have the same aim before us in the geography of plants and animals.

On the other hand, the numeration is guided by the division of the concept, which gives for our comparison the numbers in which the differences of things are realized. How many stars there are of the first, second, or third magnitude, how many human beings belong to the different races, how many are males and how many females,—all this is included in a complete knowledge of the particular.

The extent to which the statistical method prevails, and everything is counted, is another instance of the fundamental difference between modern and ancient science. Nothing seems more superficial and unimportant when contrasted with the mere concept, in which for Aristotle all knowledge lay, than these merely matter-of-fact numerical relations ; the knowledge of the concept can gain nothing by our knowing how often it is

realized, and there can be no inner relation between the number there may happen to be of individuals and their qualitative differences. Number attains scientific value only when we realize that our first task must be to apprehend the Given fully and accurately in its actual constitution, and then to understand the necessity of this constitution; its importance even for the higher problems of science is proved by the inferences which can be drawn from statistical tables, and it has become an indispensable aid to induction.

8. The numeration of particular events which occur in time accompanies the numeration of particular things which exist side by side in space, and has a similar aim. Where the *things* numerated arise and pass away in time, as do the generations of living beings, the two kinds of numeration are necessarily combined in reference to the same objects.

The numeration of like events subserves primarily the description of the particular object and of collective unities in their temporal duration and change. Where the same events recur in the same individual, or in similar individuals, such as the breathing or beating of the pulse in man, or the birth and death of individuals within a nation, the numeration of these events and the determination of their distribution in time is an aid to mere description which is unable to note down each case as individual; the number of definite recurring activities and exertions gives us the total amount of action within definite time-limits. Then conceptual arrangement appears here also, and we are called upon to count how many events of different kinds occur within given time-limits, or within a circle of like conditions, so that we may compare their frequency. We shall have to return later on to the importance of statistical statements of this kind; here they appear primarily only as a summation of particular perceptions which is made possible by the subsumption of the particular under general concepts, and which in this abbreviation enables us to take as the object of communicable knowledge the particular which we should have neither time nor strength of memory to survey in its inexhaustible fulness and with all its specializations.¹

¹ Cf. A. Meitzen, *Geschichte, Theorie und Technik der Statistik* (1886), where in §§ 63-67 the conditions of statistical numeration are carefully discussed.

CHAPTER V.

THE PROCESS OF INDUCTION AS THE METHOD OF OBTAINING UNIVERSAL PROPOSITIONS FROM PARTICULAR PERCEPTIONS.

THROUGHOUT the preceding chapter we have been led to see that objectively valid statements about particular perceived things and events rest upon universal propositions, which express a necessary connection, and from which it can be inferred that if something is, something else also is; they rest upon rules which express the connection between our subjective perceptions and things, the connection of the attributes of things amongst themselves, and the connection of their changes with other changes. On the other hand, it has also been established that belief in the truth of such universal propositions depends entirely upon the particular facts of perception, and the question which we now have to consider is—to put it quite generally—how do we pass from judgments of perception referring to the particular to universal propositions?

In the problem of determining a continuous motion or change from intermittent observations we have already approached this question; we were there concerned in finding for the particular event the rule according to which the magnitude of the changes is determined in successive moments, in deriving, that is, from particular data a proposition to hold good for all particular stages of the change; and it appeared that particular observations pass imperceptibly into inferences bearing the character of INDUCTION. In this simplest example we already find an indication of the general nature of the logical process contained in it; and it is this process which first calls for closer investigation.

I. THE GENERAL LOGICAL CHARACTER OF THE PROCESS OF EMPIRICAL INDUCTION.

§ 93.

If the object of the inductive process is described as the attainment of universal propositions from particular judgments of perception, then it

may be shown that induction in this sense is indeed a process which is universally practised according to psychological laws; but that from no point of view can a ground be found for its logical justification in the sense that the propositions inferred in this way are necessarily true, and that induction is therefore a kind of strict proof for universal propositions by means of particular facts.

The logical justification of the inductive process rests upon the fact that (according to § 62) it is an inevitable postulate of our effort after knowledge that the given is necessary, and can be known as proceeding from its grounds according to universal laws.

This presupposition is the ground for a process of reduction, through which we seek, upon the ground of the syllogistic rules, the premises as consequences of which the particular facts of observation present themselves.

The premises thus discovered are HYPOTHESES, which may indeed be refuted by contradiction between their consequences and observed phenomena, but which cannot be proved by agreement between their consequences and the phenomena, and can at most be made increasingly probable.

Within this process of reduction, which is ordinarily known as the inductive method, we must distinguish between the process of obtaining specific propositions, which possess a merely numerical generality, and generalization, in which the results are obtained by means of generic concepts.

1. INDUCTION (ἐπαγωγή) was opposed to the syllogism by Aristotle as a special kind of inference, by means of which the truth of propositions is known; and he describes it as ascending from the particular to the universal,¹ without giving us any detailed teaching about it. He finds first the process of illustrating a universal proposition by a series of examples in the practice of oratory, and more especially in the method of Socrates; and he explains that it is the popular process because it attaches itself to familiar experiences, and thus easily excites belief. Its only presupposition is, that the similar in different instances is recognised, and that a familiar word for it exists. Thus it is shown that the best pilot is he who understands the matter, and the best driver in the same way he who

¹ *Top.*, I, 12, 105a, 10 sq.: ἐπαγωγή ἡ ἀπὸ τῶν καθ' ἕκαστον ἐπὶ τὰ καθόλου ἐφοδος. Cf. *Top.*, I, 18, 108b, 7 sq. *Anal. post.*, I, 13, 80a, 40 sq. With what follows cf. Max Consrbruch, "Ἐπαγωγή, und Theorie der Induction bei Aristoteles," *Archiv für Gesch. d. Phil.*, v. 3, 302 sq.

understands it, and hence it is inferred that in every art he is the best who understands it. Akin to the *ἐπαγωγή* is the proof from the particular example (*παράδειγμα*). Athletes are not chosen by lot, therefore statesmen should not be chosen by lot.¹ In a wider sense the process of explaining a general concept, such as that of the relation between *δύναμις* and *ἐνέργεια*, by examples such as the relation between the man who knows how to build and the man who actually builds, is called *ἐπαγωγή*.²

That such an *ἐπαγωγή* is cogent as a proof is always taken for granted by Aristotle; if any one will not accept a proof from the particular, he must produce a negative instance, without which to assail the proof is mere contentiousness.³ The very major premises of our syllogisms are themselves not syllogistically proved, and therefore rest upon an *ἐπαγωγή*.

There is only one passage (*Anal. pr.*, II. 23, 24) in which Aristotle deals more fully with the process of inference contained in the *ἐπαγωγή*, by contrasting it with the syllogistic process.

If the concepts *A*, *B*, *C* are subordinate to each other, then the syllogism shows by means of the middle concept *B* that *A* belongs to *C*; the inference is *B* is *A*, *C* is *B*, therefore *C* is *A*.

But induction shows by means of the lowest concept *C* that the highest concept *A* belongs to the middle concept *B*. For instance, let *A*=long-lived, *B*=without gall, *C*=the particular long-lived animals,⁴ man, horse, mule; then induction infers—

man, horse, mule are long-lived;
man, horse, mule are without gall;
therefore that which is without gall is long-lived.

The inference is necessary when man, horse, and mule are all animals which have no gall; for where the middle concept *B* does not extend beyond *C* the minor premise may be converted, and we get—

man, horse, mule are long-lived;
that which is without gall is man, horse, mule;
therefore that which is without gall is long-lived;

i.e., an inference according to the rules of the first figure, where, instead

¹ *Rhet.*, II. 20, 1393a, 27.

² *Metaph.*, Θ., 6, 1048a, 25.

³ *Top.*, VIII. 8, 156a, 35 sq.

⁴ We should expect, instead of τὸ καθ' ἑκαστον μακρόβιον, to find τὸ καθ' ἑκαστον ἀχολον καὶ μακρόβιον. With respect to difficulties in the text cf. Conbruch; nevertheless the logical structure of the inference which Aristotle means to present is beyond question.

of a simple middle concept, we have the total of the members of the division, which, in the natural order of the concepts, occupy the last place. Thus the presupposition is that *C* consists of all the species of *B*, for induction surveys all the particulars; ¹ it is then the means of proving a proposition *B* is *A*, which cannot be proved by a syllogism following the natural order of the concepts, because there is no middle concept between *B* and *A*.

In this way is shown what Aristotle sets out to prove in the chapter, that even rhetorical inferences, and all processes of conviction however they may be arrived at, follow the syllogistic figures.

Aristotle, then, treats in a similar manner the inference from an example (*Anal. pr.* II. 24). It is shown that it would be harmful for the Athenians to make war with the Thebans by a similar instance, by the example of the Thebans, who made war with the Phocians; both are instances of wars between neighbours. Here there are four concepts: the highest, *A*, is harmful; the next, *B*, is war between neighbours; the two subordinate to *B* are *C* and *D*, war between Athenians and Thebans, war between Thebans and Phocians. Now we have the propositions, which are immediately certain (*γνώριμα*):

D is *A*—the war between Thebans and Phocians was harmful.

D is *B*—this war was a war between neighbours.

C is *B*—war between Athenians and Thebans is also a war between neighbours.

and we infer:

C is *A*—therefore it is harmful.

This contains really a double inference; from the example it is inferred:

D is *A*, *D* is *B*, therefore *B* is *A*;

then from this universal proposition it is inferred, according to the first figure: *B* is *A*, *C* is *B*, therefore *C* is *A*.²

¹ Ἡ γὰρ ἐπαγωγή διὰ πάντων. *Anal. pr.*, II. 23, 68b, 28.

² The fact of this double inference explains how Aristotle can say at one time (68 b 38, 69 a 11) that the inference from the example shows that the higher concept belongs to the middle concept (*A* to *B*), and directly afterwards (69 a 13 sq.) can state that example differs from induction by the fact that in example we infer from particular to particular, while induction does not direct its conclusion to the lower concept (the particular), but infers from all the particular that the higher concept belongs to the middle concept. Thus induction lacks the application to the particular instance which, according to Aristotle, is peculiar to the example; it is however superior to the example as affording complete evidence for the universal proposition.

In the same way it might be shown by several examples that *A* belongs to *B*, and this is what Aristotle calls in the wider sense *ἐπαγωγή*, in those passages, that is, where he does not maintain that induction must include all the particular.

Here it is clear that the first inference is not cogent, and is not justified by syllogistic theory. From one or several examples which may be presented in the propositions,

D, E, F are *A*

D, E, F are *B*,

it does not follow universally that *B* is *A*; there is absent just that inclusion of all cases which would justify the conversion of the minor premise. A generalization takes place by which that which happened in one or several cases is taken as a universal rule.

It is worth noticing that Aristotle, when he speaks of induction, is hardly ever thinking of deriving a universal proposition from the observation of particular instances in the proper sense. His examples generally have reference to concepts of species, and what he does is to combine, not particular facts into a lowest concept, but specific concepts into a more general concept, or specific rules into a general rule. That the best driver is the man who understands it, is itself a universal rule, but he treats it as particular; in the same way he takes as particulars that man, horse and mule are without gall, although these are already universal judgments. He does not inquire how these have been obtained from the observation of particular men, horses and mules; his metaphysical presuppositions make him regard the power of the conceptual form over the particular datum as so absolute, that he never considers whether he is justified in affirming of mankind what he has observed in particular men, any deviation he would attribute to the inertia or imperfection of matter, without losing faith in the thorough-going determining power of the form. For this reason he is not troubled by the objection which suggests itself as to whether the inexhaustibility of number does not make it impossible to survey all the particular; what he is concerned with is the complete enumeration of the species of a genus, and this he holds to be attainable.

When this presupposition that a genus is exhausted by the enumeration of its known species, and the previous presupposition that whatever falls under one specific concept is similarly constituted, fail, then the cogency of the inference also breaks down; it can no longer be a ground for the conceptual judgment from which further inferences may be made, but only

for the empirical summation of particular judgments by means of a common name (cf. I., § 57, p. 368 sq.). When, therefore, the inductive process is taken to mean only that what is true of all *A*'s known to me is for that reason true of all *A*'s, then the conclusion goes beyond the premises and the inference is not cogent.

2. It has been rightly pointed out that every such inference can be represented as referring back to those characteristics *a, b, c* of the concept *A*, by which the subsumption is guided. In all the instances known to me I find connected with the attributes *a, b, c*, a further attribute *d*; from that I infer that everything which agrees with these instances in possessing the attributes *a, b, c* will also possess the attribute *d*. I assume therefore that that which is alike in the known part of its determinations will be alike in the unknown remainder also. But this is nothing else than what is ordinarily called the inference from analogy (v. § 83, 5, p. 213). Induction in this sense first argues by analogy from the known *A*'s to all others which fall under the concept *A*, and then sums up all *A*'s in a common judgment. To express it differently: from the fact that in the instances known to me I find *a, b, c* together with *d*, I infer that *d* accompanies those attributes *a, b, c* necessarily. But this is only the Aristotelian inference from example, which would therefore coincide with induction.

3. Bacon is fully justified in directing his criticism in the *Novum Organon* (1 Aph. 105) against this view of the inductive inference. That induction, he says, which proceeds by simple enumeration is a childish matter, and infers rashly, and is always liable to be refuted by a negative instance. It is therefore unsuitable for a scientific method.

As opposed to it, Bacon then attempts to give the rules of the true scientific methods, by which we may pass safely from immediately given particular facts to universal propositions, and thus to determine the true concept of induction.

In order to estimate rightly the instructions which he gives here, we must bear in mind what is the end to which they are intended to lead. Knowledge, he says, is the knowledge of causes; hence the universal propositions (axiomata) which are to be gained are causal laws. Now, according to Aristotle, there are four kinds of causes: material, efficient, formal, and final causes. Final causes are at once excluded from the investigation. Material and efficient causes have only a subordinate importance; the most important kind of cause is the formal. Form is that

which constitutes the essence of a phenomenon, the ground which remains the same throughout varying fortuitous conditions, for the fact that a thing is what it is. There are forms of concrete things, of gold, silver, man, horse; but these forms are not simple, but compounded from the forms of particular qualities, or, as Bacon calls them, natures. The form of gold is compounded from the forms of yellow, heavy, malleable, etc. Therefore we must first find the forms of the simple and ultimate qualities or natures, and then from their manifold combinations we shall get the forms of particular things.

In these views Bacon shows himself still the disciple of scholasticism. Although he waged such bitter war against Aristotle, he nevertheless inherited from him the presuppositions of his methods; the concept and its attributes have an immediately real significance, and as the concept is compounded from its attributes, so the concrete thing is compounded from its various natures. But he is also influenced by the physico-atomistic theories of his time, for he distinguishes in particular perceptible attributes between the quality of sensation appearing to us (warm, white, etc.), and that which corresponds to it objectively. This objective element he looks for in the structure of bodies, in the way in which the smallest particles move and are related to each other. For example, he considers the cause of the colour white to be the mixture of a transparent, finely divided body with air, on the ground that powdered glass and snow are white. For this reason he can explain the concept of the form by that of the law, the significance of which lies in the fact that it determines that arrangement or motion of the smallest particles.¹

On the basis of these presuppositions he sets about describing the methods according to which we have to determine the form of a sense-perceived attribute, *e.g.* of heat; *i.e.*, to determine that which really brings it to pass that bodies are warm, however different the substances may be (where there are different material causes), and however different the occasions (where there are different efficient causes, friction, ignition, etc.).

According to the presuppositions, then, the form of heat must be something which is everywhere where heat is, and nowhere where heat is not, which is present in a higher degree where there is more heat, and in a less degree where there is less heat.

¹ Concerning Bacon's concept of form, cf. the penetrating investigation of Hans Heussler, *Franz Bacon und seine geschichtliche Stellung*, and Hans Natge, *Ueber Francis Bacon's Formenlehre*.

The method of proceeding is as follows : In the first place, a table is drawn up in which all the instances are entered in which heat appears—sunbeams, volcanoes, fire, warm-blooded animals, etc., etc. Secondly, a table is drawn up containing instances in which heat is absent. In itself this would be a never-ending business ; but it is sufficient to note those cases which are similar to the first in other respects, but differ from them in the absence of heat : thus the moon is opposed to the sun, etc. Finally, a *tabula graduum* is prepared, which contains all the instances in which there is a more or less of heat. In each of the registered instances, various other qualities or natures are connected, or not connected, with that of heat ; in the sun, the *Natura cælestis*, light, etc., are connected with heat ; in the moon, the *Natura cælestis* and light are not connected with heat.

The problem now is to discover that nature which is everywhere where heat is, nowhere where heat is not ; which is present in a higher degree where there is more heat, in a lower degree where there is less heat. The process employed for this purpose is that of *exclusions*. From those natures which accompany heat, all are excluded which do not satisfy the conditions ; and after a complete process of exclusion—hopes Bacon—that nature will be left which is the desired form. So he begins : *Rejice naturam cælestem*, because there are heavenly bodies which are cold, and terrestrial bodies which are warm, such as terrestrial fire. Away with light because there are bright bodies which are cold, and dark bodies which are warm ; away with a special heat-substance, because warm bodies can impart heat to others without losing in weight, and so on. In his illustration, however, he does not complete this process of exclusion, because he is not in possession of a knowledge of all the natures present.

The application of his instructions obviously presupposes that there is a definite, finite, known number of simple so-called natures, and that every observed phenomenon is a definite combination of a number of such elementary determinations or natures ; that one of these natures is the form of heat, *i.e.*, the ground which is always present when there is a subjective sensation of heat. Given this presupposition, then there must be a quite simple process of elimination, tedious, perhaps, and cumbersome, but quite certain, and capable of being carried out mechanically like a calculation.

4. In order to get a clear idea of the Baconian induction, we may represent it schematically, denoting the various natures, including heat *w*,

by the letters of the alphabet. The instances of the first two tables will then be represented by the following combinations :—

<i>a b c d w</i>	<i>a c d f</i>
<i>a b d f w</i>	<i>a d f g</i>
<i>b g h w</i>	<i>d g h x</i>
etc.	etc.

What we now have to do is to find that letter which always appears in the first column, and never in the second, and that will be done by striking out of the total list of letters every one which appears in the second column, and is absent anywhere in the first.

Represented as logical inference, the process will take the following form :—

The form of heat is either *a* or *b* or *c*, or . . . *x* or *y* or *z*.

The form of heat is not *a*, for

the form of heat is everywhere where heat is ;

a is not everywhere where heat is,

therefore the form of heat is not *a*.

The form of heat is not *f*, for

the form of heat is not present where heat is not present ;

f is where heat is not present, therefore

the form of heat is not *f*, etc.

Whence we obtain by combining the negative conclusions :

the form of heat is neither *a* nor *c* nor *d*, nor *e* nor . . . *z* ;

therefore the form of heat is *b*.

That upon which the whole process, therefore, is grounded is a disjunctive major premise with many disjunctive members ; the minor premise is a copulative negative judgment, which excludes all members of the disjunction but one ; the inference passes by the *modus tollendo ponens* to the member which remains.

The particular members of the copulative minor premise which are combined by it are derived by special syllogisms, which are all in the second figure, in Cesare or Camestres.

Thus we really have a completely syllogistic process ; the induction of Bacon is in no way opposed to the *Organon* of Aristotle. His endeavour to obtain certain propositions which are not liable to the dangers of ordinary induction leads him back to the same logical rules which he had described as incapable of building up science.

That which is obtained by an induction, in the sense of Aristotle, is only the enumeration of the members of the disjunctive major premise, in so far as it states that a number of empirically discovered natures exhausts the totality of the concept. But it is just this major premise which Bacon is unable to establish, nor has he given any rules as to how it is to be found, and how we are to obtain certainty about it; he recognises a weak place, which he promises to remedy later on.

But all which follows in the particular steps of the process of inference is as purely deductive as any process of inference in Aristotle; inferences are drawn from a principle which is not gained from experience, but is assumed *a priori*, and from the concept contained in it. The principle is that every quality perceptible by sense has one formal cause, which consists in one of the natures; and it is a part of the concept of cause to be always immediately connected with the effect, and to be proportionate to the effect.

From this principle follows the disjunction of the major premise, by means of which it is affirmed that *one* nature, to the exclusion of all others, is the desired form; from this concept there follow the major premises of the particular syllogisms by which the exclusion is carried out.

Thus, although Bacon lays such stress upon experience, and insists that scientific investigation must take perception of the particular as its foundation, he does not really derive any universal proposition from merely particular facts of experience by themselves. The facts of experience only give him the minor premises; they can only be employed as members of a process of inference when accompanied by universal major premises which can trace their descent from the scholastic metaphysics.

5. The question as to the nature and justification of a method of obtaining universal propositions which should start from particular perceptions alone without the aid of general principles could not be formulated clearly and completely until Locke and Hume had propounded the thesis that all knowledge and all mental content whatever has its origin in experience alone, and that therefore all universal propositions must have the same origin. The critical point in the question was introduced when Hume undertook to derive entirely from particular sense impressions just that concept which had played the chief part amongst Bacon's presuppositions, the causal concept.

But there are two meanings in which the question, "how do we pass

from particular impressions to universal propositions?" may be asked, the psychological and the logical. It may be asked: How does it *happen* that from particular perceptions alone we arrive at the belief in universal propositions and at conclusions about that which is not perceived? or it may be asked: What *right* have we to derive a universal judgment from particular conceptions, and by what right do we infer from known facts to unknown facts?

In other words, we must distinguish between induction as a psychological fact, and induction as a logical method.

6. It is in the first sense that Hume has interpreted the problem, and his treatment is distinguished by its complete clearness and consistency. In accordance with simple psychological laws associations are formed which give rise to habits of imagination; qualities which we have seen several times connected with another quality associate themselves in our imagination in such a way that when we think of the former the latter also presents itself, and when we come across the former again we expect the latter also. In the same way events which we have seen succeed each other repeatedly give rise to an association, and when the first appears we necessarily expect the second. Hume might have added that not even a repetition is needed to give rise to an association; one case, if only it has made a lively impression, is sufficient to make us expect a similar event in a similar case. Burnt children dread the fire after a first experience, and refuse the second spoonful of a bitter medicine; repetition merely strengthens and confirms the association which a single event was sufficient to establish. Or we may express this process differently by saying that there is a powerful and ever-acting tendency to generalize every particular proposition which experience offers; we are always inclined to expect similar from similar, and we meet the events of every new day with a host of *anticipations*, which are grounded upon single or repeated experiences; we are always guided by such generalizations in seeking the useful and avoiding the harmful. All experience in the ordinary sense, all the rules which guide the action of men, and even of animals, may be traced to this tendency. If induction is nothing more than the process of forming, from a number of propositions of perception referring to particular things, a universal judgment which we take as a ground for expecting similar results from similar things in the future, then we certainly do not need to learn the process of induction; we practise it from childhood without instruction and without being conscious of a

rule, just as we digest and breathe. We are dealing with a psychological and natural law of generalization, the validity of which is unassailable upon psychological ground ; and when in many instances new experiences constantly fulfil our expectations and strengthen the associations, there results quite naturally and in an increasing degree what Hume calls belief, the subjective confidence that our anticipations will continue to be justified in the future also.

7. But Hume is acute and logical enough to see that the universal rules thus obtained cannot possess objective validity, that we cannot pretend to regard the laws of association amongst our ideas as laws of real things, and that the feeling of subjective psychological constraint must not be confused with an objective knowledge of the necessity of nature ; upon this foundation, therefore, no science can be erected, but only the recognition of the impossibility of knowledge.

However firmly established, therefore, the psychological fact may be that we generalize in consequence of the laws of association, nothing is determined by it as to our *right* to the generalization and the trustworthiness of the assumptions based upon it. If we simply have in view a collection of particular facts there is nothing to show us how ninety-nine cases in which we have found a raven to be black are to decide anything as to whether the hundredth is also black, since this hundredth is as much a particular, independent instance as the first ; the accumulation of observations which agree can do nothing directly but heighten the subjective probability with which we are inclined to expect in the hundredth case what appeared in the ninety-nine preceding cases ; but this subjective probability has no kind of significance with reference to reality.

8. This sceptical result of Hume's forced his successors in the doctrine of empiricism to look around for a guarantee by which to justify generalization ; nevertheless they are frequently obliged to be content to show that our universal propositions are as a matter of fact largely obtained by generalizations from experience, instead of showing that some objective necessity justifies this process as a logical method of obtaining universal propositions of certain validity.

In one respect J. S. Mill holds the same views as Hume. For him nothing is given but particular sensations, and these sensations are originally subjective states of feeling. But there must be some way of proceeding from these to science in the full sense, and this way is to be shown by inductive logic ; this will be, moreover, the only way in which

we can pass beyond immediate experience to the knowledge of something which we do not experience immediately.

Induction, as he defines it,¹ is that operation of the mind by which we infer that what we know to be true in a particular case or cases will be true in all cases which resemble the former in certain assignable respects—the process by which we conclude that what is true of certain individuals of a class is true of the whole class, or that what is true at certain times will be true in similar circumstances at all times.

But he goes on to add that this process of inference presupposes a principle, a general assumption with regard to the course of nature and the order of the universe, namely, that what happens once will, under a sufficient degree of similarity of circumstances, happen again, and not only again, but as often as the same circumstances recur.² This proposition, that the course of nature is uniform, is the fundamental principle, or general axiom of induction.

Every particular so-called induction is therefore a syllogism, of which the major premise is this general principle, and which can be expressed as follows :—

Under similar circumstances, the same event will always happen.

Under circumstances *a, b, c*, *D* has been found ;

Therefore under circumstances *a, b, c*, *D* will always be found.

It is clear, although Mill has not sufficiently noted it, that, regarded only in this aspect, the particular case proves just as much as a whole series of cases, and that I can draw exactly the same conclusion from a single observation as from many similar observations.

But now the question arises as to the origin of the universal major premise and the consequent significance of this syllogism ; and here comes in again Mill's doctrine as to the nature of the syllogism of which we have already spoken (I. § 55, 3, p. 359). The universal major premise cannot explain the inductive process, for it is itself obtained by induction ; it is indeed one of the latest and highest inductions grounded upon preceding partial inductions. The more obvious laws of nature must have been already recognised by induction as general truths before we could think of this highest generalization. Hence we can only regard this highest major premise as a guarantee for all our inductions in the sense in which all major premises contribute something to the validity of their syllogisms ; the major

¹ J. S. Mill, *System of Deductive and Inductive Logic*, bk. iii. ch. 2.

² *Ib.*, ch. iii. p. 331.

premise contributes nothing to prove the truth of the conclusion, but is a necessary condition of its being proved, since no conclusion can be proved for which there cannot be found from the same grounds a valid universal major premise.

In other words, we really infer only from observed cases of uniformity to other cases; because we have found a uniform relation between a certain number of phenomena, we infer that it will be so also with every other class of phenomena; but, according to Mill, this latter conclusion—a real Aristotelian inference from example—is only certain if we can infer from the observed uniformities to general uniformity.

Upon what ground can we infer from a number of instances of observed uniformity to universal uniformity? Mill enters more fully into this question in chapter 21 of the same book on the Law of Universal Causation. He there tells us that men first begin to perceive uniformity within particular provinces; they learn that fire burns, food nourishes, water drowns, and they learn it by means of an *inductio per enumerationem simplicem*. After they have perceived a number of such uniformities they generalize their observation, and assume a general uniformity, and this assumption enables them to look for and find uniformity even where it is not obvious.

One would think that since the *inductio per enumerationem simplicem* is an unreliable process in particular inferences, inasmuch as it is often deceptive, the inference to a universal uniformity must be doubly unreliable. First we make the uncertain inference from particular cases of a given kind to all cases, assuming what is at most only probable, that the unknown are similar to the known; then from the particular results thus obtained we derive a second generalization, which would itself be uncertain, even if it were based upon none but certain data, and which is doubly uncertain, since it has for its ground not facts, but uncertain generalizations.

Mill does indeed try to show that the *inductio per enumerationem simplicem* is more reliable in proportion as it refers to a more extensive subject-matter. "When a fact has been observed a certain number of times to be true, and is not in any instance known to be false, if we at once affirm that fact as an universal truth or law of Nature . . . we shall in general err greatly; but we are perfectly justified in affirming it as an empirical law, true within certain limits of time, place, and circumstance, provided the number of coincidences be greater than can with any probability be ascribed to chance." But if the subject-matter of any generalization is so widely diffused that there is no time, no place, and no combination of

circumstances, but must afford an example either of its truth or of its falsity, and if it be never found otherwise than true, then it is an empirical law, co-extensive with all human experience, a point at which the distinction between empirical laws and laws of nature vanishes. Hence the causal law is to be regarded as the most extensive, and therefore the most certain induction.

It need not be pointed out that this latter comparison between the greater and lesser certainty of induction has a meaning only when we assume the uniformity of nature to be actually forthcoming, and are considering the conditions under which it may be known; it has no meaning so long as we are still endeavouring to obtain the certainty of this uniformity. It is true that when we are dealing with many, and to some extent counteracting, conditions, it is more difficult to perceive the uniformity which is there; but what has that to do with the question whether we have any right at all to assume a universal uniformity?

According to Mill's own views, we have no such right from the standpoint of the *inductio per enumerationem simplicem*. He himself observes that at the beginning of investigation the uniformity of phenomena appears within certain spheres, but not in others; hence these latter are "negative instances"; and it would be absolutely inconceivable that, in view of the absence of law throughout so large a sphere of phenomena, men should ever reach the idea of universal uniformity if they had had nothing but the facts of experience, which Mill himself admits could for the most part be brought under general laws only by the application of methods which are derived from the principle of universal uniformity.

Moreover, Mill himself denies the universal validity of the law of causation by the limitations which he sets to it. He recognises that there is nothing in our experience which could yield a sufficient, or indeed any ground for believing it to be impossible that in some distant region of the fixed stars events might succeed each other by chance, and not according to any fixed law; he finds it possible to conceive of the present order of the universe coming to an end and a chaos following, in which there would be no fixed order. The uniformity in the succession of events must be received not as a law of the universe, but of that portion of it only which is within the range of our means of sure observation, with a reasonable degree of extension to adjacent cases.

By these limitations the universality of the induction is really destroyed; if it depends upon what lies within the range of our sure observation, then inference from the known to the unknown is abolished in principle; if we

never know where the limits lie in space or in time beyond which we are no longer to be able to infer, then we can lay down no universal propositions; we are confined to *narrating* what has happened until now, to narrating that so far as we have been able to disentangle phenomena they have shown uniformities, to narrating that our conjecture that what has happened a hundred times will happen again has often been confirmed, but for the rest we must leave it to chance, or to the gods, whether it will happen again in the next hour or in a remote stellar region. When we lay down a universal proposition we venture upon unsafe ground, and there is ultimately no motive for assuming such a proposition beyond mere subjective association.

Taking away with one hand what he gives with the other, Mill shows in the uncertainty of his views the helplessness of pure empiricism, the impossibility of erecting an edifice of universal propositions on the sand-heap of shifting and isolated facts, or, more accurately, sensations; the endeavour to extract any necessity from a mere sum of facts must be fruitless.

The only true point in the whole treatment is one in which Mill as a logician gets the better of Mill as an empiricist; namely, that every inductive inference contains a universal principle; that if it is to be an inference and not merely an association of only subjective validity, the transition from the empirically universal judgment all known *A*'s are *B* to the unconditionally universal all that is *A* is *B* can only be made by means of a universal major premise, and that only upon condition of this being true are we justified in inferring from the particular known *A*'s to the still unknown *A*'s. But then the universal major premise cannot be obtained simply by means of a summation of facts, for this by itself can yield no more than it says, that in a certain number of cases *A* was *B*, and as pure matter of fact contains no reason for passing beyond these *A*'s to other *A*'s; it must have some other origin than in previously perceived facts, and our right to make use of it must have some other ground than the narration of cases which have been observed so far.

9. It has sometimes been thought that the theory of probabilities might be called in to establish, if not absolute certainty, yet a probability approximating as near as we like to certainty, for a general proposition when it has been confirmed in a given number of cases; and we are instructed to reason somewhat in this fashion: If *a* and *b* did not belong necessarily together, but coincided by chance, then the judgment would be true that *a* is either accompanied by *b*, or not accompanied, meaning that there is

an equal probability for either case ; the probability for either would be $\frac{1}{2}$. Suppose, now, in 100 cases where *a* is present, *b* is always there ; if we assume their coincidence to be fortuitous, the probability that they will accompany each other 100 times in 100 cases is only $(\frac{1}{2})^{100}$; but if we assume that they are necessarily connected, this result would be necessary, therefore certain, and thus the second assumption is incomparably more probable than the other.

But in stating the alternative between chance and necessity this deduction presupposes the existence of necessary connections, and can merely help us to find them with more or less probability in the particular case, and to determine which of the given coincidences may be regarded with most probability as the sign of necessary connection. It cannot, therefore, dispense with the hypothesis of a uniform order, in inferring by induction to a presupposed proposition, and here also this hypothesis is not the simple product of facts ; finally, as in every induction, the process leads only to probability, not to certainty.

10. We might, finally, take the purely logical point of view, and say that it is an identical proposition, that all *A*'s are constituted alike ; that like things have the same attributes, that under like circumstances the same event will take place ; it is a part of the concept of likeness that one thing can be simply substituted for another.

This proposition is no doubt true, but it fails to apply to the given. For the question is just this, whether what seems to us like is really like ; whether the constancy which prevails in our concepts is realized in that which we perceive. Absolute likeness in all attributes and relations is an ideal which we could not see realized in perceptible things until we had thoroughly penetrated them and learned to know them in all aspects ; but as soon as this came to pass any ground for an inference leading from the known to the unknown would disappear ; nothing would remain but simple subsumption, and before we could undertake this everything which induction endeavours to infer must be directly observed. As the presupposition for a process of induction, therefore, this self-evident proposition is useless ; in every induction it is assumed that we are justified in inferring from a group of a few perceptible attributes which serve for subsumption under a specific or generic concept to one or more other predicates which are not directly perceptible, or which we can dispense with perceiving directly just because the connection which we find in a number of individuals between the conceptual attributes and other predicates justifies us in assuming that

they are necessarily connected. The proposition that all ravens are black can indeed be inferred from a single example, if I determine to give the name of raven only to those birds which are exactly like my pattern raven in all respects, so like that I can substitute any one for the first; the proposition is then as analytical as the proposition that all triangles are triangular. But then before I can apply the word raven to any bird, I shall first have to show that it resembles the first in all respects, and if I once know that, I have no further need of inference, since I can learn nothing fresh about it. But I may regard the proposition "all ravens are black" as obtained by induction: then my concept of raven contains primarily those characteristics which are taken from the form and anatomical structure, etc.; in many cases and without exception I have found the black colour of the plumage combined with this form, and am now to believe that black is so inseparably connected with the characteristics of the form that I can lay down the universal major premise "whatever has the form and structure of the raven has black plumage," and may be certain beforehand that every bird with that form must also have black plumage. But it is evident that hundreds of thousands of black ravens would not make it impossible that I should ever come across a white one; they prove only that so far as my observation extends ravens are black, not that the combination of white plumage with the structure of the raven is *impossible*.

If we should try to employ the principle that like is true of like as a basis for induction, we should find it useless, not only because absolute likeness cannot be known, but also because in many departments of reality it is the rule for the objects, to which I must apply my concepts if I am to make any use at all of them, to differ individually.

11. We may take the question as we will; we shall never find in the given a sufficient ground to yield us the conviction that because so many perceived *A*'s have without exception the attribute *B*, or because event *B* has followed so many times upon circumstance *A*, therefore it must *necessarily* be so. The innumerable exceptions by which many attempted generalizations of this kind are met do, as a matter of fact, refute even to superfluity the assumption that a universal law can be derived with infallible certainty from similar cases, however great their number. For a long time it is for most Europeans a good induction that all men are white; it is a good induction that all men have five fingers; for thousands of years it was a good induction that all metals are heavier than water. We cannot

even guard against exceptions, which may subsequently overthrow a universal proposition by insisting upon our observations being as extensive as possible; at the best our field of observation is minute as compared with space, and is always limited in time by insurmountable barriers; nevertheless, both theoretically and practically, the value of the universal propositions we are looking for consists to an important extent in the fact that the future is to conform to them.

12. On the other hand, it remains just as certain that our knowledge of the perceived world has arisen and developed in no other way than by these inferences, for which there is, strictly speaking, no sufficient ground and the results of which have been actually confuted in thousands of cases. These inferences were, indeed, made before any question was raised as to the assumption upon which they were made; men continued to make them without being disturbed by the numerous cases in which they turned out badly; and unless such attempts had been constantly renewed, and had become gradually more successful, the universal highest principle, without which no conscious method was possible, could not have been established and applied, although even this principle is in no way strictly justified if we regard it merely as the result of such inferences.

How is the riddle to be solved? Does it not seem humiliating, and a bad omen for the value of all logic, that the greatest and most important part of all our knowledge should have arisen in an illogical manner, as it were by chance, and in defiance of strict logic? that in demanding the proof of a proposition logic is a preacher in the wilderness, with whom no one troubles himself?

If we look more closely, however, we shall find that logic has played a more important part in our empirically gained knowledge than might appear; it needs only to see rightly what that part has been.

13. It is true that human experience begins first with those associations which make us expect to undergo again what we have undergone once. It is these associations which make the infant expect milk from its mother, and not from its father; which make the child believe that the apple he sees tastes good, and desire it, and which make him fear the bottle holding a bitter medicine; and as one part of these associations is confirmed by frequent repetitions, while another part is destroyed by contradictory experiences, the universe becomes divided into one sphere in which we feel at home, and are accustomed to expect results with certainty, and another composed of phenomena which are changing, variable, and fortuitous.

It is characteristic that from this point of view of mere associations the belief which is based upon association can hardly be shaken, even by exceptions, especially when this belief is not merely theoretical, but has grown up with the wishes and needs of men. Chance is of such every-day occurrence within a wide sphere that it is not found strange that it should sometimes occur even within the sphere where order is the rule ; by some personification of the capricious power of chance men easily overcome the difficulties which further reflection might find in exceptions. There is even a peculiar attraction about the exception ; it is an object of wonder, a *θαῦμα*, and the miracle is accepted with the same easy credulity as the supposed rule.

The whole history of popular ideas about nature refutes the assumption that the idea of a universal order can in any way have arisen of itself, by the mere passive reception and association of particular perceptions. There is no doubt that every one infers from known cases to unknown, but it is just as certain that this process, so long as it starts only from that which presents itself, cannot lead to the assumption of universal uniformity, but only to the thought that law and disorder bear sway in wild alternation. But for strict empiricism there is nothing but the sum of particular perceptions with their coincidences on the one side, and their contradictions on the other side.

That there is more order in the world than it shows at first sight, is only found out when order is looked for. The first impulse to look for it starts from practical needs ; where we want to attain ends we must find reliable means, which are certain to possess an attribute or to bring about a result. But practical needs merely form the first occasion for reflecting upon the conditions of a real and certain knowledge ; apart from them there are motives which drive us beyond the level of mere association. It is not with equal interest, or rather equal want of interest, that we confront events in which like combines itself with like, or like with different ; the former correspond to the conditions of our thought, the latter do not ; in the former our concepts, judgments, and inferences have a significance for reality, in the latter they have none. Thus even the satisfaction, which we first feel without thinking about it, contains the stimulus to the wish to find realized in the whole world of the perceptible that necessity which is the fundamental element, and the aim by which we guide our thought.

14. Thus, as we have shown in § 62, the universal presupposition by which we are guided in our mental elaboration of the particular proposi-

tions given by perception is that the Given is necessary; and since necessity signifies for us the same as the invariable and universal connection of a ground with a consequence, we get as the postulate of our search for knowledge that every particular perception is an instance of a general rule, a conclusion which follows from subordination to a universal major premise. This presupposition has reference both to the co-existence of the permanent attributes which we find in a particular object, and to the connection between the changes in the same or different objects; the concepts of things, in which we synthesize certain perceptible attributes at first as subjective images, have objective significance just in so far as the connection is necessary, and as there is a general rule according to which these attributes exist together in the particular case; the particular event is necessary when it happens according to a rule which prescribes that under certain conditions a certain change will take place.

Hence it follows that we are forced by the nature of knowledge to apprehend all particular objects and facts with which observation presents us as instances in which a universal rule expresses itself; the problem of induction is to find this universal rule, and to formulate it in such a way that the Given shall everywhere correspond to it.

In other words, we are dealing with a process of reduction, which constructs the premises from which the particular perception follows with syllogistic necessity, whether it expresses the co-existence of attributes in a thing, or a change, or the succession of different changes; and the problem is to determine these major premises in such a way that they may be in harmony with all the perceptions known to us.¹

15. From this it follows in the first place that the propositions gained by induction are never *proved* in the strict sense, but from a logical point of view are only hypotheses; further, that the fundamental principles even of induction are based upon the rules of the syllogism, by which it is determined whether the premises assumed lead necessarily to the conclusion. Just as in the division of a number by a divisor what we want to do is merely to determine by what number the divisor must be multiplied in order to give the dividend, and the division of 36 by 4 consists solely in going through the products of 4 as contained in the multiplication table in order to find the one which is 36, multiplication being for that reason the proof of

¹ It has been shown quite clearly by Jevons, *Principles of Science*, I. 139 sq., that induction as a logical method is a reverse operation, and is related to deduction as division to multiplication, or the integral to the differential calculus.

division, so also methodical induction always presupposes the knowledge of deduction, and can only seek the premises which, according to known rules, give the result, and the proof of it is that the result really follows of necessity from these rules. It differs, however, from division in that the proof does not prove the correctness of the premises, but only their possibility; for it is conceivable that every conclusion should have different premises from which it may follow.

Thus induction necessarily takes the form of a hypothetical experimental method, which tests an assumed proposition by the agreement of its consequences with given progressive perceptions, and so determines whether it may be accepted as the rule to which the particular conforms; and it must at once relinquish its assumed propositions when their consequences contradict observed facts.

In this respect Bacon is right in telling us to proceed by way of exclusion, and has hit upon the most essential feature of the logical process of induction, that only negative conclusions are strictly necessary.

If a perception does not agree with what is at first the assumed hypothesis of a universal proposition, then, assuming the process of inference to be correct, one of the premises is necessarily false; but the most comprehensive agreement of the hypothesis with facts can never show it to be necessarily true, it can at most make it probable. One case in which *A* is not *B* refutes the proposition that all *A*'s are *B*; while one thousand cases in which *A* has the predicate *B* are not sufficient to prove the proposition, it is impossible for an *A* not to be *B*.

When, however, any one hypothesis breaks down because the inferred universal proposition is opposed by exceptions, our universal presupposition that the Given is necessary is not on that account destroyed; it is only the special assumption with reference to the necessary connection between a special ground and a special consequence which fails.

16. It is because the obtaining of universal propositions, expressing a necessity, from particular facts can only be a process of reduction, which is justified by the universal assumption that the actual is necessary, because as reduction it must by its nature be a hypothetical and tentative process, that we are also justified in beginning the inductive process before we have ensured the completeness of our observations. It is true of induction what we said of classification (§ 77, p. 166), which indeed merely represents a special aspect of induction; just because we are dealing, not with the summation of the particular, but with knowledge of the necessity

determining each particular, this necessity must be able to manifest itself under favourable circumstances in a single instance. A single experiment is sufficient to justify the chemist in pronouncing the universal proposition that two substances in certain proportions enter into a combination having such and such qualities.

17. From this view of the true nature of empirical induction it also follows that the number of instances from which a universal proposition is obtained makes no fundamental difference in the logical process involved, and that the character of the process is obscured when the colligation of a number of similar instances is put forward as its essential feature. We should rather assure ourselves from the beginning that the distinction which Aristotle makes between the *ἐπαγωγή* as a process which undertakes a complete enumeration and the inference from the particular example disappears in our view; the fundamental process of reduction is the same in both cases, and this leads to the inconvenience that the term induction, which has established itself for the process of deriving universal propositions from particular perceptions, can only be used in the wider sense in which Aristotle sometimes uses it, and not in the narrower sense alone, according to which it denotes an inference from a number of similar instances to a universal proposition.

The importance in various ways which attaches to the comparison of a large number of instances as the basis of an inductive process must be shown in the fuller treatment of the subject. In this general explanation it is enough to point out a distinction which is often insufficiently observed, but which certainly appears in logical processes, according to the nature of the universal proposition to be obtained—more accurately, according to the meaning of its universality. This universality¹ may be merely *numerical* and comprehend only like instances, which are not conceptually distinguishable but only separate in space and time; or it may be a *generic* universality, to which are subordinated specific instances which differ from each other, but which meet in a general concept. In the first case the subject-concept contained in the universal proposition is fully determined, an *infima species*; in the other case it is a generic concept, admitting of a number of modifications.

The distinction may be illustrated by an example. That oxygen and hydrogen combine in certain proportions to make water is a universal proposition in so far as it is true in all particular instances and for all oxygen

¹ Cf. § 7, 10, I., p. 47; § 42, 2, I., p. 266 sq.

and hydrogen wherever it is forthcoming ; that carbon and oxygen in certain proportions yield carbonic oxide and in different proportions carbonic acid is also a universal proposition in the same sense. But the concepts contained as subjects in these propositions are absolutely determined, and have no further species below them. If, on the other hand, I lay down the proposition that all the elements combine chemically in certain proportions, then I have for my subject a general concept, and the proposition is obtained by inferring that what is true of all known species of a genus is true of the genus itself. In the former case that which was found in particular instances was extended to all completely similar objects in all space and all time ; in the latter, to all which agree in a general concept, although not similar throughout. The former may be called induction of specific laws ; the latter, generalizing induction or generalization.¹

The two kinds of induction have not always been distinguished with sufficient care ; in accordance with the precedent set by Aristotle, induction has generally been understood to mean generalizing induction.²

II. INDUCTION AS THE METHOD OF FORMING VALID CONCEPTS ABOUT REALITY.

§ 94.

Every universal proposition which ascribes to the things contained

¹ The expression "generalization" is used commonly for all the different ways of arriving at a universal judgment ; its etymology justifies us in confining it to those cases in which the process is conditioned by a generic, as distinguished from a specific concept. The term "generalization" then coincides with the narrower use of the term "induction."

² Benno Erdmann, *Zur Theorie des Syllogismus und der Induction in the Philos. Aufsätzen*, E. Zeller zum fünfzigjährigen Doctor-jubiläum gewidmet, p. 197 sq. ; and *Logik*, 1, p. 568 sq., has lately attempted to establish another view concerning the nature of induction and its relation to the syllogism. He begins by distinguishing two kinds of induction ; in the one we infer from a part of the extension of the subject to the whole extension of the subject, in the other from a part of the content of the predicate to the whole content of the predicate (the kind of inference generally known as inference from analogy). Leaving the latter for the present, the former may be represented :

S_1, S_2, S_3 etc., are P , therefore all S 's will be P .

The question is, what is the logical justification for this inference ? how far is the generalization contained in the inference one which is necessary to thought ? We see at once that the universal proposition which is inferred can have only a probable validity, and differs in this from the conclusion of a syllogism with valid premises ; the question is, therefore, whether the special cases given can contain conditions which make probable statements about the superordinate universal, or, more accurately, probable predictions about the still unknown S , a necessity of thought.

These statements are necessary only in so far as we may assume that in the subjects

under a concept A a certain predicate as necessarily belonging to them, presupposes the objective validity of this concept, *i.e.*, that its attributes belong to each other with objective necessity, and affirms that with these

S_3, S_4 , etc., which are not given but are included in the conclusion, the same causes of the reality of the predicate P will be present, as in the S_1, S_2 , which are given, and that these same causes will bring with them the same effects; that these causes are contained in that which is common to all S 's, in the nature of S . The assumption can be more exactly expressed in the two propositions:

- (1) the same causes will be given,
- (2) the same causes produce the same effects.

The second of these propositions may be accepted as immediately evident. Causes are events in so far as experience teaches us that their actuality is connected with the actuality of other events in such a way that when they occur these others also occur. We learn from experience that an event which we regard as a cause is not followed by different effects, but that the effects are definitely fixed for every cause. The causal law says: Every event which is actual requires sufficient causes for its actuality; it may also be formulated: Every cause produces an effect which is determined by it. Thus the second proposition follows as an immediate consequence of the causal law.

Although this exposition leaves the question open as to what character belongs to the causal law, I am content to point out that according to it we can only know from experience that there is anything of the nature of a cause (*cf. Logik*, I, p. 582: the causal law does not tell whether events are given as causes), and that only experience teaches us that the same events have the same effects. Thus, according to Erdmann, the proposition that the same causes produce the same effects is only a generalization from experience, which can only teach us in the first place that hitherto when an event A has occurred another event B has occurred, and that up to the present time A has always been followed by B , and not by C or D at random. To obtain from this a universal proposition which will be true for the future also, we have need of generalizing induction; hence according to Erdmann himself the proposition is only probable; it cannot be accepted as an immediate consequence from the causal law in the above sense, since it merely brings to it minor propositions which are inferred from experience, and are therefore merely probable; it does not depend for its validity upon the evidence of the causal law itself.

But for the present we will allow the second proposition to stand, and turn to the way in which the first is grounded; of this it is rightly said that it must have the really decisive function for the inductive inference in Erdmann's sense. After various possibilities have been rejected there follows the proposition: We must allow that we assume the same causes to be given in the unobserved similar S 's, merely because they have constantly appeared in the given actuality, S_1, S_2 , etc. Thus the thought itself is an inductive statement, it is the most general amongst all the general principles obtained by induction, for every other presupposes it as the basis from which it is itself obtained.

In other words, Erdmann's theory is primarily no other than that of J. S. Mill. Every particular induction depends upon a general presupposition, and is syllogistically inferred from it; for this reason therefore we cannot speak of an antithesis between the inductive and the deductive process, of an essential difference between the two. The distinctions made on p. 585 do not affect this point. The particular induction is a necessity of thought only in so far as it follows syllogistically from the general presupposition. But this general presupposition is primarily only the expectation that the same causes will be given, and this expectation is not a necessity of thought, as we are

attributes constituting an essential concept the predicate is necessarily connected.

The statement that the attributes of a thing necessarily belong to-taught on p. 585, but has its origin in experience alone. Thus, we should have necessary inferences from a principle which is not necessary. In one important point Erdmann certainly goes further than Mill; he calls the presupposition a postulate of foreknowledge, and says that our thought receives from the actual the problem of learning to know beforehand, upon the ground of what we know by experience, the experience which is possible and to be expected. Thus the presupposition is not what Mill represents it to be, a mere result of previous experience; but it is made for the sake of an endeavour, a problem which we set before our thought.

With this I agree completely; but it does seem to me that this problem is too narrowly limited, if it is to be confined to mere foreknowledge of the future. No doubt for all practical purposes this is the main interest, but for purely theoretical knowledge this important distinction does not exist. Here what is most important is the endeavour to understand things, to find in them a comprehensible order; and the particular is understood when it is recognised as necessary, and it is recognised as necessary when it is the consequence of a general law which determines both past and future. The presupposition that the particular must admit of being presented as the representative of a general concept, as the fulfilment of a general law, is not merely a postulate of foreknowledge, but a postulate of the whole endeavour to know; and the process of induction consists in finding, upon the ground of the presupposition, these concepts and laws in the particular form required by the constitution of the actual. We should have attained the object of induction if, without reference to the future, we were to represent what is merely known hitherto as such an expression of necessity.

It is nothing more than this which is affirmed when I say that induction is a process of finding universal major propositions when the conclusions are given,—a reduction and therefore the reverse process of the syllogistic deduction from given major propositions, that all induction consists in framing hypothetically universal propositions and comparing their consequences with the Given. Erdmann himself says that verification by experience is the justification of the process. But how is this verification brought about? Obviously by showing that every new experience agrees with the general proposition when this is applied to the instance in question. Thus the confirmation presupposes a comparison of the consequences of the general proposition with the particular given, while the consequences themselves can only be deductively developed. Represented schematically:—

To S_1 is P , and S_2 is P , etc., we add as universal major premise: All S 's are P . If a new S_n comes in view, we infer from the universal major premise: this will also be P , and this conclusion is confirmed. In the simplest cases we lose sight of this process, because it is so simple and takes place so easily; but when we apply the law of falling bodies to the pendulum in order to show that its motion is determined by the law, the deductive calculation by means of which the agreement between the observed motion and the law is ascertained is quite clear.

I cannot therefore consider my own views to be refuted, since Erdmann's theory in its consequences leads to the same result. The attempt to show that the comparison of induction to a reversed mode of calculation fails, assumes what I never affirmed, that they are the same in all respects. I have indeed expressly noted the difference (p. 309).

I must, however, note a few objections to Erdmann's exposition. He aims at reducing the presuppositions of induction to a causal concept which merely relates events as cause and effect (although he remarks incidentally that this view may perhaps seem too

gether is itself the result of a reduction, which refers their actual co-existence to a necessity contained in the unity of the thing as the ground of these connected attributes.

narrow). But then his examples should at least have agreed with this formulation. On p. 570 it is stated that various hexagonal crystals have a double refraction, and hence it is inferred that all hexagonal crystals have a double refraction; but the quality of being hexagonal cannot be regarded as an event the actuality of which is regularly accompanied by another event, that of refracting a ray of light. Nor in the proposition (introduced as an instance of a false induction) that all birds learn to talk can we call a parrot an event with the actuality of which the actuality of learning to talk is regularly connected. The formulation of the ultimate presuppositions of all induction is certainly too narrow.

We see this still more clearly when we consider the other branch of induction which infers from particular predicates to the totality of predicates—what Erdmann calls “ergänzende Induction.” Here, from the fact that certain partial predicates P_1, P_2, P_3 , etc., of a total predicate P , belong to a subject, we are to infer that P belongs to it. Example: this body has the colour, the extensibility, and the specific gravity of magnesium, therefore it is magnesium. On p. 583 we are told that in proportion as we find more P_1, P_2 , etc., together, we may be assured that these P_1, P_2, P_3 have not come together by chance, but obey a cause contained in the nature of P by which they are combined in one whole. Here the previous concept of cause is quite inapplicable; the nature of P is not an event any more than colour, specific gravity, etc., are events. The attempt to treat the two branches of induction as parallel leads, when represented in this way, to obscurity. We must point out especially that the predicate-concept P cannot be called a “universal of content” with respect to its attributes, and treated as parallel to the general concept S in its relation to the particular S ’s included in it. This way of treating them as parallel leads to a logical error. On p. 570 there is given as a negative “ergänzender Inductions-schluss”: G is not P_1, P_2, P_3 , etc.; therefore G is not P . But we need no proof to show that the first premise suffices to exclude the subsumption of G under P ; the absence of one attribute of a concept makes subsumption impossible, and it is therefore superfluous to go through all the different premises.

Finally, I must in self-defence touch upon one special point. On p. 603 sq. Erdmann endeavours to show that at least two premises (S_1 is P , S_2 is P) are necessary to an inductive inference, while I had stated (p. 310) that it was not essential to the nature of the process itself to start from a number of instances, since the presupposed necessity might under favourable circumstances reveal itself in a single instance.

I cannot see why, according to Erdmann’s own premises, the conviction that the same causes produce the same effects, and that the same causes will be given, cannot attach itself to a single fact in such a way that in similar cases we should expect the same. In the ordinary course of thought it is certainly so, but even for the most cautious consideration one case must be sufficient for the application of these presuppositions, for similarity does not necessarily presuppose many to which a new instance is similar, there may be similarity with only one. Moreover, since the result of an inductive inference is after all only a hypothesis, there is no reason why these hypotheses should not be founded upon one instance.

So soon as I have grounds for the assumption that a certain phenomenon is conditioned by a certain thing, and for the assumption that I shall come across other similar things which I shall recognise as similar, then the inference that what is true of one will be true of all similar to it is fully justified according to Erdmann’s own presuppositions. This is the case in the example I have used (Erdmann employs for his refutation another example which I have not used, and of which the conditions are

In this way the significance of the subject-concepts in propositions which are meant to express an objective necessity differs from the merely logical significance of the concept as a subjective product, and in conse-

essentially different): the chemist can infer from one experiment that two substances in certain proportions will enter into a combination having certain qualities. He has means of distinguishing whether or not he has before him the same chemical substance, and knows that chemical substances of the same kind are exactly alike in their qualities, and when the experiment is carefully prepared he has ground for assuming that no unknown variable causes have been at work; such are the "favourable circumstances" of which I spoke. Erdmann here objects that a particular case S is P cannot determine whether the conditions under which P is predicated of S are essential or accidental; the assumption that they are contained in the nature common to all S 's must be grounded upon at least two instances. This is true when S is a universal concept generic to the particular S 's, which therefore have, in addition to their common nature, specific or individual differences, upon which the predicate observed in a particular S may depend; or when the predicate in the particular case may depend upon varying and accidental conditions; but it is not true when S is an infima species of such a kind that all particular S 's are exactly similar. It is for this reason that I have myself always pointed out (§ 94, 3, 4; § 95, 2, 3, etc.) that in order to exclude accidental circumstances it is generally desirable to make a number of observations; and have represented the possibility of obtaining a general proposition from a single experiment as an exception occurring under favourable circumstances. When therefore Erdmann shows (p. 604) that an isolated observation of the number of stamens in a floral specimen does not suffice for a general proposition, he refutes nothing which I have said. I grant also at once, since I have myself stated it elsewhere (e.g. p. 320), that the "favourable circumstances" consist of knowledge obtained elsewhere, from which I am able to infer that within the province of chemistry the particular subjects of my experiments will be exactly similar, and that so far there is a deductive element in the process. But that does not affect the fact, that on the strength of a single observation a general proposition is believed, which cannot be deduced in this definite form from other propositions, solely upon the presupposition that events are determined according to law and the Given by universal rules; and that there is therefore no fundamental difference of process between the inference from the single example and the inference from several examples. For the inference from several examples also presupposes, according to Erdmann's own showing, that the causal connections are the same, and makes use of several examples only in order to ascertain what can be accepted as the same causes. In practice it is always found in the inductive process that we are unable to avoid making hypotheses, upon the strength of a single observation, concerning a law upon which they are based, and that to a large extent the multiplication of experiments serves only to test and verify our conclusions and to exclude errors. I cannot therefore regard my view of the inductive process as refuted in any point by Erdmann. The antithesis between induction and syllogism as two different methods of proof is justified while, as with Aristotle, induction as *ἐπαγωγή διὰ πάντων* is a means of proving a proposition; as soon as we recognise that this is impossible, that we can obtain only hypotheses by which to explain the given deductively, then the principles of the syllogism become regulative for the inductive process also (cf. Riehl, "über Galilei," *Vierteljahr. f. w. Phil.*, xvii. 1).

In his general view of the theory of induction, Dr. Venn (*The Principles of Empirical or Inductive Logic*, London, 1889) is a follower of J. S. Mill; but in his careful and acute analysis of all the presuppositions involved in inductive inferences, he not only supplements in a valuable way Mill's *Logic*, but also corrects its doctrines on essential

they were logically complete, *i.e.* if their attributes were fully determined, they would not exhaust the content of the whole possible perception of the object, but would merely include those features which first present themselves; and the subsumption of the given object under such a concept merely states that I find the attributes contained in this concept in my perception, and therefore connect the given with a familiar idea.

2. When, however, I come to employ such a concept in forming a judgment which is intended to express real, objective necessity, hence invariable universality and applicability, I presuppose that the subjective comprehension of attributes has an objective ground, that the attributes belong together necessarily and represent one of the fixed forms which the classification of the given has in view. I assume that it possesses not merely the purely logical universality of being applicable to an indefinite number of particular ideas, but objective universality in the sense that by virtue of the order of nature these attributes belong together, and the same complex will therefore recur wherever the conditions are present, and that the same order of nature separates it definitely from other complexes.

The application of such a concept to a universal judgment expressing necessity (*e.g.*, all gold is malleable) itself presupposes a process of reduction in which I assume that the actual co-existence of certain attributes is in some way grounded in the nature of the things in question (or else in their relations), and thus explain the fact that I find the same attributes repeatedly combined by an assumed necessity.

It is this assumption which alone makes it possible to regard the particular thing as the representative of an essential concept which is objectively valid, of a concept which does not merely signify that I once had occasion to combine certain attributes subjectively, but which asserts an objectively universal ground for the combination.

3. When the perception as it recurs continues to present the co-existence of that which is already thought of together in one concept (which is perhaps first formed by chance and merely as subjective), then it gives rise to the assumption that the combination of the attributes is objectively grounded, and that their co-existence will therefore always recur in the same way. All water which I find to be like in its perceptible attributes to what I know already, every piece of gold or silver which I examine, is a confirmation of that assumption, and strengthens my right to regard my concept as an essential concept. It is not, therefore, for the subjective

necessary they must follow from the subject with either internal or causal necessity. They must therefore be grounded in the fact either that different modifications of the attributes of the subject-concept, or that different causal relations, bring about different predicates; and this is so both when the different predicates appear in the same thing in consequence of its change, and when they appear in different things subsumed under the same concept.

The problem, therefore, of induction as a means of finding essential concepts, is to derive the different characteristics which appear in things subsumed under the same concepts from laws of development, or causal laws.

In this way the differences between individuals, which are disregarded in the traditional logic, become important in the formation of our concepts. The Darwinian theory has the credit of having made us aware of this deficiency in logic and having pointed out the necessity of explaining the specialization of general concepts by means of general principles.

1. Every process of reduction, by which we try to find for a judgment of perception of the form this particular is *b* the premises from which it necessarily follows, presupposes the subsumption of the subject under a concept *A* without which a universal judgment is impossible. The primary form of the syllogism in which the perception appears as conclusion is:

All *A*'s are *b*. This is *A*; therefore it is *b*.

It is possible here, either that the concept *A* already contains the attribute *b* in itself, the major premise being an analytical judgment; or that the concept *A* does not contain the attribute *b*, and then the major premise states that the predicate *b* is necessarily connected with the attributes constituting the concept *A*, hence that it necessarily belongs to the given particular because it has the characteristics of *A*.

Now the concepts under which we can, at the beginning of a methodical process, subsume the objects given to perception are at first only the subjective products which have arisen from the apprehension of one particular, or by abstraction from several particulars, and the nature of which we have examined in § 42 (I. p. 265). It is natural, therefore, that even if

of induction; for it seems to me to show that he also, in spite of his opposition to Jevons, does not attach the chief importance to the so-called simple induction as opposed to deduction, but sees in the whole process a method of showing a hypothesis which has been framed in some way to be more and more probable, by means of processes which are of an essentially deductive nature.

they were logically complete, *i.e.* if their attributes were fully determined, they would not exhaust the content of the whole possible perception of the object, but would merely include those features which first present themselves; and the subsumption of the given object under such a concept merely states that I find the attributes contained in this concept in my perception, and therefore connect the given with a familiar idea.

2. When, however, I come to employ such a concept in forming a judgment which is intended to express real, objective necessity, hence invariable universality and applicability, I presuppose that the subjective comprehension of attributes has an objective ground, that the attributes belong together necessarily and represent one of the fixed forms which the classification of the given has in view. I assume that it possesses not merely the purely logical universality of being applicable to an indefinite number of particular ideas, but objective universality in the sense that by virtue of the order of nature these attributes belong together, and the same complex will therefore recur wherever the conditions are present, and that the same order of nature separates it definitely from other complexes.

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3. When the perception as it recurs continues to present the co-existence of that which is already thought of together in one concept (which is perhaps first formed by chance and merely as subjective), then it gives rise to the assumption that the combination of the attributes is objectively grounded, and that their co-existence will therefore always recur in the same way. All water which I find to be like in its perceptible attributes to what I know already, every piece of gold or silver which I examine, is a confirmation of that assumption, and strengthens my right to regard my concept as an essential concept. It is not, therefore, for the subjective

formation of concepts, or their purely logical completeness, that it is essential for them to be abstracted from a number of similar particular perceptions, but for our belief in their objective significance, by virtue of which they are capable of expressing the nature and differences of definitely determined and separate forms of beings.

4. The fact that our earliest concepts cannot exhaust the full content of possible perception, together with the further fact that in the natural course of thought general concepts are formed side by side with the most specific, involves the appearance of perceptions combining a further predicate with those attributes of the concept which serve for subsumption, or determining more accurately an attribute already included in it. We learn that a bee has a sting and a snake a forked tongue after the subsumption of the particular object under the concept "bee" or the concept "snake" has already taken place, and we learn it first from a particular instance; in the same way, though it is part of the concept gold that it is heavy, it needs more careful observation to teach us that a piece of gold is heavier than a piece of iron, lead or silver of the same size.

That in this *A* & *b* occurred which was not included before in its concept is for mere perception a simple fact; for the systematizing impulse in its search for necessity it serves as a stimulus to the question whether this new predicate is necessarily connected with the attributes of the concept or not.

The question will first lead to comparison; if all the bees which I can examine have a sting, and all the snakes a forked tongue, then the assumption is confirmed. If there should be a bee without a sting, or a snake with a tongue not forked, it would be refuted, and the universal judgment by which we desired to explain the particular fact invalid: there would be no necessary and universally valid connection between the attributes of snake and a forked tongue. Thus, on the one hand, our concepts, once established, are always being enriched by such new attributes as are always found together with the original ones; and, on the other hand, progressive perception is always breaking up formed concepts, and separating from their complex some attributes as not necessarily belonging to the others, because the latter have been found without the former. A man who had never in his life seen any but white sheep would at first suppose the colour to belong to the other attributes of the animal, and would regard his concept, that of a white sheep, as one of the fixed forms of nature; on coming across an animal like a sheep in all respects but that of colour his

assumption would be destroyed, and the concept framed would not be confirmed as was expected ; invariable connection between the colour and the other attributes is absent, and the first result is that the colour is removed from the concept as a variable and therefore unessential attribute. There appears the particular judgment that many sheep are white, but that others may be black ; according to the view of the older logic the colour is only contingent, and the concept has nothing to do with it.

Thus the process is everywhere a tentative one, forming from a co-existence of attributes, once or repeatedly perceived, the assumption that these attributes belong together and constitute an essential concept, which makes all its elements necessary as constituent parts of it ; and this assumption continues to stand until refuted by some fact which forces us to remodel the concept. The more extensive our field of observation becomes, the less probable is the assumption that any negative instance has escaped our observation.

5. The process of induction, therefore, is first employed for that formation of concepts which must be the basis for all further progress ; and in some departments it meets at first with no important difficulties in this work of laying the foundations. As regards many simple substances, we find constant confirmation of the assumptions that their attributes are always combined in the same way and clearly separated from other groups of attributes ; and in this way there is formed a fixed kernel of objectively valid concepts, together with the justified hypothesis that we shall find the same constancy in all similarly formed concepts. No chemist is afraid that when he discovers a new element it will behave differently from all the others, and manifest variability in contingent attributes ; and the inductive hypothesis that the chemical elements yield us these distinct fixed concepts, in which everything is necessarily determined by the whole, by the nature of the substance, is always being confirmed anew.

6. Even these concepts have not, indeed, been too easily discovered. The earliest and most straightforward induction, that the characteristics which occur together in these pieces of gold or silver will always be together in the same combination, and will thus confirm the necessity of the combination, has yet a difficulty to overcome ; and this lies in the variability of all observable things which we learn from a more extended experience, and the importance of which for the establishment of the concepts of things we have pointed out in § 72, 14, p. 86 sq., and § 77, 6, p. 163. A complex of perceptible attributes could be definitely accepted as the

real essential concept of a thing only if it should remain invariable ; when a change occurs, and water becomes solid or gold fluid, then the necessity by which the attributes of the substance are held together in its habitual state cannot be unconditional and contained only in the attributes themselves, the assumed concept breaks down. Instead of being the expression of the *essence* of things, it becomes only the expression of their *chance condition* ; and the same principle according to which the black or white colour of the sheep was described as unessential or accidental compels us also to describe the solidity or fluidity of water as accidental and not belonging to its essence. But then we are in danger of having nothing left but accidental states ; for if we put aside the variable as accidental, what perceptible qualities remain as the kernel of the concept, which are common to both steam and ice ?

7. The only safeguard against this failure of our first attempts is to accept change itself as necessary, and to see whether we cannot substitute for the judgments which aim at constructing—*e.g.*—the concept of a substance by means of the necessary and invariable connection of certain attributes, others which are the expression of change itself as necessary and as proceeding from the nature or the relations of the thing. The concept will then become (see § 77, 6) a system of universal propositions, which represent the changing states of a thing as the necessary consequences of certain presuppositions, and in this way connect them together ; and these universal propositions are also to be obtained by way of reduction, which lays down a universal major premise as a ground for what is actually observed.

Two alternatives are open to us in this process of reduction. Either the universal propositions may be constructed in such a way as to express merely the connection between the fixed attributes and changing qualities of the same thing, in which case the necessity of the change proceeds from the nature of the thing itself and is thus internal (§ 33, 3, I. p. 198) ; or we are compelled to have recourse to external relations, to regard the change as brought about by external causes, and its necessity as the necessity of external causality.

8. We cannot ascertain which of these alternatives to take from one particular case, but only by comparison of many cases, in which the changes supervene upon a given state of a thing which is expressed by a previously formed concept. The question is, whether this comparison is favourable to the hypothesis that the necessary ground of the changes lies

in the nature of the thing itself, or to the other hypothesis, that the change depends upon external causes.

Organized beings seem in their development to afford an illustration of the first kind of change. All individuals of the same kind pass through the same series of like changes in similar intervals of time, and so afford ground for the assumption that the subsequent state follows necessarily upon the preceding, and that these sequences are grounded upon the unity of the being itself. Thus the essential concept of an organic species cannot indeed be exhausted by the attributes expressing any one given state, whether that of maturity or of some earlier stage of development; but it might be exhausted by the law of development, which tells us what particular forms of the individual succeed each other in an invariable series, from the germ to the state of maturity, and thus confirm the assumption of a purely internal necessity. In establishing this law of development induction would have constructed the full and exhaustive concept, and the particular individual would prove itself to be the realization of this concept by the fact that this development does actually take place in it.¹

If, on the other hand, we should try to regard the changing states of a quantity of water in the same way, the attempt would fail. It is true, that with changes of temperature, beyond certain limits, the aggregate state changes, and so far we must presuppose a necessary connection between the two, leading to the concept that it is the nature of water to be ice below 0° Centigrade, fluid water between 0° and 100°, steam above 100°; but the corresponding changes of temperature and aggregate state do not always proceed in the same way, and in the same succession, from previous states. The state of fluid water does not follow of itself from the state of ice; on the contrary, the change can be explained as necessary only by causal relations, it is not produced by the thing itself. Hence, in this case the system of universal propositions, which is to express the essential concept of a changing object of perception, must include causal relations,

¹ That the development of the organic individual itself depends upon the external conditions, and is modified by variations in these, may be left out of consideration here, where we are concerned only to illustrate the logical opposition between those concepts which must be constructed by means of laws of development, and those which need the help of external causal laws. In reality, the two points of view combine in the completely accurate determination of the concepts; that water solidifies at 0° is grounded primarily on its own nature, and is so far comparable with a law of development; it is only the becoming cold which depends upon external relations.

which connect the states of the object with external conditions ; and the complete concept more accurately stated will be that fluid water, when cooled below 0° , changes into ice, when heated above 100° into steam.

Those changes which follow from mixture and chemical combination occupy a position between laws of development and external causal relations. If we disregard both the external conditions, which may promote or hinder the combination of two substances, and the variability of the combination itself, then the change in each substance, which follows from the combination, though it cannot be regarded as the development of one alone, is yet comparable to development, in so far as its ground lies only in the nature of the combining substances, which both persist in the combination in such a way that the attributes of the combination are inseparable common attributes of the substances combined. One subject does not merely undergo action by which it is changed, from another which remains distinct from it, but both together constitute a new unity of which the attributes seem to be grounded only in the nature of the combined substances, in a way analogous to development ; on the other hand, there is also a causal relation in so far as the change occurs for the one substance only when the other is added to it. It is true that the complete development of the causal concept on the one hand, and the atomistic theory on the other, put an end to a distinction which continues only so long as in the first stages of induction we confine ourselves to concepts which are combined from perceptible attributes and changes.

The peculiarities of chemical action explain why exhaustive essential concepts of substances must include their chemical affinities, while chemical constitution forms the fundamental conceptual formula for all composite substances.

Thus, to complete the inductive process, which begins with the formation of concepts, it is necessary to find laws of development, external causal laws, and the chemical laws which lie between them ; but the finding of these is conditioned by the first preliminary beginnings, in which we referred attributes invariably given together to a necessity of co-existence, because only in this way can we obtain subjects for laws of development and causation. From the variability of objects we get only the general rule that simple, fixed, conceptual formulæ, expressing merely a co-existence of perceptible attributes, are primarily true only as the expression of certain perceived states or stages of development in the thing, and not as exhaustive essential concepts ; they should rather be regarded as meant in

the first instance to serve as a basis for our investigation of those laws of change by which alone the essential concept can be made complete.

9. We are next concerned, not with the particular processes by which such laws are found and verified, but with a further and difficult question which is connected with the fundamental principles of that induction which aims at forming concepts, and which is more especially suggested by the organic world.

We have pointed out above that when, in forming our concepts, we come upon attributes which vary with the individual in a complex in other respects fixed (such as varying colour in different species of animals), we are easily satisfied by calling these attributes unessential and accidental, and therefore not to be included in the concept. The small quantitative differences in individuals, which cause particular specimens of a species of plant or animal to be distinguishable from each other, are treated in the same way; they matter nothing to the concept, which is sufficiently elastic in its attributes to admit of small variations without injury to its validity. Whether the leaves of a plant are more or less hairy, whether the bones of one horse are somewhat stronger than those of another, whether particular organs are relatively more or less strongly developed, the concept remains the same; a fixed law binds all the parts together.

In this easy disregard of small differences, and the convenient distinction between essential and unessential attributes, we trace the original æsthetic and teleological element contained in the Platonic doctrine of ideas and the Aristotelian doctrine of forms. The concepts of organic forms are ultimately types which we think of as intuitable by sense, original models of which the particular individuals are more or less successful, and therefore more or less beautiful, copies; when nature reproduces them in somewhat different substances, there are, of course, slight deviations, just as there are in the copy of a human work of art, which nevertheless reproduces the total impression in such a way as to be easily recognised by intuition.

10. But this expedient cannot continue in face of the stricter logic of modern science. The given must be necessary,—necessary in its full determinateness, in the totality of all its parts, as it is for the most accurate observation. If two things, *A* and *B*, falling under the same concept *M*, show deviation from each other, like the black and white sheep, then the colour of the black sheep must also be regarded as necessary, and a ground sought for it; we cannot have a predicate, *b* or *w*, belonging now

to *A* and now to *B*, without any connection with the attributes of the concept *M*, an indifferent appendage which might as well be something else.

Here we must face the dilemma: either the different attributes of *A* and *B* are entirely determined by the attributes constituting their concept, which cannot then be absolutely the same for *A* as for *B*; or the conceptual attributes are the same, and then the variations must proceed from external causes, by which they are made necessary. They are not unessential even in the latter case, for in this reaction upon external causes the essence of the thing is again revealed.

11. If we take the first case, then it may be that the unity of the concept disappears altogether, and we have really two different concepts; we make a special specific concept of black sheep; the remaining attributes of the one concept are, it is true, similar to those of the other, but the very fact that they necessitate another predicate reveals their difference. If, on the other hand, the unity of the concept is to be maintained, it can only be by expressing the concept in a formula, in which the variation of one or more constituent attributes, which are not absolutely determined, but admit of a certain breadth of interpretation, is connected with variation in others. If the black colour of a sheep were always connected with some peculiarity of organization in other respects, and white with some other peculiarity, then the resulting concept, instead of having none but constant and absolutely determined attributes, would contain some which are variable within certain limits although connected by a constant relation; it would resemble, not the concept of the circle in which everything is absolutely determined, but the concept of the ellipse, of which the eccentricity is variable but in which one law governs all relations between its points. What Darwin describes as "correlation of growth" indicates some such formula, combining the variation of one attribute with variations of others, and thus enabling us to find necessity in them; if, *e.g.*, cats with blue eyes are invariably deaf, then the colour of the eyes is no accidental freak, but is connected with modifications in the organ of hearing.

12. When no ground can be found for such formulæ, then the deviation of particular predicates in instances of the same species points to external causes which deviate from those acting upon other specimens; the greater hairiness of particular individuals among plants, for instance, points to a drier situation. Such variations may be called accidental in relation to the concept, which asserts nothing as to the presence of this or that external

cause, but need only, in order to be exhaustive, contain the information that if cause *A* takes effect, then modification α must follow, if cause *B* then modification β .

So long as we are treating a concept merely as a subjective product, the addition of different attributes to the same general concept, and its consequent division, matters nothing if only they are compatible ; it then seems a matter of course that any possible colour may be added to the attributes expressing the structure of the horse in order to specify it. But when we attribute objective significance to the concepts, and try to understand the Given in its complete determination as necessary (hence all that co-exists in a thing as necessarily co-existing), then the general concept enters into a new relation towards its specializations.

13. As soon as we have made it clear to ourselves in what sense, having in view only the general presuppositions of induction, we can speak of accidental variations in the things of a species, or of the comprehension of attributes which differ within the unity of the concept, a wider perspective opens out before us. The same principle by which we are forced to distinguish in particular things between the attributes determined by the concept and those which are accidental, in order to include differences in individuals under one concept without giving up our assumption of the necessity of the given, assails the stability of our concepts themselves, threatens to dissolve their internal connection, and to change into external causal necessities that essential necessity for which we were looking. For no fixed line can be drawn between the differences belonging merely to individuals, which we must neglect in order to obtain an infima species, and the specific differences which we disregard in ascending to a genus; and even if there were such a line, yet the generic concept would be related to the specific differences in exactly the same way as the specific concept to the differences of individuals, as soon as we attribute objective reality to the generic concept and necessity to the co-existence of its attributes. Just as the attributes of the sheep are found sometimes together with black colour, sometimes with white, so the attributes of each genus *abc* are found in one species together with difference *d*, in another with difference *e*; but if both *abcd* and *abce* are to represent a concept, it cannot be that the same *abc* necessitates in the one case *d*, and in the other an *e* which is incompatible with the *d*. The possibility and ground of variation must, on the contrary, lie either in *abc* itself, in such a way that $a_1 b_1 c_1$ produce the attribute *d*, and $a_2 b_2 c_2$ the attribute *e*; or opposite external

causes must supervene upon the same abc , and so give rise in one case to d , in the other to e .

14. In the first case it is possible to have a real development of the concept, by means of which the specific concepts may be seen to be differentiations proceeding from the nature of the general concept; just as all possible ellipses proceed from the formula of the ellipse. If the generic concept contains abc , e.g. if the concept of the mammal contains a certain construction of the skeleton, a certain arrangement of the digestive apparatus, a general form for the teeth, etc., and also a formula according to which every variation in the teeth necessitates a variation in the digestive apparatus and the extremities, then such a concept might be schematically represented in such a way that each concept could be developed into a disjunction of the specializations, a into $a_1 a_2 a_3 a_4$, b into $b_1 b_2 b_3 b_4$, c into $c_1 c_2 c_3 c_4$.

We must now find a formula according to which, when a is determined as a_1 , b becomes b_1 , and c becomes c_1 , so that the only combinations possible are $a_1 b_1 c_1$, $a_2 b_2 c_2$, etc., each representing an essential concept; in this way the species would be *idealiter* contained in the genus, and the general concepts would have the same objective validity as the specific, though they would not exist in individual specimens, because the universal a could be actually present only as a_1 or a_2 , etc.¹

In the same way we may ascend still further to the highest generic concepts of the animal and the plant. But knowledge of the ideal comprehension of the species within the genus gives us only a hypothetical law; if there is an abc , of which the attribute a is determined as a_1 , then the other attributes are $b_1 c_1$; it tells us nothing as to the existence of the

¹ The merely logical or mathematical combination of these disjunctions, which would assume that there was no connection between the variations of a and those of b and c , would yield no less than 64 species, in which the concept abc developed itself; and if the number of attributes were increased, and their merely quantitative and therefore unlimited variations included, we should have the possibility of endlessly many specifications in which everything would be compatible with everything else. But such an assumption does away with that inner connection which is requisite to the concept of the thing, as well as with all possibility of induction; where everything can be everything we have only the particular and nothing universal. If Darwin's theory of the variability of organisms were not limited by his assumption of the dependence of certain variations upon others, it would then be represented by the merely mathematical calculation of the combination of all possible variations, which admits of no inference from one case to another because it presents us with no concept which is valid in reality, but only with subjective combinations. Here again we find the difference between the merely logical combination of attributes and that which is determined by the idea of the essential concepts of things. Cf. § 42, 4 and § 43, 8, I., p. 283 sq.

forms possible in these concepts. The question : how have these possibilities become actual? in what way has any one of these specializations, which has others in its train according to the law, been actually brought about? denotes the revolutions which have been introduced by Darwin's teaching even into logic, and which extend in their consequences over the whole province of substantial concepts.

15. The first result of the Darwinian theory seems to be merely destructive. It does away entirely with the Aristotelian basis, upon which, more or less, our logical theory has stood until now, more especially in obtaining its classificatory concepts, and denies the objective validity of the specific and generic concepts by which the classification of the organic world proceeded under the assumption that the whole organic world was constructed according to forms which could be fixed in definitions, and in such a way that each individual could be attributed to this or that species according to unmistakable characteristics. On the other hand, attention has been drawn to the gradual transitions between those differences which are peculiar to the individual and which have always been neglected in the formation of concepts, the differences of varieties to which a doubtful recognition has been accorded, with a tendency to refer them to external causes of climate, situation, etc., and the differences which are usually accepted as specific, and employed for the determination of species; and in so doing the Darwinian theory has assailed the distinction between the *διαφορὰ εἰδοποιῶς* and the *συμβεβηκὸς*, and represented it as an arbitrary one. By pointing out the innumerable transitional cases which confuse the boundaries of the species, and the impossibility of carrying out any classification in such a way that every individual can be confidently assigned to a species, it has substituted for the "discretion" demanded in forming concepts the continuum of imperceptibly small differences as being alone objectively valid; and by disputing the invariability of organic forms, and asserting that different forms have gradually come into being through small deviations from common forms, it has destroyed the chief assumption upon which the Aristotelian doctrine of the concept was based—the assumption that a significance independent of time attached to concepts, as timeless forms always realizing themselves in the same way. It is only for the present moment that, the intermediate members having disappeared, a part of the organic world falls into separate spheres in such a way that individuals of one sphere seem more similar to each other than those of different spheres; in another part we find continuous transitions

between individuals which are just as remote from each other, or even more so, than those which in the former case belong to different spheres; and if we should try to establish sharply limited concepts even within the first province, they would possess only an ephemeral validity, and the restless activity of variation would sooner or later compel us to re-model them like coins that have been withdrawn from the currency. They can possess no other significance than that of being a temporarily convenient arrangement, enabling us to make a general survey of all which is known to us.

No more impressive way could be found than this of showing the merely subjective, and—in relation to the totality of being—accidental importance of general concepts, which makes any universal judgment impossible; and the theory of the tendency of organisms to vary in all directions repeats the Heraclitean *πάντα ῥεῖ* in a way which seems to mock all our attempts to obtain from past experiences any universal proposition of which we could guarantee the truth for even one day, or to find in the co-existence of different attributes any necessity which would exclude other possibilities. There might be a history of this eternal flow of variation in individuals, if time and memory sufficed to write it, but there can be no knowledge of a universal which governs the formation of its waves. What remains as universal proposition, the so-called laws of variability and inheritance, is not capable of replacing the concept which has been banished; placed side by side in this crude form, the two propositions that the offspring are not like the parents, and that the offspring are like the parents, contradict each other. They are not, indeed, universal propositions expressing a necessity which enters into every case; on the contrary, the first one, strictly interpreted, contains only the negation of all necessity, inasmuch as the direction of the variation is left wholly undetermined; while the second merely states a tendency, *i.e.* a possibility, which at best only happens frequently, *ὡς ἐπὶ τὸ πολὺ*. All that remains as universal proposition, the multiplying capacity of every organic form, refers to just that which is a matter of indifference to the concept, the number of individuals corresponding to it; the struggle for existence also has immediate reference only to the numbers, since its direct effect is only to destroy and not to create. The only general concept which finally remains is the widest of all, that of the organic in general; but we are deprived of the means of predicating anything definitely about it with universal validity.

16. Although, therefore, Darwin's theory, in the form in which he has presented it, negates in principle within its sphere the presuppositions of all induction, of all subordination of the particular to a universal proposition; yet, carried a step further, it is fertile even in the sphere of logic by disclosing the inadequacy of the traditional doctrine of the concept and of induction, and by forcing us to revise these doctrines; and its importance in this respect is quite independent of whether its assumptions are confirmed or refuted.

It brings into evidence the impossibility we have already pointed out of treating the variable attributes of individuals as something external in relation to the concept; of drawing a line between that which is alike in all individuals and that which differs in different individuals, in such a way that the latter is really removed from scientific notice, and only finds a place at best in the form of occasional notices; and of being led by the subjective psychological motives, by which we are guided in forming concepts by abstraction, to overlook the claims of logical necessity. It is not, indeed, possible for the general concepts which are denoted by words to arise at all without this overlooking of differences and determinations peculiar to the individual; but that has nothing to do with their objective significance. Here Darwin has the great merit of having put an end to the neglect of all which would not conform to the traditional logic, and compelled us to consider the manifold of things in their full concrete determination; in this way, by showing that the determinations and differences of the individual need scientific explanation no less than common attributes, he has given a right direction to the induction upon which our formation of concepts is based.

It is possible to assume with Leibnitz that everything is individually determined according to its nature, and that therefore there exists for every particular thing a different law for the mutual relation of its attributes and its development; it is possible to say with Spinoza that the *modi* of one and the same attribute, which is everywhere essentially alike, are determined only from without by external causes in different ways; either view is at least consistent. But it is arbitrary to come to a halt at a given point up to which the inner determination of the concept prevails exclusively, and then to hand over what remains exclusively to external determination by causal relation. If Spinoza and Leibnitz are wrong a way of mediation can be found rationally only by endeavouring to think of everything as determined both by internal and external necessity; at any rate only the

most comprehensive consideration can determine the share which each has in the development of the particular individual. For if external conditions are capable of causing a malformation which we think we must regard as an accidental encroachment upon the order of nature, and neglect in determining the concept, then the normal construction must also be partly due to external influences which differ from the disturbing conditions; on the other hand, the direction taken by the malformation is partly prescribed by laws of development.

17. Another aspect of the importance of the Darwinian theory for logic lies in the problems which it raises as to our view of the relations between particular concepts. For a long time logicians had been content to regard concepts which were co-ordinate to each other, and subordinate to higher generic concepts, as something simply given,—*essentiæ rerum sunt æternæ*; by this scholastic proposition they excluded every question as to why just these concepts are realized in the world. The divisive judgment breaking up a genus into so many species which together exhaust its extension, is true simply because there actually are these and only these species, which agree in certain generic attributes; it was only by assuming such an eternally persisting and finite number of valid concepts that Aristotle could speak of an *ἐπαγωγή διὰ πάντων*. It makes no difference, from the point of view of principle, whether we assume that the existing species are all known, or that because of the narrowness of our horizon they are only partly known; the important point is that a finite number of specific concepts are assumed as the exhaustive expression of the manifold of the given, and that it is supposed that nothing must be said about these infimæ species except that they are subordinated in a particular way to higher concepts, and distinguished from each other by certain attributes. The whole relation, including not merely the fact that a universal specializes itself, but that it specializes itself in just this way, is taken as an ultimate presupposition, and is not even an object of investigation which should ask about the why of this relation and derive it from more remote grounds. No one, however, has seriously tried to develop logically from the concept man the different races, nor its various species from the concept rodent.

This *ignava ratio* is now called upon by the theory of evolution to explain the plurality of the species themselves by some universal ground, to say why it is permissible to comprehend similar and yet different individuals in specific concepts, and these again in genera, and then from these grounds to deduce the reason why just these forms and no others, no more

and no fewer, are forthcoming as species of a generic concept. What makes the solution of this problem possible is the principle that everything which is has first gone through a process of becoming; that the history of reality corresponds to the logical development and differentiation of concepts; that the laws by which becoming is controlled have under given circumstances developed differences in what was previously undifferentiated, and have made necessary just these particular differences. The only point which is opposed to scientific principles, because it excludes the possibility of scientific knowledge in the form of general propositions, is the way in which Darwin frequently speaks of the variation as something quite capricious, as a tendency which strives, so to speak, in all directions at once, thus making the basis of his edifice an absolutely irrational element. As soon as we inquire about the laws of this variation, and try to find rules according to which it *must* follow—either (according to one school of Darwin's successors) chiefly through inner development, or (according to the other school) through external influences,—then the application of his principles is assured. It is true that we now have three unknown elements to determine, the universal essence upon which the organic is grounded, then its laws of development, and, finally, the external causes which determine it in this or that direction; but the difficulty of a problem is no proof that it has not been correctly formulated.

18. The solution of this problem would include a reconciliation between the Platonic-Aristotelian view, and the conception of mechanical explanation which refers everything to external causes; the general concept would contain the possibility of specialization in definite directions, but it would depend upon external circumstances which of the various possibilities became actual. And in another aspect the causal concept, when consistently followed up, attributes by means of the concept of force everything which a subject can become by external relations to its own essence, and thus treats it as in a certain sense a development of its being, though also conditioned by external circumstances; thus in this way also we are justified in desiring to think of the specializations of general concepts as determined both by an inner necessity of development and by the external necessity of causality.

19. These considerations, to which we are first led when we investigate the logical application of the Darwinian theory, are not confined to the sphere of the organic; we find questions of exactly the same sort in chemistry. Just as irrational as the plurality of the hundreds of thousands

of species of plants and animals registered in descriptive botany and zoology is the plurality of elements enumerated by chemistry. Although none of the difficulties are seriously felt here, which in the former case prevent us from forming sharply limited lowest concepts, although here, if anywhere, we have to do with objects which readily conform to the claims of discrete specific concepts, yet those of the chemists who are philosophers have not been willing to allow that with these elements we have come to the end of everything, and that all that is left for us to do is to see how we may bring them under higher genera which have only a subjective value, whether as metals or metalloids, according to their atomic value, or according to some other principle. The question has always remained : in what relation does this plurality stand to that which is common to them all, to the universal essence of matter? and this has given rise to the idea of explaining chemical atoms by homogeneous material atoms of the second power, which unite in different combinations, thus returning really to Aristotle, for whom matter is one, and only the form differs. But if we try to think of this theory as carried out, there is only one way open to us ; under different conditions of spatial distribution or of the motion of this last homogeneous substance, a different grouping must follow from the law which expresses its being, and the permanent relations which are now contained in the atoms before us must at some time have become what they are. And if we are inclined to call these idle speculations, we must remember that they arise in exactly the same way as every other induction, even those which seem to be forced upon us by nature itself ; the principle that we must explain the given as necessary is satisfied only by working backwards from the plurality of phenomena to ultimate presuppositions which are common to them all.

And thus we see that, however far apart they may seem at first, Darwin in the problems which he places before logic is striving towards the same end as Hegel : that completeness in the concepts applicable to reality upon which the justification of induction depends is only to be attained when the law is found according to which the plurality of determination arises. Hegel believed that in the essence of the concept he discovered, not only the necessity of development, but also the one law according to which the differentiation of the concepts proceeds, and produces from itself the content which progressively determines itself. From the point of view demanded by Darwin's suggestions we have only to presuppose that the laws, according to which the real plurality of distinct beings

has come to be, must be knowable in the form of universal propositions; *what* these laws are, and to what presumably original elements they are to be applied, is not to be discovered deductively, but only by way of induction.

20. The first result which these considerations yield for those methods of induction which are to lead to concepts is the general rule that when a tentative proposition, All *A*'s are *b*, is not confirmed, the comparison of many instances leading to the result : the *A*'s are in part *b*, in part *c*, then we must not infer that *b* and *c* have no connection with the conceptual attributes, but that on the contrary we have to discover the ground of the difference. This can only be done by again trying the possible hypotheses ; the ground of difference will lie either in modification of the attributes of the concept itself, or in external causal relations ; we have to see which of these hypotheses breaks down, which is confirmed, and to what extent the one or the other is to be accepted.

The complication of the problem of forming concepts with that of finding causal relations does not prevent us from investigating the methodical principles of its solution ; just because the whole process is a hypothetical one, the circumstance that we must begin with merely provisional concepts cannot be any hindrance to our formulating universal propositions about causal relations, and observing whether perception of the particular agrees with their consequences.

III. THE PROCESS OF OBTAINING UNIVERSAL PROPOSITIONS ABOUT THE ACTION OF CAUSES.

§ 95.

I. In formulating universal causal laws we first aim at predicating of certain perceptible things that by a certain activity they necessitate changes in other things, and we thus presuppose the popular concept of an EFFICIENT CAUSE.

The logical process is one of reduction. It starts from the chief postulate that perceptible events are necessary, and begins by conjecturing in a particular case that the change β in thing *B*, which follows immediately upon a change α in thing *A*, is brought about by *A*.

This conjecture is uncertain, because there is the possibility that the coincidence in time of α and β is due to chance. It will be confirmed by the comparison of several cases in which, on the one hand, $B\beta$ followed $A\alpha$, while on the other hand β was wanting in *B* when *A* α was absent.

From this follows, in the first place, the assertion that $A\alpha$ is the cause which regularly and necessarily produces $B\beta$.

But such a proposition does not attain to complete logical exactness without the determination of the quantitative values of $A\alpha$, $B\beta$, for which it is true; and it gains increased certainty from the establishment of a formula which states the effect β to be proportional to the efficient action of the cause. Where such a formula can be found and confirmed in all known cases, it may be stated as a causal law. *

II. In reality the presupposition of such simple causal connections between A and B proves to be a fictitious case, and the hypotheses referring to it meet with exceptions and deviations from the rule which point to the co-operation of a plurality of presuppositions, upon which the result depends.

The problem is, to find this plurality, to express in a partial law the contribution which each one yields to the result, and to formulate the law according to which the partial results combine to make the total result. This also takes place by means of a hypothetical process of reduction.

When we cannot show different co-operating perceptible causes in the original sense of the term, the distinction appears between the CIRCUMSTANCE and the EFFICIENT CAUSE, and the problem arises of eliminating the indifferent circumstances, and of determining for those which have an influence the law according to which they modify the effect.

In so far as there are circumstances which counteract the effect of an efficient cause, we need for the complete expression of every causal law a statement of the NEGATIVE CONDITIONS; *i.e.* a statement of those circumstances the absence of which is a condition of the result.

III. The simple and fundamental causal laws, to which the analysis of the given connections between different causes and circumstances leads, are therefore hypothetical assumptions, which can never be directly compared with the observed reality, because it nowhere shows absolutely simple connections. Only the consequences which follow from the combination of those simple laws can yield an expression of what actually happens in the form of universal judgments.

1. As we have shown in § 73, the concept of causality contains a synthesis, which, like that in the concept of substance, has for its primary and original aim to connect the spatially and temporally continuous changes of different things, by referring them to one ground of unity. In every judgment which expresses efficient action there is presupposed the

reference of distinguishable changes to different subjects, which at the beginning of the logical process can have only the phenomenal significance of § 91.

If we presuppose that there is given to perception the motion—or, more generally, the change—of an individual thing *A* which is given in this way by spatial limitation, and standing in immediate spatial and temporal connection with it the change of a second thing *B*, then the judgment that *A* causes the change of *B* seems to be a judgment of perception in the same sense as the judgment that *A* or *B* changes, which is valid as a judgment of perception because a universal necessity of thought constrains us to refer to one and the same thing the continuous changes of sense-given qualities which take place within the same spatial limits. Thus it seems possible, first, to ascertain efficient action in particular cases, and then from these particular causal judgments to obtain universal causal laws.

2. Against this we must set the objection that the application of the causal concept is not unambiguous in the same way as the application of the concept of substance. The assumption that the subject of a continuous series of changes suddenly vanishes to make room for another, in which the change proceeds further, is inadmissible; but the perception of the consecutive changes of two things is always open to the interpretation that what takes place is mere succession without inner connection, that the ground of the change in *B* is to be found only in itself or in an unnoticed third, not in the action of *A*; that, as it is ordinarily expressed, the coincidence of the two changes is chance, *i.e.* not necessarily brought about by the relation of *A* and *B*. If a bomb explodes just as some one touches it to throw it away, or if a man falls down in a fit just as some one is speaking to him, there is the same continuity of events which generally gives rise to the thought of efficient action; and if a causal judgment could be unambiguously and infallibly grounded by the particular perception by itself, then this succession would have to be apprehended as efficient action and its result, just as much as the breaking of a dish by a blow. But it is open to us to see in such cases only a casual coincidence of two events, only a temporal succession of changes which occur without inner connection as the result of inner development or of independent causes.

3. We are guided in our decision by the fact that we see one set of successions occur regularly, but not the other. Thus, if it is to claim validity, every causal judgment presupposes a universal proposition, and we move in a circle in so far as the universal proposition must be derived

from the particular cases, while the particular case can only be really established as a case of causality by the universal proposition.¹

The logical process by which we succeed in establishing the universal proposition is none other than that which we have described, in another application, in the previous section. We are naturally inclined to refer every connection of changes to an efficient action as its inner ground, and in this way to explain it as necessary; but in one set of cases this hypothesis is refuted by the fact that the same circumstances do not lead to the same event, while in another set it is confirmed because when the same circumstances are present the same changes always succeed each other.

4. Just because it is only possible to determine causal relations in this hypothetical manner, the procedure calls for careful consideration of the sense in which a causal relation is assumed and employed to connect phenomena. The analysis of the causal concept has shown it to us in different stages of logical elaboration, from the popular view which finds expression in the meaning of transitive verbs to the determination of the concept of invariable forces which under different conditions produce different results according to certain laws; and it has also pointed out how this elaboration of the causal concept goes hand in hand with that of the concept of substance. The logical process by which we arrive at universal causal propositions must take a different form, according as we ask in the popular sense what are the changes produced by certain perceptible things in exerting their activity upon other things, or what is the law of action of the forces which belong to the substances.

5. The investigation of causal judgments must always start in the first instance from the point of view taken by ordinary, unschooled apprehension. The problem is to determine in what way the things given to perception as the most obvious phenomenal unities regularly produce changes in other things; and here the changes in the object of the action are also taken as a unity at first.

6. From this point of view, to find the causal connections between daily events seems an easy and simple problem, which ordinary experience solves satisfactorily without the aid of any scientific methods. That to eat food satisfies hunger, and to drink water quenches thirst, that fire burns wood, and a stone is set in motion when thrown, that salt is dissolved by water, and cooking makes an egg hard: these propositions seem so easily

¹ Cf. § 47, 12, I. p. 314 sq.

discovered and so securely established that the way in which they have been obtained recommends itself as a model of causal induction ; all we have to do is to analyse the process as we observe it, and to reduce it to generally applicable rules.

It is really this popular concept of efficient action which J. Herschell in his *Preliminary Discourse*, and after him J. S. Mill in his *Logic*, have had in view in following the plan of Bacon's *Novum Organum* and applying the rules therein laid down for the investigation of forms to the investigation of such causal relations. The causes which are to be sought are those perceptible things, and changes of things, upon which other things and their changes regularly follow ; the cause is the invariable antecedent, the effect the invariable consequent. The assumption¹ is, therefore, that the same antecedents are invariably followed in time by the same consequents ; the problem is, so to disentangle the course of actual, observed events that it may be resolved into constant connections between given antecedents and given consequents. The result of the process consists in universal propositions, which state that whenever an *A* appears it is followed by a *B* ; when that has been ascertained we call *A* the cause, *B* the effect of the cause. To avoid the objection that in this case night would be the cause of day, Mill adds a distinction : it is not sufficient that *B* should have followed without exception upon *A* during the whole course of our experience ; we must also be able to believe that *B* follows *A* *unconditionally*, i.e. independently of any other determining circumstances. Because we do not believe that day follows night independently of the rising of the sun, while we do believe that the rising of the sun above our horizon, and the absence of any dark body between us and the sun, would have day as a consequence whatever other circumstances might be present ; therefore we call not the night, but the sun, the cause of day. In the same way another distinction is employed to remove the objection that frequently what we call effect occurs simultaneously with the cause, and that therefore the cause cannot be its antecedent : the *beginning* of the effect, we are told, may be simultaneous with the cause, but the caused phenomenon as a whole follows the appearance of the cause, and its beginning certainly does not precede the cause.

7. Starting from these assumptions, we get first two simple methods by which to select from the circumstances which precede or follow a natural

¹ We have noticed on p. 300 the ground upon which this presupposition is based for Mill.

phenomenon those which are connected according to an invariable law : the METHOD OF AGREEMENT and the METHOD OF DIFFERENCE.

In the first method we proceed in this way : we compare a number of instances in which a natural phenomenon a is given to observation ; in each instance we note all the antecedents of a , and also all the phenomena which follow a . Now we can say :

(1) that whatever does not precede a in *all the instances* cannot be its cause, whatever does not follow a in all the instances cannot be its effect ; thus we eliminate all the circumstances which are not causally connected with a ;

(2) On the other hand, whatever precedes whenever a appears will be the cause of a , or will contain the cause, or will be a part of it ; whatever follows whenever a appears will be the effect of a or will contain it.

Thus the process ends as a purely mechanical enumeration. Let the instances in which a presents itself as consequent be expressed by

$$A B C—a b c$$

$$A D E—a d e$$

$$A F G—a f g$$

where bc, de, fg represent circumstances accompanying a : now we eliminate from amongst the antecedents B, C, D, E, F, G because they do not always precede when a follows, and only A remains as the circumstance in which all the instances agree ; A will therefore be the cause.

By a corresponding process the effect is sought of a given circumstance which is taken as the cause ; we compare all the consequents of A , and that which is common to them all, a , is the desired effect.

The method of difference, on the contrary, compares two instances, which differ only in the fact that a is present in the one and absent in the other. If two instances are to each other as bc and abc , and if we find in the second an antecedent A which is absent from the first, then A must be the cause of a or a part of the cause of a ; if we find in the second instance a consequent a which is absent in the first, then a must have the effect a . This method is applicable wherever a new element is introduced into a known set of conditions and has changes following upon it ; we are then convinced that these changes are brought about by the newly introduced circumstance. This is the method of experiment.

Similar to these two, and really only a special application of the method of difference, is the method which Mill calls the METHOD OF CONCOMITANT VARIATIONS, and which has for its axiom that any phenomenon of which the

modifications are invariably followed by modifications in another must be the cause of the latter, or connected with its cause. In fact, the *plus* or *minus* of the antecedent and consequent constitute the two different instances which are necessary for the method of difference.

8. But it is just this application which reveals those deficiencies in the original formulation of the problem and its solution which are easily overlooked in the schematical representation of the methods, but which make themselves apparent even in the examples which are given.

If we disregard for the present the question how far we are justified in assuming that every phenomenon has only one kind of cause, if we overlook the difficulties caused by the frequent presence of counteracting conditions, and the impossibility of really enumerating all the antecedents and consequents of a particular event—even then the formulæ leave us entirely uncertain, in the first place, as to what we are to understand by these antecedents, and, secondly, in what sense universality belongs to these symbols *A* and *a*.

The examples which Mill quotes are, first, the contact of an alkaline substance and an oil as antecedent, the production of soap as consequent; then, the deposition of a solid matter from a liquid state as antecedent, crystallization as consequent; a shot through the heart as antecedent, death as consequent; the sun as antecedent, the gravitation of the earth as consequent, etc.; and this collection is enough to show how heterogeneous are the relations to which the methods are to be applied. Now it is a substance which, in combination with another, yields a new substance, now it changes in the aggregate state of one and the same substance, and in the example of crystallization that is treated as effect which closer observation shows to be only a particular form of the event itself; again, as in the instance of the sun and the earth, no relation of sequence in time can be established at all, and an ever-present body is taken as the cause of a perpetual motion.

Mill himself afterwards corrects this inaccuracy by saying that the causes which we have to look for and determine are not certain substances which may have all possible qualities, but one definite quality of the substances. But even then we are still left in doubt as to what we are to regard as antecedent and consequent, for qualities in the sense in which the word is here used are themselves permanent. If we are to take as the cause of the poisonous action of various metalloxydes their property of entering into insoluble combinations with the tissues of the body, which prevent the

possibility of chemical change, then there is no meaning in saying that this property is the invariable antecedent in such cases of poisoning; the antecedent is only the fact that such substances have been brought into contact with the body under certain conditions, or that they have been swallowed.

Again, what is the generality which we may attribute to our A and a ? Mill's examples show that by his methods he aims at reaching at once propositions of considerable generality; propositions such as that alkalis and oils combine to produce soaps, that in the transition from the fluid to the solid state crystals are formed. Here, then, we are not dealing with an investigation merely of a number of instances, which differ only in the combination of circumstances and in which exactly the same a is to be shown as cause or effect; it is an investigation of instances in which different phenomena occur, having only one element or attribute a in common, *i.e.* falling under the same generic concept a ; like that of Bacon in seeking the causes of heat, whiteness, density. Bacon's procedure was so far comprehensible that he undertook to determine the proper and formal cause, the essence of these qualities, which is everywhere the same, wherever heat, whiteness, etc., appear; but in Mill's application what we have to determine is not only the really fundamental qualities from which an effect results, but also the merely occasional antecedents which are accessible to observation, and of which it certainly cannot be expected that they resemble each other in the same measure and have in common a characteristic which is as easily recognised as the attribute a in the investigated instances; in Bacon's example, for instance, the perceptible antecedent of heat is now the rays of the sun, now friction, now burning. Mill's own example of crystallization shows the inadequacy of the methods, for if we ask what is common to the states from which crystalline bodies issue, we find that there is ultimately nothing but corporeality in general, since crystalline structure is formed from gaseous, fluid and amorphous solid states of the most various substances; the method of agreement would show as the invariable antecedent of this phenomenon only some material substance, and for this there was no need of a complicated method. The result would be similar if we conducted our inquiry as to the cause of death of a living being; the instances in which a living being dies have no antecedent in common—at any rate for the one way of investigating antecedents—except life, and life would appear at the end of the elimination as the cause of death.

The logical error in this formulation of the method of agreement is to be found in the assumption that the invariable antecedents of a number of phenomena to which the attribute a belongs in common, must also possess in common a recognisable attribute A , so that the whole process appears ultimately to be the very uncertain inference from analogy that the causes of events which in some way resemble each other must also resemble each other. On the other hand, the method of difference, as Mill states it, suffers from a similar logical fault—it is incapable of proving a universal proposition when we are inquiring about the cause of a given phenomenon. Take Mill's example: the two instances are, a healthy man in certain circumstances, and the same man in circumstances otherwise the same who is shot through the heart and dies; because the two cases differ in nothing but on the one hand cessation of life, on the other hand the shot which supervenes upon the other circumstances, it follows from the method of difference that the shot was the cause of death. In this case that is, of course, beyond doubt, but it does not prove the universal proposition that the cause of the death of a man is a shot through the heart; and yet the methods are to be employed to obtain causal laws, and to instruct us how to discover, not merely the effects of given causes but also the causes of given effects. This method, however, is only capable of finding the effect of a given cause, and, as Mill himself rightly points out, its usefulness for this purpose is limited by the difficulty of knowing whether two instances are really quite alike in everything except one single circumstance.

We do not mean by these objections that Mill's exposition has no firm basis, nor that fruitful inferences cannot be made in the ways indicated by him; we desire only to show that both the questions which are to be answered in this manner, and the limits within which an answer is possible, must be more precisely defined, and that to discover real causal laws is much too complicated a business to be achieved in such a simple manner.

9. If we are to avoid the inaccuracies of Mill's doctrine, it is most important to start from an unambiguous concept of causality, and we must be careful not to confuse generalization with simple reduction.

Turning back to the beginning of the section, we find that we had first to deal with a particular case in which the change a (regarded as a whole) of a thing A stood in immediate temporal and spatial connection with another change β (also regarded as a whole) of another thing B . The first question to be decided was whether the change β in B (the explosion of the

bomb) coincided in time with the change α in A (contact with the hand) only by chance, or whether it was caused by the change in A . In the former case it must nevertheless—according to our universal presupposition—have occurred necessarily, and therefore have been produced either by a necessity of sequence in the states of B , which was contained in B alone, or by some third agent.

10. By itself the particular case can settle nothing as to this; we need comparison of several cases. If we know that the same B has never, when we have observed it apart from A , undergone this particular change β , while the change β always followed in B whenever it was preceded in immediate connection by the particular change α in A , then we infer that it is the action of A which has caused the change in B . This assumption explains all instances, both positive, in which a change occurs, and negative, in which none occurs; while the assumption that B has changed of itself would involve a repetition of chance coincidence between the same change in B and the same change in A , which becomes more improbable in proportion as we observe more cases, and have less ground to assume such spontaneous variability in B . When we notice that a bell never vibrates and rings when it hangs still and without being touched by another body, and that it always vibrates and rings when the hammer strikes it, then we infer that the blow of the hammer is the cause of the vibration and ringing.

11. Thus the rule according to which we make our inference in this case must be formulated: If a body B shows a definite change β in several instances when a change α in another body A has preceded it in spatial and temporal connection, and, on the other hand, does not show this change when not so preceded, then we must assume that A by its action α has caused the change β in B .¹

If considerable experience had enabled us to be sure that B never underwent any change, or only changes which followed some definite rule, or changes of another kind than β —then a single instance in which we observed that an unusual change β followed in B after a change α in a spatially connected A would justify us in assuming that it is this A which caused the $B\beta$ (the so-called simple method of difference).² Then the

¹ This is what Mill calls the combined method of agreement and difference.

² Simple only if we overlook the fact that the conviction of the invariability (from inner grounds) of B needs a long observation—more accurately, a number of particular observations.

universal proposition is true : "Whenever *A* and *B* come together in this way, this change in *B* follows"; and this proposition would merely be confirmed if (by the method of agreement) we found the same change in *B* following the same action of *A* in a number of instances.

In this way we always infer that when a persisting state of rest of one or more bodies is interfered with, and a change follows, this interference has caused the change; a single experience is enough to convince us that a spark has exploded a heap of powder, and that touching a hot body caused a blister; and we at once believe ourselves justified in expecting that the same thing will happen again under the same conditions.

It is this which gives its peculiar power of proof to the arbitrarily instituted experiment. When a new body is introduced into a complex of known bodies which are at rest with respect to each other, and are as far as possible protected from the intrusion of unexpected agents, or when a change is arbitrarily brought about in their relations, and a change follows at once, then we are certain of the causal relation in proportion as we were familiar with the previous constitution of the bodies. We are still more certain if mere repetition of the experiment at arbitrarily selected times always gives the same result.

12. These considerations are primarily useful as excluding the idea of accidental coincidence, and as treating the observed instances as instances of causation; and, in accordance with our assumption that the causation is necessary, they are reduced to a universal proposition which states, at first, that this *B*, whenever it enters into the same relation with this *A*, will undergo the same change.

But this universal proposition is at first only a hypothesis, and rests upon the assumption that *B* in its causal relations is invariable, and is influenced in its dependence upon *A* neither by inner development nor by external circumstances. This hypothesis could be much strengthened if other bodies exactly like *B* behaved in the same way; we should then have an induction in the narrower sense, mediated by the specific concept *B*, and stating that all *B*'s manifest the same change β when an *A* enters into relation with them in this way. The belief that this bell is made to ring by the hammer is strengthened if all the bells known to me act in the same way when they are struck by a hammer, or, more generally, by a hard body; and I should be still more inclined to lay down the universal proposition: When a hard body strikes a bell, it makes it vibrate and ring, meaning thereby to express a causal necessity.

On the other hand, the conjecture that touching the bomb caused its explosion is refuted by the fact that bodies of exactly the same kind frequently explode of themselves, while others do not explode when touched in the same way. Hence the coincidence was probably accidental—*probably*, because there is still the possibility that this one was differently constructed from the others.

In this way the propositions arise, pointing out causal connections in the earliest and popular sense. They start from the perception of continuous change in two things; this arouses the thought of efficient action itself, and further comparison of the same or similar things confirms it.

Meanwhile the assumption of efficient action is easily extended to regions within which *immediate* spatial connection is wanting; we know in the same way that the sun illuminates and warms, dries and bleaches; instead of immediate spatial proximity we have a definite spatial relation, which admits of causation just as immediate contact did in the other instances.

13. The assumption which we began by making in 11, that B did not manifest the change β so long as it was not preceded in spatial connection by the change α in A , seems to justify the conclusion that only $A\alpha$ causes the change β in B , that the former is the *conditio sine qua non* of the latter, therefore that the universal proposition is also true that whenever $B\beta$ occurs $A\alpha$ must be the cause. We should have the means of stating, not merely, "when $A\alpha$ occurs, $B\beta$ follows," but also the other proposition, which is not included in it, "when $B\beta$ occurs, $A\alpha$ has preceded."

But for this conclusion it would be necessary not merely for this assumption to hold good within our experience, but also for us to have a comprehensive knowledge to the effect that a $B\beta$ never occurred at all without $A\alpha$ preceding. But such a negative proposition cannot be justified by our limited experience; we may, indeed, infer from positive instances to a necessity connecting $B\beta$ with $A\alpha$, but not from the mere absence of positive experiences to an impossibility.

Moreover, the assumption often fails to apply as our experience grows wider. The possibility that the same perceptible phenomenon may be brought about by different causes has been made familiar by our daily observations; in one set of cases we were forced to assume that $B\beta$ was caused by $A\alpha$, in another set that it was the effect of $C\gamma$ or $D\delta$. Thus the attempt to infer that any definite cause is always connected with a

given effect stands from the first upon an uncertain footing, because our experience tells us that $B\beta$ is connected in the way which we must accept as signifying a causal relation, not only with $A\alpha$, but also with $C\gamma$ and $D\delta$. We may establish the fact that when a hammer strikes a bell it will ring; that a man who is shot through the heart will die suddenly; but not that when a bell rings it has been struck by a hammer, and not that a man who dies suddenly has been shot through the heart. Induction which proceeds in the way described can only aim at proving that certain events have a certain consequence, not that a particular phenomenon presupposes particular conditions. The hypothetical judgments which tell us that when $A\alpha$ occurs $B\beta$ follows cannot be converted. Only the most comprehensive comparison of specific propositions, telling us by what causes the changes in B are brought about, can be a ground for the belief that $B\beta$ is caused *only* by $A\alpha$.

14. Suppose that such a proposition, that a certain activity α of an A causes a change β in B , had never met with an exception within the range of our observation, it would still be nothing more than a crude preliminary assumption. What we have to do is to secure for it the most logical accuracy of expression which is possible, and to endow it with all the strictness of a universally valid proposition, of which the particular applications are the particular instances observed. It is upon those stages in the process which we have considered so far that popular inferences of causal connections are based; the scientific and methodological problem in the narrower sense only begins now.

15. First with respect to the elements contained in the proposition. All external events, which we can perceive, are *quanta*, and can be completely and accurately expressed only by quantitative determinations. All change, motion, heating, cooling, etc., in an A take place in a certain amount; and in the same way the effect, which consists in the change in B , is quantitatively determined. In many instances the A 's and B 's also, of which the proposition speaks, are capable of quantitative distinctions, even when they are of exactly the same kind; and only when these are taken into consideration can we say with conceptual accuracy what is to be understood by A and B , and their changes α and β . Hence the proposition must say how great an α of how great an A produces how great a β in how great a B . Neglect of quantitative determinations is the chief deficiency in Mill's *Logic*, as in that of Bacon. If, indeed, we are to speak accurately we cannot say that eating stills hunger and drinking

thirst, for a mouthful or a sip is no good ; nor can we say that arsenic kills or quinine reduces fever, for it depends upon the dose ; it is inaccurate, again, to say that common salt is dissolved by water, for it is not true that any quantity of salt is dissolved by any quantity of water.

The quantitative determination of all events not only calls for the accurate determination of the limits within which the conjectured proposition is to hold good ; it also affords a very efficient test of its correctness.

16. With reference to the first point, the logical determinateness of the proposition necessitates first an investigation as to whether every value of A and α produces a change of the kind β in B ; whether there is a limit below which $A\alpha$ has no effect, or limits beyond which the effect becomes qualitatively different. A few observations are enough to convince us that heat expands a certain amount of quicksilver, and that cold reduces its volume again ; but this rule cannot be stated as an absolutely universal proposition ; there is a degree of heat at which the effect becomes quite different,—is evaporation, and a degree of cold at which solidification takes place. If the proposition is to be determined, we must first of all state the limits. Where all possible gradations in the cause and its continuous decrease and increase are presented in the course of nature, these limits can be found by mere observation ; in other cases, we must have recourse to experiment, which varies the quantities and shows us the corresponding changes of effect.

17. Within these limits we now have to determine whether and how change in the effect is connected with change in the cause, thus establishing the law of the effect in the narrower sense, assuming that both are measurable. When it is found that a gas changes its volume with increase of pressure, then we have to discover the appropriate values of pressure and volume, and to see whether any mathematical relation obtains between the values—the same problem which occurred (§ 90, 5, p. 274) in finding the formula for a continuous change from intermittent observations.

In the simplest case these values are proportional ; and because the effect is the measure of the efficient action of the cause, we have ground for assuming that where this simple proportionality manifests itself the whole observed change in the cause A has been efficient, and that the change in B is to be attributed entirely to the action of the cause A .

Our proposition is then for the first time fully determined, and can claim to include every particular case in such a way that it can be derived

from it in its complete specialization ; we can now speak of a *law* in the strict sense.

18. The formula of the law is still capable of further verification by the continual confirmation, by means of observations and experiments, of the values following from it ; in proportion as the cases in which the observed and the calculated values agree increase in number, the probability increases that the causal law has been discovered.

But we still have only a proposition, of which there appear as the consistent consequences a number of observations, which, though they may be great, are yet very small in comparison with the total number of cases possible. That it is true without exception can never be shown empirically, and is only assumed on the presupposition of constancy in the order of nature, and of the objective validity of the concepts of *A* and *B*.

Instead of the unattainable proof that an exception is impossible, we must substitute the proof that it is improbable that within the range of our experience a case should have escaped us in which *A* α took place, and *B* did not undergo the change β . When this consideration has also brought us to a satisfactory result, then the causal connection between *A* α and *B* β has been inductively proved as completely as the logical nature of the process will allow, and is as fully determined as it demands.

Only the causal connection between two phenomena as wholes, however, not the forces which must be attributed to *A* and *B*, nor yet their mode of action. The necessity has not yet been referred to the nature of *A* and *B*, and their relation to each other ; we have merely established a constant occurrence as the expression of a necessity of nature in general.

19. But it is not often that an inductive process leading to causal laws can be carried out in this clear and simple form. That a change β in *B* should be the unmixed and whole effect of an isolated cause *A* and its activity α , and that between the quantitative differences of *A* and *B* there should exist a simple relation of proportion, applying exactly in every observed instance, is a fictitious case. In it *A* and *B* are isolated from the rest of the world ; it overlooks the possibility that other things may act upon *B* as well as *A*, and may modify its behaviour ; that there may be amongst them causes which counteract *A* in producing the effect, that the same *B* may vary from inner development or external influence, or—where a universal proposition is to be true of all similar *B*'s—that the individual *B*'s may differ, and each respond to the action of *A* in a different manner.

Moreover, the method, as described in this fictitious simplicity, is only applicable where both the action of the cause and the effected change are perceptible, where both are accessible to immediate observation and measurement; it is not applicable where they are withdrawn from immediate perception, or at any rate measurement.

20. These considerations would lead us to expect that this simple process of reduction would soon break down if it demanded conformity to its rules on the part of the actual world, and the manifestation of these isolated connections in reality. And in accordance with the character of the whole process, this break-down will make itself known by the fact that the assumptions made come upon contradictions in the Given, the observations do not agree with the presuppositions; that the universal proposition: $A\alpha$ causes $B\beta$, is confronted with an instance in which $A\alpha$ was present, and $B\beta$ absent, or that the formula which makes differences in β depend upon differences in α fails when the same $A\alpha$ has different $B\beta$'s, and different $A\alpha$'s the same $B\beta$ for consequence. Even the impossibility of finding a simple proportionality between $A\alpha$ and $B\beta$ is at variance with the assumption that they are related simply as cause and effect. If, for instance, the exhalation of oxygen from a plant were not simply proportional to the strength of illumination, the conjecture that a direct and simple causal relation existed between illumination and the exhalation of oxygen, such as exists between pressure and change of volume in a gas, would be refuted, even though a quickening of the chemical processes, increasing in a complicated relation with the strength of light, should point to the fact that the cause produces a part of the effect, or a part of the cause produces the effect, or part of the effect.

But where there is no fixed relation at all, different $B\beta$'s following the same $A\alpha$'s, and *vice versa*, then pure logical theory would proceed as in respect to the differences in attributes of objects falling under the same concept; we could merely infer that the universal proposition is not true, and that the assumption of a fixed connection is erroneous, that the results occur accidentally in relation to the assumed causes. But the assumption of necessity in all that happens does not leave this way of escape open to us; from a purely logical point of view, it is the first to present itself, in reality the most improbable as soon as a considerable number of instances have shown $A\alpha$ and $B\beta$ as connected; the occurrence of $B\beta$ must be based upon some law, and the effect must measure the action of the cause. If then the change $B\beta$ occurs after $A\alpha$ at one time, and not at another, or if

at one time a larger or smaller $B\beta$ follows $A\alpha$ than at another, or if the effect is not proportional to the cause, then, because a connection between A and B is probable upon other grounds, we must make one of two assumptions. Either our presupposition, that the same A has acted in the same manner upon the same B , is false, in which case the discrepancy must be explained by the fact that the A and B which appear alike are not really alike, but fall under different concepts, or, at least, under different quantitative degrees within the same concept, or represent different stages in development, or else the difference must be due to the fact that there are further causes co-operating, and that the result is determined by the combination of several laws, and then the problem consists in breaking it up into the partial effects of the particular co-operating causes. The rule that everything which does not produce a result every time must be eliminated as not being a cause would, if consistently applied, prevent us from obtaining any results by way of induction.

21. This analysis of the effect is again only possible by a process of reduction, which assumes a rule according to which the particular partial effects combine, and so form the one phenomenon as it is given; thus in addition to the particular laws we need a further law according to which the results of causes which act together combine.

This law of the composition of the effects of several causes admits of a simple formulation when the partial results are only quantitatively differing magnitudes of the same kind; e.g., motions in the same direction, which, assuming that every cause acts upon a body in motion in the same way as upon a body at rest, may be simply added together.

22. The analysis of a given result into the partial effects of different causes is rendered difficult by the fact that we are frequently unable to find within the range of observation efficient causes, in the sense in which we have regarded $A\alpha$ as an efficient cause; in other words, a second A^1 which should present itself to observation as an efficient cause by means of a visible change α^1 proceeding simultaneously with α .

The fact that water boils at one time when it is heated to 100° , and at another when heated to 90° , proves that the boiling of water cannot be regarded as a simple effect of heat, according to a law stating the invariable connection of the phenomenon of boiling with the presence of a certain temperature. But when we compare the two cases, we find nothing perceptible which at all resembles the heat-giving fire as an efficient cause, or which acts upon the water by a visible activity; no perceptible change

goes on in any surrounding body, which, by its presence the first time, and absence the second, would explain the difference.

23. It is upon this that the distinction between circumstances, and efficient causes in the narrower sense, is chiefly based. According to etymology, circumstances are primarily surrounding bodies; when distinguished from efficient causes, they were so called because *by themselves* they do not exercise the particular effect in question. With reference to a particular effect, they are then divided into circumstances which are indifferent, and circumstances which have some influence in so far as difference in them modifies the effect of a particular cause. In a wider sense even the changing states of bodies acting upon each other, and the differences of the individual things falling under the same concept, are also called circumstances; and these again are sometimes indifferent and sometimes modify the result. Circumstances which may modify or hinder the result of a cause appear as conditions of a given effect, and where we distinguish conditions from efficient causes (in another meaning, as conditions of forces, § 73) that body is called the cause of which the perceptible change has a change for its consequence, while those other bodies, or states of bodies, are the conditions, variation in which makes the result different, although they cannot by themselves produce a perceptible change.¹ But this distinction can only be maintained so long as we keep to the popular concept of cause and effect. Further elaboration, which takes us back to invariable constant efficient forces, banishes the distinction and takes as the equivalent of the cause coming into action at a given time a change in the totality of the conditions; though even then we may still distinguish between the permanent conditions and that of those which, being introduced by some change, complete the total complex at a given point of

¹ It is easy to understand why in popular language the answer to the question as to what any given event is due confuses causes and circumstances at random. That N— died when struck with a stick on the head was due to his thin skull; that an old tree was blown down yesterday was due to its being hollow, etc. Where, as in these instances, the efficient cause is fully understood, the ordinary expression which makes the circumstances responsible for a result larger than appears in the plurality of cases is quite correct; it extends the accusation even to that which is not, by ascribing the falling of fruit to the want of rain, or a theft to the absence of the police, though it is not meant that the rain which does not fall breaks the fruit from the tree. But as a question of language it should be noticed, with reference to remarks sometimes made about this way of talking, that such circumstances or negative conditions are not usually denoted by the term causes, but by more general expressions.

time in such a way that an effect can arise. (From this point of view the cause of the spontaneous fall of an apple from the tree is composed of its distance from the attracting earth and its consequent tendency to fall, and the organic changes which gradually weaken the cohesion of the twig and the fruit ; when these have reached a certain degree, all the conditions of the fall are completely present.)

For the present, however, we prefer to adhere to popular usage, all the more because the investigation of causal connections always starts from causes active in a perceptible manner, and because the application of the causal concept is here the most obvious and original ; moreover throughout large departments of knowledge even exact investigation cannot get beyond formulating causal laws which connect perceptible changes in certain bodies with changes in others.

24. Since all induction presupposes a definite meaning in the propositions which it seeks, we will illustrate the above distinctions by a few examples. Water boils at a higher temperature in low elevations than on the mountains ; the circumstance which explains the difference here is that the pressure of the surrounding atmosphere is greater in the former instance, less in the latter. The surrounding atmosphere by itself, and change in its pressure, does not (at any rate within the range of our common experience) make water of the ordinary temperature boil ; not until the fire is introduced does boiling occur, and for this reason the atmospheric pressure is described as circumstance, and the heating by fire as cause. A spark causes powder to explode, but wet powder does not explode ; the circumstance that it is wet, in other words, the presence of water, prevents the explosion. By itself, however, water produces no perceptible change ; with respect to the powder, it is only circumstance, not efficient cause ; its absence is condition, but not cause of the explosion.

A blow makes a bell ring ; but if it is touched on the rim by an inelastic body, the sound is muffled ; this body itself causes nothing, but the circumstance that it is there diminishes the action of the hammer, its absence is the condition of the full sound. A given remedy acts or does not act according to the circumstances, *i.e.* according to the permanent or changing disposition of the patient, etc.

Of still greater importance are the circumstances where we have to do with chains of action : how a change which is directly brought about by a single cause will produce further changes will depend in quite another sense

than the preceding upon the circumstances—*i.e.* upon the possibilities of future action given by the collocation of certain bodies. In setting fire to a house it may be that we directly cause, in the ordinary sense, only the burning of a wisp of straw; but the given circumstances, proximity of combustible bodies, wind, scarcity of water, etc., may cause the conflagration of a city to follow from this fire through the continual transmission of the action to new bodies. If we wound a man by a shot we directly cause nothing but the movement of the trigger with the finger; the explosion of the priming is the act of the spring which is released, the explosion of the powder the act of the priming, and this explosion causes the motion of the bullet, which causes the wounding of the tissues. Each of these stages is conditioned by special circumstances, and upon circumstances it depends also whether the wounded man will bleed to death or will be bandaged, whether the wound inflames or not, whether in consequence of this wound he escapes another danger or falls into one which he would otherwise have avoided. This concatenation of effects proceeding from each other, which is so important for practical life, we will disregard for the present, and limit ourselves to instances of immediate causation, where the action of the cause and the genesis of the effect can be regarded as single events in two single things.

25. What are the problems to be solved by our methods of discovering causal laws, in view of the fact that the effect of a given cause varies according to circumstances?

Let us return to our first case, that in which comparison of a number of instances has given us the proposition “when $A\alpha$ occurs, $B\beta$ occurs,” and in which we are able to establish a formula according to which the differences in β correspond to the differences in α .

The question to be answered now is, whether $A\alpha$ gives rise to $B\beta$ according to this rule under all circumstances, and whether therefore it can be described as by itself the complete and whole cause; and if not, which circumstances are indifferent, which have influence.

For the elimination of indifferent circumstances we have the canon that instances are to be compared which, while alike in everything else, differ in the fact that one circumstance varies quantitatively, or is present at one time and absent at another. If the result of the causes remains the same, then this circumstance is indifferent.

With reference to circumstances which are not indifferent, the problem arises of determining their contribution to the observed effect in the form

of a law. For this it is necessary that each particular circumstance should be varied by itself whilst everything else remains the same, and that from our observations the formulæ should be constructed according to which differences in the effect are connected with differences in the circumstance.

These formulæ lead us at once to another meaning of causal dependence than that which belongs to the causal laws which we were before looking for. When expressing the action of different circumstances in partial laws, we are no longer engaged in determining the dependence of one perceptible change upon another perceptible change, but only in ascertaining the modification which varied circumstances bring about in the action introduced by a perceptible event; and since these varied circumstances will generally be bodies at rest, not engaged themselves in any motion or change, we may be brought to ascribe efficient action to them also in so far as they alter the result by their mere presence. Thus we arrive at the thought of causes which take effect, without themselves manifesting any perceptible change.

The result of the whole process would be a connected complex of laws, expressing the dependence of an effect $B\beta$ which follows a cause Aa , partly upon the cause Aa , partly upon the many circumstances amongst which it acts. Each particular case then appears as the application of different laws taken together, according to a rule which determines the combination of partial consequences in a total result.

26. The obstacles to this process lie first in the difficulty of finding or constructing instances admitting of comparison, in which one and only one circumstance varies. The presence of the earth cannot be eliminated in terrestrial observations and experiments; if we attempt to vary it by observing the same events at different points in the earth's surface, or at different elevations, then a host of other circumstances are changed at the same time; the circumstance cannot be varied *alone*. Still more difficult is it to overcome the second difficulty. What is to be regarded as circumstance, and brought under our investigation? When the immediate intuitable connection, which causes one thing to appear to us as acting upon another, is not present, an unlimited range is given to conjecture; to the innumerable perceptible things which co-exist with a phenomenon, and are in some sort of spatial relation to it, must be added innumerable others, which are perhaps not perceptible, and which betray their existence only by their disturbance of events which we expect to happen according to a preconceived law.

A direct and exhaustive process, leading us straight to our goal, is therefore not to be expected ; at best we can only be guided by analogies in conjecturing that some circumstances will be indifferent, while others will exert some influence ; and if there still remain discrepancies, they point to unknown sources of disturbance.

27. A simple example may serve to illustrate the treatment of the different kinds of circumstances. When a bullet hanging freely on a string is pushed on one side, it moves from its former position, rises in an arc up to a certain point, then turns back and swings for a time to and fro, diminishing the length of its movements until it finally comes to rest. We know that the push was the cause of the movement because the bullet never enters into motion of its own accord, and always enters into motion when it is pushed, and because when the push is stronger the motion is stronger and longer. In this way the causal relation between the push and motion is first established with all attainable certainty, in the ordinary sense according to which one change—the motion of the pushing hand—is connected with another—the swinging bullet—as its necessary consequence. At the same time the whole continuous course of oscillations, until the bullet is again at rest, may be taken as one whole, as the effect of the push, inasmuch as it is a homogeneous continuous motion of one and the same thing.

But if we are to find the strict causal law, we must first of all measure the event itself, and establish quantitative relations between cause and effect. If we had an exact measure for the strength of the push, and caused this to vary, then we should find that the stronger push drove the bullet higher, and caused it to continue to swing for a longer time ; that nevertheless the particular oscillations—at any rate for any but a very minute observation—will occupy equal times, and that therefore the whole time of the event will be measured by the number of oscillations. Experiments made with the same pendulum in the same manner, and differing only in the strength of the push (which might be measured by difference in the elevation from a state of equilibrium), may now yield a formula expressing a definite relation between the strength of the push and the number of oscillations of a given duration until the pendulum comes to rest ; and we should be inclined to state it as a universal law, that a bullet suspended by a thread responds to a push in this way.

But now the question arises : Is the whole course of the event really dependent upon the push alone and a simple function of it? Are our

assumed conditions so closely determined as to necessitate the result following always in the same way? This would be the case if the law, as first formulated, was confirmed under all circumstances. What we have to do is to vary the circumstances. But what are the circumstances?

In the first place, we have the solid bodies around, and the direction in which the blow is imparted. That these are indifferent, we learn because the event, so far as can be ascertained, is the same whether the bullet be caused to swing within the room from north to south or from east to west, and whether any other bodies are removed from the room or not. It takes place in the same way, again, by day or by night; the position of the sun or the stars has no influence. But finding that the air offers resistance to the movement of the hand, we are led by analogy to the thought that the surrounding atmosphere might have some influence; we must either vary this ourselves, or compare cases in which it is varied from other causes. If it should prove that when the barometer stood high the number of oscillations in response to a given push was smaller than when it stood low; if it should prove that in proportion as the air is rarefied the duration of the single oscillations diminishes while their number increases; then the air would be shown to be a circumstance having influence, and we should have to find the formula according to which increasing density of the atmosphere affects the motion.

A second class of circumstances would be found in the changes of which the object is capable; in the first place, the quantitative changes. The original law of the action was obtained for a given length of the thread. If we shorten the thread, a change appears in the result, the oscillations become more rapid; if we lengthen it, they become slower; and at the same time the total time occupied by the oscillations alters. A series of observations will now show the values l_1 l_2 l_3 for the length of thread corresponding to t_1 t_2 t_3 for the duration of each oscillation, and on comparing them we shall find that the various t 's are proportional to the square roots of l , and shall obtain the constant by which the length of the second-pendulum is determined. In this way we determine the connection between length and duration of oscillation; further comparison would give us the effect which the length of the pendulum has upon the duration of its motion altogether, and upon the total number of oscillations in response to a blow of a given strength. If we diminish the bullet while keeping the thread the same length, the effect upon the duration of the particular oscillation will not be perceptible, though it will be upon the number of oscil-

lations until the pendulum comes to rest ; a further law would determine the dependence of duration of oscillation upon the size of the bullet.

A third class of variations, which may be called circumstances in the widest sense, are the differences in objects which fall under a common concept. If we compare bullets of equal weight and equal size, but of different material, no difference would follow in the result ; it is therefore a matter of indifference what material they are made of.

But the formulæ thus found, which determine the influence of the atmosphere, length of thread, and weight of the bullet, upon the time of oscillation and the whole duration of the phenomenon, yet give us no certainty as to whether they are absolutely valid, and whether they contain all the conditions upon which the phenomenon depends. If no further conjecture draws our attention to some unnoticed circumstance, then nothing remains but either to vary at hap-hazard, or to wait until a divergence from the law appears somewhere, and then to seek the circumstance to which it may be ascribed.

If, for example, we should find in another part of the world deviations from what was to be expected from established laws, this would point to some circumstance not yet noticed ; there must be some circumstance which differs in the two places. What we now have to do is to eliminate from the innumerable circumstances which differ—in, *e.g.*, Cayenne and Paris—those which are indifferent, and to discover those which are of influence—those to which it is due that the same pendulum swings more slowly in Cayenne than in Paris.

Here the unlimited number of the circumstances prevents there being any direct method, and the ground of the difference would hardly have been discovered so soon had not the mathematical theory of the pendulum shown that gravitation, which is measured by the fall of a body during one second, enters into the formula by which the duration of a pendulum oscillation is determined, and so directed attention to the possibility of a variation there.

If we had to proceed entirely by a comparison of observations, nothing would remain but to make conjectures as to the ground of the difference, and to test them by comparison with other cases ; or else to discover from a large number of observations what are the circumstances which vary in proportion to the time of oscillation. Even this latter investigation would not by itself give us a result, however many observations it might be based upon. Some of the observations would suggest that the length of

the pendulum is a function of the geographical latitude, but others—those made at a considerable height above the sea—would form exceptions interrupting the regular series. Only by comparing a number of observations alike as to elevation but differing as to latitude, and a number alike as to latitude but differing as to elevation, could we construct the twofold influence of these circumstances and reduce it to formulæ, the common result of which would be the actually observed lengths of the second pendulum; but it is clear that such a selection amongst the observations must be guided by some conjecture in order to be possible, so that here again the hypothetical nature of every such process appears. Here again only experiments at any latitudes and any elevations could be decisive.

28. In reality, then, every particular event which we attempt to refer to a causal law appears as the complex result of a number of more special laws, which make the amount of effect depend, not merely upon the causes first perceived, but upon a larger or smaller number of subsidiary causes. It is expressed, therefore, not in a hypothetical judgment of the form, "if Aa is then $B\beta$ is," but in one of the form, "If A is a , and C is γ , and D is δ , etc., then B is β ," where β is a function of $f'(a)$, $f''(\gamma)$, $f'''(\delta)$.

29. In the instance just analysed we are dealing with merely quantitative changes in the duration of oscillation, and in the motion in general; but the result appeared every time that the pendulum was pushed or moved from its position of equilibrium; we could always distinguish between the one efficient cause and the modifying circumstances which only alter the amount of the action.

Other instances in which the effect fails according to the circumstances, whilst it occurs in other circumstances, oblige us to add the negative conditions to the above formula, so as to make the expression of the causal law as accurate as possible. The proposition that a spark causes gunpowder to explode is incomplete; we must add the negative condition that the powder is not wet. The proposition that water under the pressure of an atmosphere boils at 100° is also incomplete; the negative condition that the vessel is open must be there. Thus our causal law in its completed expression takes the form:

If A is a and C is γ and D is δ , and E and F are not ϵ and ϕ , then B is β .

We must not, of course, forget that an exhaustive statement of all nega-

tive conditions is from the nature of the case impossible ; we can never be certain that we know all the agents the presence of which would hinder the action of given causes ; we must confine ourselves to stating those which are known to us. To this extent the most exact formulation of such causal laws is only an approximation, and is true only for those cases in which none but the known agents are present ; strictly speaking, we ought to add to every such law the general clause : provided no counteracting causes intervene from elsewhere. These laws are strictly true only for an assumed and relatively simple combination of causes and conditioning circumstances. We may perhaps expect that in the great majority of cases this assumption will be entirely realized, and the result correspond to our law ; should the opposite occur, we should not be satisfied to regard it as valid only *ὡς ἐπὶ τὸ πᾶν*, but should be forced to inquire as to the ground of the deviation, and to find another partial law. A number of previously unknown causal connections have been discovered by observations in which a law was found to fail which had previously been regarded as reliable ; the most famous instance is the discovery of the atmospheric pressure by Torricelli.

30. In the knowledge that actual events depend upon many conditions, and in the recognition of the fact that events which are alike for our perception may have different causes, we find a ground for regarding the methods of agreement and difference which Mill presents as fundamental as being useful merely as heuristic methods, not as the means of establishing a causal law. Let us look again at the method of difference. It compares two instances of antecedents ABC and BC , of which the first has for consequence a phenomenon a , which is wanting in the second. We may not infer from this (as Mill himself allows) that A was the whole cause of a , and that the other antecedents BC contributed nothing to the effect, and did not belong to the conditions of a ; B and C could be eliminated only by showing that they might be absent and a yet occur. Thus a single pair of such observations can never establish more than that the circumstance in which they differed will be a part of the cause. If we sow seed in a dry bed, and water one half and not the other, the plants will grow in the watered half and not in the other ; but it does not follow that the water alone was the cause of growth, and that soil and warmth and light, which produced no growth in the dry bed, did not belong to the conditions of growth. Thus the method of difference shows us a circumstance which is part of the cause, but what we want to do is to include the whole circle

of agents which are necessary to produce the effect, and this can only be done by eliminating the indifferent circumstances and discovering those which co-operate.

It is the same with the method of agreement. From the fact that only *A* was present in all the instances in which a phenomenon *B* occurred, it does not follow that *A* by itself is the cause of *B*; it may be that it has produced this result only in connection with other things.

One more result follows from these considerations. Every methodology which, starting from the purely empirical standpoint, insists chiefly upon extending perceptions as far as possible, and regards the inductive process as primarily a summation of like perceptions, not only makes a demand which would prevent our ever passing beyond preliminaries to the beginning of induction; the way which it directs us to take is an impossible one because the greater the extent covered by the phenomena the less is the probability that they will manifest connections of simple regularity. The actual process of science has been a tentative one, which starts with a limited range of observation and corrects its first assumption by means of further comparison. A comprehensive knowledge of facts is valuable only as facilitating combinations which lead by analogy to hypotheses, and as preventing the over-hasty formulation of definite universal propositions.

31. Looking back upon what has preceded, it becomes obvious that the fundamental laws by which we can express the causal connection of phenomena exactly, *i.e.*, with a mathematically accurate statement of the magnitude of the effect for every value of the cause, do not generally represent the way in which phenomena actually behave; that, on the contrary, they are valid only for instances of a fictitious simplicity which are never forthcoming in reality. They cannot therefore be directly verified by observation, much less obtained by a simple summation of observations. They are hypothetical not merely in their logical form, in so far as they state that if *A* is true *B* is also true; nor yet merely in the sense that their unconditionally universal validity is only of the nature of a probable hypothesis, not of a strictly proven proposition; they are also hypothetical in the sense that they refer to instances which are assumed although they are forthcoming in no experience. The whole science of mechanics rests upon the law of inertia, which alone enables us to give a consistent interpretation to events according to their causal relations; but no observation is capable of showing us that a body, when set in motion, continues its motion in a straight line with uniform velocity; the law does not express the form

of an actual event. It is the same with the law of parabolic projectile motion, etc. The actual events never conform to these simple formulæ; the laws by which we can express what actually happens are all of them deduced by means of different combinations of the fundamental formulæ, corresponding to the actual changing combinations of their elementary conditions.

Here another point must be noticed. In the mathematical formulation of the "exact" laws, especially when we are dealing with constant forces such as those to which we have been brought in our consideration of "circumstances," we are always obliged to fall back upon the differentials of space, time, velocity, etc. But we can never observe any but finite magnitudes which present themselves as integrals within certain limits. Here again, therefore, the law which can be seen to agree with what actually takes place appears as a deduction from the simple and original causal laws; and the agreement itself—owing to the possibility of errors of observation—can be only approximately ascertained. Once more, therefore, we see that the problem of finding strict causal laws can only be solved by hypotheses which pass beyond experience, and by mathematical construction.¹

IV. LAWS WHICH ARE NOT CAUSAL LAWS.

§ 96.

We must distinguish between the causal laws treated of in § 95, and those universal propositions which are also frequently described as "laws," but which are merely descriptive, either containing the formula of a particular actual event, or of what goes on in a class of things, or else expressing the regular connections between different phenomena which exist as a matter of fact within our experience.

By themselves the former cannot be regarded as the expression of a necessity, nor do they affirm any necessity except in so far as it is assumed that a constant way of happening is conditioned by a constant ground. The latter do indeed point to a causal connection, but they also stand in need of more definite explanation and of being derived from proper causal laws. In distinction from these they are usually called merely empirical laws.

1. The conclusions just arrived at, as to the value of methods which we

¹ Cf. F. Braun, *Ueber Gesetz, Theorie und Hypothese in der Physik* (1886), p. 5 sq.

have already found to be inadequate, are less important than the extension we have made in the import of those universal propositions which are intended to express necessity in what happens; for this carries us on to judgments of an essentially different character from those which were first sought concerning the actions of particular given causes. Those propositions which state the influence of circumstances upon the action of a given cause can no longer be described as causal laws, in the same sense as the law that pressure diminishes the volume of a gas; they express only relations of dependence between a change and certain conditions, but by themselves they give no information as to the causes active therein. That the same pendulum oscillates more rapidly in a higher latitude than it does at the equator is so firmly established that we may speak of a law of the dependence of the time of oscillation upon the latitude; this law might be established without anything being known as to the actual ground of the acceleration, for mere difference of place can never by itself be described as an active cause.

This leads us to determine the various meanings in which we are accustomed to speak of laws, for the ambiguity of this term has sometimes been to blame for serious confusion.

2. The mere description of a given event led us in § 90, 5, to formulæ which expressed the relations between differences of time and differences of space or quality, by means of a concept to which the particular stages in the event were subordinated; and such a formula may be called the law of this given event in so far as it states the fundamental principle which is added as major premise to the particular values belonging together, and of which they appear as the necessary consequences. Observation of a single falling body would justify us in the proposition that it moves according to the law $s = \frac{1}{2}gt^2$; what it tells us is that it moves as if this formula determined for it the space it should traverse during every interval of time; the law is true from beginning to end of the motion.

3. Nearest to this description of a particular event of given duration comes the description of an event which recurs indefinitely in a given subject, an event which, from another point of view, may be regarded as one connected whole. We may call it a law that the earth turns round its axis with uniform velocity every twenty-four hours; so far as memory extends, the motion of the earth has been determined by this formula. But here another thought creeps in, and we expect the earth to move according to the same law in the future also; the use of the expression "law" involves

a tacit inference from the form of the event as perceived until now to its continuance in the same form for an indefinite time,—an inference which is in no way justified by the mere fact of what has occurred previously, any more than the inference that a man who has lived eighty years will complete his eighty-first year has any certain basis. Such a basis it would have only if it could be shown that no cause of change is present, and that therefore the same grounds which have necessitated the event so far will also last for an indefinite time ; otherwise the so-called law contains merely the description of a fact, together with the indefinite expectation, grounded upon mere habit, that it will occur again.

4. Kepler's first law is itself only a description of what actually takes place in certain subjects, with the difference that it is also an empirically universal judgment in so far as it predicates motion in an ellipse not merely of one planet, but of all which were then known. It expresses only the actual form of the event, and conceals the ground of its necessity ; only by assuming invariability in the heavens may we justify the expectation that it will be just as true in future. As an expression of necessity it lacks the hypothetical form, which would represent this mode of happening as the necessary consequence of some ground ; it is a purely categorical judgment, one, moreover, which predicates no attributes, but only a temporal activity or relation. The categorical judgment that all ravens are black, and all gold yellow, contains the ground of the predicate in the essential concept of the subject ; by assuming the necessity in reality of the combination of certain attributes, I extend the empirically universal proposition that all the gold known to me is yellow into one of unconditional universality ; this categorical judgment is equivalent to a hypothetical. But the judgment that the planet Mars moves round the sun in an ellipse, within the time of 687 days, states, in the first instance, only the fact that it has moved so hitherto, and connects with this fact the expectation that it will continue to move so in the future ; but this judgment cannot derive its predicate from the nature of the subject, and hence the extension to all time, which is contained in the expectation, is not motivated by a necessity expressed in the judgment itself, but by the tacit assumption that the ground of the motion of the planet is invariable,—an assumption which has no firm footing so long as we know nothing as to the ground of this motion.

If Kepler's first law is taken to mean that it is true of all planets, both known and unknown, then it contains a two-fold induction : by help of a

definition of "planet" it includes a generalization and also affirms a predicate which is true of all things belonging to a class ; it is a judgment universal as to time and genus, but it is still merely categorical.

5. The law of falling bodies is again only a generalizing description of the form of an event recurring in many things ; it does not say that they fall, nor under what conditions they fall, nor why they fall, but how they fall ; it says that whenever a body falls freely it describes spaces which are proportional to the squares of the time. It is a law, in the first place, because it regulates all the stages of the event according to a formula, and in the second place, because it is assumed to be true for all bodies, without exception, hence to have generic universality.

It is the same with the law of multiple proportions in chemistry ; it expresses a universally valid form of the event : whenever two substances combine chemically, they do so in definite relations of weight, etc. (If we should say : if two substances combine, they do so, etc., this would be no hypothetical judgment connecting a real ground with a real consequence ; "if" here means only a general connection, for the consequent states, not a result of the antecedent, but only the particular mode of the event expressed in it.)

6. In this class of laws we have had to do only with descriptions of the way in which particular events take place in particular things, or in whole classes of things ; to them we must now add those other universal propositions which state relations between what is different, without being causal laws in the sense referred to in § 95. Every partial law, which makes the modification of an effect depend upon given circumstances, belongs to this class ; but it extends much further than to these instances, and passes over gradually into the class of judgments which state necessary connection between the attributes of one and the same thing.

7. When we find that water boils upon a high mountain at a lower temperature than it does below, we have established the influence of a circumstance which consists primarily only in position upon the action of a cause ; how elevation above the level of the sea manages to exert this influence may be entirely concealed from us. When we find that the inclination of the magnetic needle becomes greater as we move from the equator towards the pole, there is no question of a perceptible active cause which places the needle in a certain direction ; we can show nothing which causes by its change the change of direction, such as a magnet, which, if placed near the needle, would alter its direction ; com-

parison of different cases merely gives us a regular connection between the situation of the place and the position of the needle, which we attempt to state in one law, by means of a process of reduction, deriving all particular cases from one major premise.

Observation, again, does not show us the cause of the motion of the tidal waves in the ocean. We may begin by determining in a merely descriptive law the regularity of the recurrence of the event, and say that the tide is repeated every $12\frac{1}{2}$ hours; but comparison of the times with the changing position of the moon with respect to the earth gives at once the general rule that in every place the tide stands in a definite temporal relation to the culmination of the moon, and thus leads to a law of the relation between the tide and the position of the moon. Comparison of the varying heights of the tide with the position of sun and moon with respect to each other shows a regular relation between the maximum and the syzygies, and between the minimum and the quadratures,—a law, therefore, which states the connection between changes in the height of the tide and the changing positions of sun and moon with respect to each other. But, directly, what it shows is only the temporal connection; at the bottom it is again only a descriptive law concerning the regular accompaniment of one change by the other, and it is essentially different from the causal explanation which deduces this connection from the attraction of moon and sun upon the waters of the earth.

If we examine Kepler's second law, we find that in one aspect it is merely a formula for the motion of the planets. But this formula states a relation between diminishing velocity and increasing distance from the sun, and between increasing velocity and diminishing distance from the sun, and it may therefore be taken also as the law of the relations between velocity and distance from the sun; it does not, however, cease to be a mere description of what happens, without any statement of the ground why it happens.

Many every-day experiences, as well as scientific propositions, are expressed in such laws of relation. That bodies when dropped fall to the ground is no more a causal law than that water flows down-hill, or stands at the same level in tubes which communicate; in the first of these propositions the dropping is regarded by no one as the active cause, and in the others, although the inclination of surface or the communication between the tubes is a condition of the phenomenon occurring, it is not the ground of it.

8. Physiology, again, abounds in general propositions which have only the character of empirical laws.

In the first place, there are the laws which describe events in the individual. The propositions stating the development of an organism of given form from the spore, the successive stages in the division of a germinal cell, and in the growth of the particular organs, all describe a succession of given events which recurs regularly in all individuals of the same kind. That a seed when laid in the ground will sprout, will unfold its cotyledons, will strike down with its roots and up with its stalk, that the roots will branch and the stalk bear successively leaves and blossoms, that the male parts of the plant bear pollen, which settles upon the pistil and there gives rise to further processes which lead to the ripening of the fruit, and that from this fruit there is again developed a similar plant,—all this is primarily description of a succession of events which repeats itself in many similar individuals. In the same way it is an empirical law that the bodily temperature of mammals and birds varies only within narrow limits, or that when they breathe an exchange of gases takes place in their lungs.

The general laws in which we give descriptive expression to such regularities of event do not, in the province of physiology, admit of the same strict form as in mechanics; the great variation in individual specimens, even when they belong to the same inferior species, and the more or less rapid change in one and the same individual which is inseparable from the vital processes, prevent any accurate quantitative determinations which would be always similarly applicable, as in the fall of bodies or the motion of the planets, either with respect to the temporal course of the changes or to the form and size of the particular organs. Where proper generalizing induction, which connects the phenomena observed in conceptually different things, has not yet come into play, the variation between individuals manifests itself in the fact that all we can do is to state certain limits within which the events in similar individuals vary, limits of the size to which they develop, limits of periods occupied by the particular stages of development, etc. The expression of the common element takes the form of an average; we determine the average duration of human life, the average duration of its state as an embryo, the average time at which puberty commences, the average frequency of pulse and of breathing at different ages, etc. (see below § 101, 5).

Those laws, again, are purely empirical which—as laws of relation—determine the connection between corresponding changes in the same

subject, concerning the causal relation of which we are still uncertain ; or the connection between corresponding deviations in the individual from an average type. That the larynx enlarges and the voice breaks at the commencement of puberty, or that red hair is generally combined with a fair complexion, are such general propositions.

To these we must add those propositions which express temporal connection between vital phenomena and parallel events in the surrounding material universe. That the blossoms of certain plants open and shut at certain hours of the day, that periodical changes in animals take place in certain months and are early or late according to temperature, are propositions establishing actual coincidences between the course of the sun and the temperature on the one hand and organic events on the other ; and that the hairiness of certain kinds of plants or the colours of the blossoms change with the situation are also relations which can at first be only empirically established. Over a wide range want of knowledge as to definite causal relations restricts us to these merely empirical uniformities.

9. It is more difficult to determine the exact import of such propositions. In what sense are they universally valid, and to be regarded as the expression of a necessity when they are confirmed without exception in the range of our experience ?

Starting from the postulate that the given is necessary, and from the further assumption that it is determined either by the inner necessity of the essence, or by the outer necessity of causality, or by both together, then all that we find at first in descriptive general laws is an indication that there are grounds for the regularity of events and an instigation to look for them ; but they also form a ground for the assumption that these grounds have been constant within the range of our experience until now, and that they are contained in the constancy and similarity of things themselves and their relations. According to this assumption we are then able to establish them as universally and invariably valid at first within the province in which we have observed such uniformities ; even while we do not know why bodies fall we are perfectly justified in assuming that everywhere upon the earth's surface they fall in the same way whether we see them or not, and that they will continue to fall in the future so long as no other change takes place upon the earth than such as we have already experienced ; this confidence, however, is based, not upon the mere summation of the particular cases, but upon the assumption that bodies are governed by an invariable necessity which is common to them all and

grounded upon their given attributes and relations. But the certain validity of the law is limited to the sphere within which it is found, to the surface of the earth; so far we have not sufficient evidence to warrant us in inferring that all bodies, at whatever distance from the earth, would in the same way press towards its centre. The descriptive laws of physiology also we can accept as universally valid, if we assume that no change occurs either in the nature of individuals or in the conditions of their life.

Where the propositions which express empirical uniformities connect different phenomena, they point more definitely to a causal connection, either to a direct dependence of the one phenomenon upon the other, or to the fact that they are both connected consequences of the same ground. For that a mere temporal coincidence, or the exact agreement in the periods of independent events, should be repeated so often is so highly improbable that we find ourselves forced to assume that there is a real dependence. That the period of the tide should be accidentally the same as that of the culmination of the moon, that the period of the highest tide should be accidentally the same as that of the syzygies, is possible *in abstracto*, but it is in the highest degree improbable; the far more probable assumption is, either that sun and moon produce the tide, or that their motion is due to the same grounds as the motion of the tide.

10. In physiology it is obvious that we can attribute universal validity to descriptive laws only if we assume a circle of constant conditions, even though we may as yet be unable to discover what is contributed by each of them towards the possibility of life and development. That a germ will develop and pass through a series of successive stages is true only if we assume the presence of suitable nutrition, suitable temperature, etc.; like the physical laws, this general law really represents an instance of fictitious simplicity in so far as it disregards the manifold disturbances of unfavourable conditions.

If we presuppose these constant conditions then to the mere description of the development is added the thought that this development is necessary, in so far as it lies in the nature of the germ to evolve from itself these successive forms, it appears as the *causa immanens* of its changes. By virtue of this nature of the particular individual each particular state forms the ground in reality for the transition to the state following it; those causal concepts which in § 73, 26, p. 167 sq., were distinguished from the concepts of transient causation find their application here.

The comparison of the course of development of particular organic

individuals shows, however, deviations from the general law of the average ; and to some extent we have empirical laws of relation for these deviations, which in their turn point to an external causality modifying the development. Physiology aims at forming into strict causal laws these propositions of interdependence which first appear in an indefinite form ; it aims at establishing experimentally the influence of different light or different temperature, etc., upon the growth of plants, or at showing that the variation between sexual and non-sexual reproduction, which there was formerly a tendency to refer entirely to grounds contained in the organism itself, is directly dependent upon nutrition. Thus every actual organic development may be regarded in the twofold aspect of immanent and of transient causality ; the difficulty is to ascertain for particular individuals to what extent the peculiar phenomena which they manifest are to be referred to the disposition of the particular individual and to the law of development thereby determined, and to what extent to external influences.

11. Thus causal explanation must always be called in to complete our knowledge. Where we cannot find efficient causes such as are treated of in § 95, we must extend our hypotheses as to the working of causes further ; and we first have recourse to those forms of action in which a thing, without visible change on its own part, by its mere presence and spatial relation, conditions the motion or change of another thing, or counteracts and prevents a change which would be introduced by other causes. The most careful empirical comparison of all observed instances in which bodies fall tells us nothing as to the "why" of the fall ; this may just as well have its ground either in a tendency of bodies downwards, or in a pressure acting *a tergo*, as in a force belonging to the earth which sets all neighbouring bodies in motion. The only question is, which hypothesis explains all the connected phenomena of fall, pressure, equilibrium, etc., and whether a law can be formed for this action to which all the phenomena will conform.

Most of the partial laws which contain the influences of circumstances lead to similar assumptions of latent active forces ; the real causal connection can only be held to be established when the actual things playing the part of causes, as well as the form and law of their action, are known. It is an empty rhetorical phrase to speak of natural laws as if the mere formula exercised a magical power over phenomena, commanding them to assume a form which does not follow from their own nature ; laws can

never be the grounds for what actually happens, they only express the constant manner in which real things behave. It may be that to a large extent we shall not get beyond establishing propositions which merely allow us to regard one event as a sure *sign* of another; but to just that extent we shall fail to reach the final aim of knowledge.

12. Thus the general problem is to reduce merely empirical uniformities to propositions which state the necessary connection of the state of a thing, in part with its previous states, in part with the relations in which it stands to other things in certain states, and to state it so that we may understand a given mode of being from the nature of the thing itself. We shall not now pursue this question further; we have shown in § 73 how the concepts of causal connection, upon which the necessary hypotheses are based, must change their form.

V. GENERALIZING INDUCTION.

§ 97.

As soon as we pass beyond merely empirical generality we are only justified in comprehending several specific propositions into one more general proposition by means of a generic concept, under which their subjects or hypotheses fall, if we assume that like consequences must have like grounds.

A proposition so obtained is to be regarded as valid only if the predicates are in all cases completely identical, or if their differences correspond to modifications of the attributes of the generic concept.

GENERALIZATION is a means both of eliminating the indifferent attributes still contained in the formulæ of the specific laws, and of thereby giving the laws as precise a shape as possible.

1. Both in investigating causal laws, as in § 95, and in establishing the uniformities mentioned in the last section, we start first from a limited number of observations, of which the objects are concrete things and their definite relations in any particular case. The first step was to find for the same, or for perfectly similar things that differ only quantitatively, that formula which corresponds to their mode of action at all times. In this way we obtain propositions of which the subjects are partly fully-determined particular things—the earth, the moon, etc.,—partly infimæ species, whose universality is not generic.

But even unpremeditated experience leads to a number of propositions which have *generic* universality, and which assert the same predicate of dissimilar things agreeing only in one or a few attributes. That all bodies which are heavier than water sink in water, that all fluids assume a horizontal surface, that all birds are warm-blooded, are such propositions of experience. If they aim at being more than mere narrations of our experience hitherto, then what they tell us is that the predicate is necessarily connected with the attributes of the general subject—concept.

2. We need not explain at length that the essence of this kind of induction consists in the formation and import of the generic concept; the process which takes place in it is that described by Aristotle: all things which fall under the specific concepts *A*, *B*, *C* have a predicate *P*; *A*, *B*, *C* are the species of the genus *G*; from this it is inferred that the proposition all *G*'s are *P* is true. But we have already pointed out that we cannot assume, as Aristotle did, that the species of a genus are all known, especially where classification depends upon preceding induction. If we look more closely, what really takes place is the following process: If *A*, *B*, *C* agree in manifesting predicate *P*, then we may conjecture that predicate *P* has its ground in that in which they are alike, not in that in which they differ; by comparison and abstraction we now find an *E* and an *F* in which *A*, *B*, *C* agree, and assume that *E* and *F* make the predicate necessary; if from *E* and *F* as attributes we form the generic concept *G*, then it is assumed that *G* represents the ground of the predicate *P*, and that, therefore, all other as yet unknown things, which fall under the same generic concept *G*, must have the predicate *P*.

Or, to express it differently, the attempt is made to represent the particular propositions *A* is *P*, *B* is *P*, *C* is *P* as consequences of a common major premise; this is possible by subordinating them to a middle concept *G*, to which the predicate *P* belongs.

3. An assumption is made here, which we have not yet examined—the assumption that like consequences flow from like grounds; the assumption which Newton expressed in the first two *Regulæ philosophandi*: *Natura nihil agit frustra, et frustra fit per plura quod fieri potest per pauciora. Natura enim simplex est et rerum causis superfluis non luxuriat. Ideoque effectuum naturalium ejusdem generis eadem assignandæ sunt causæ, quatenus fieri potest, uti respirationis in homine et in bestia*, etc.

This assumption is not empirical, for, if we consider it more carefully, we find that all our reference of sensations to things is based upon it.

Like the postulate that the given is necessary, it is an assumption demanded by our logical needs ; but, as Newton indicates by his cautious *quatenus fieri potest*, it is an assumption which merely regulates hypotheses, and not a principle to which we could promise unconditional confirmation. It is always confronted by the possibility that what is like, or, at least, indistinguishable for us, follows from different grounds.

4. It is important here again to distinguish between general propositions of a more indefinite character and strict laws which determine the predicate completely. When it is shown experimentally that in a chamber emptied of air all known solids and fluids fall with equal velocity, we are justified in affirming that all differences in their chemical constitution, size, form, etc., are indifferent, and that the ground of their behaviour is to be found in that alone in which they all agree, and that it will be such as always to give rise to the same quantitatively determined motion ; we have a strict generic law. But when it is said that all metals conduct electricity, the predicate is not absolutely determined, for the power of conducting differs in different metals ; the ground of a predicate which differs quantitatively cannot lie simply in that in which all metals are alike. A strict law could be established only if there appeared in the attributes constituting the concept of metal quantitative differences proportional to the power of conducting ; then we could assume that the power of conducting was grounded in these attributes by themselves. But if that cannot be shown, it is open to conjecture that the power of conducting does not merely depend upon that in which all metals are alike, but that it is also influenced by those characteristics in which they differ. To this extent the general proposition will be uncertain, and cannot be regarded as the expression of a simple necessity ; we have a mere inference of analogy, in which we expect that every body which agrees with the known metals in the attributes constituting the concept of metal will also resemble them in the power of conducting.

Thus it is only where an absolutely identical predicate, or a predicate which is proportional to quantitative differences in the attributes of *G*, belongs to the different things *A*, *B*, *C* which fall under a genus *G*, that generalization stands upon firm ground, and can infer from the known species of *G* to the whole genus, according to the same principle by which we refer like sensations to similar things.

5. These considerations have another bearing when we attempt to make the expression of the general propositions perfectly precise. If, for

example, we have a number of specific laws that gold, silver, iron, lead, glass, etc., sink in water, then the combination of them to one general proposition serves for the elimination of those attributes of the subject which are indifferent and without influence upon the predicate, and it shows this as the consequence, not of the whole subject, but of a certain attribute of it; logical abstraction serves the same purpose as would be served by the separation in reality of different circumstances in investigating their contribution towards an effect. That gold sinks in water is indeed a true proposition, but, if stated as a law, it contains more than is necessary; it includes in its subject attributes which contribute nothing to the predicate. We find the particular ground why it sinks when we learn from comparison that all bodies which sink in water have in common a specific gravity which is greater than that of water; and, further, that in all fluids those bodies sink which have a specific gravity greater than that of the fluid.

From this point of view generalization is not merely an external comprehension; it is also a means to logical completeness in the expression of particular laws. The judgment: If *A* is, *B* is, has complete logical precision only if *A* does not merely *contain* the ground of *B*, but *is* the ground; and the value of such a precise expression shows itself in the fact that it yields the most general contraposition. From the proposition that gold sinks in water, I infer that what does not sink is not gold; from the proposition that all bodies with a greater specific gravity than that of water sink in water, I infer that what does not sink has not a greater specific gravity; I negate a much more general predicate, and thus obtain a much more useful major premise, excluding a far greater number of possibilities.

6. When the special laws combined by a generalizing induction have been collected from the whole range of our experience, then Newton's rule of explaining like by like is a reason for conjecturing that the general proposition contains the only ground of the predicate. When we find that all *A*'s are *B*, and that *B* appears under no other conditions than *A*, then we have a right to convert the proposition and say: whatever is *B* is *A*, or only the *A*'s are *B*. It is true that our inference is not absolutely certain; it might be that in consequence of the limitation of our field of observation our *A*'s contain some superfluous determination, and that *B* by itself might be the consequence of some other condition. When we find that all known ruminants have cloven hoofs, and that cloven hoofs have

never been discovered in any other animals, we have a sufficient ground for conjecturing that these two attributes are necessarily connected ; yet no one will maintain it to be impossible that some other organization of the digestive apparatus should co-exist with cloven hoofs. Whether such an inference is trustworthy or not can only be finally decided with a certain amount of probability by mere analogies, and by passing under survey large provinces ; in one set of phenomena we invariably find fixed distinctions, and exclusive connections between certain predicates and certain presupposed conditions, *e.g.* in mechanics and chemistry ; in others, *e.g.* in the organic world, the manifold combinations and the variation between individuals teach us greater prudence in closing our concepts, and in formulating general propositions.

7. Where the propositions obtained by generalizing induction are confronted by exceptions, a revision is needed of the concept reached by abstraction. For the way in which this is to be done we may refer to § 94.

VI. INDUCTION IN THE PROVINCE OF PSYCHOLOGY, AND ITS PRESUPPOSITIONS.

§ 97 *a.*

The methods which aim at discovering general propositions, whether causal or empirical laws, may be applied in principle also to the province of PSYCHOLOGY, *i.e.*, to the phenomena of the particular individual consciousness, and to the events of the external world which are given together with them.

More especially is the assumption of causal relations between events in consciousness and external changes justified by the general presuppositions of empirical investigation ; while the theory of PSYCHOPHYSICAL PARALLELISM is neither made requisite by the concept of causality or the principle of the conservation of energy, nor capable of being carried out, owing to its consequences.

A process quite parallel to the induction of natural science is, however, opposed partly by the impossibility of measuring psychical phenomena, partly by the variability of psychical subjects in consequence of their development, and partly by the great differences between individuals which are to some extent connected with this development.

Except, therefore, within the sphere of psychophysics in the narrower

sense, we cannot hope to establish exact general laws, by which the concrete temporal course of successive events in consciousness would be determined on all sides in an unmistakable way.

The development of psychology shows that it is first within the total content of consciousness, which can be comprehensively surveyed by memory and association, that fixed connections, which are independent of the moment at which they happen to enter into consciousness, are recognised between ideas, volitions, etc. ; and within certain limits these show regularity in our psychical life on the whole. This is the basis from which we must start in our analysis of particular moments of consciousness, and find by observation or experiment the corresponding connections in them.

It is in this way also that we can alone find firm ground, where the differences of individuals will fall into the background, and that which is common to all will preponderate.

Knowledge of the connections to be thus found refers partly to the dependence of events in the individual consciousness upon previous conscious events in the same subject, partly to the dependence of the particular subject upon the mental events in other subjects, partly to the dependence of psychical life upon the states of the organism, more especially of the brain.

With reference to the last point, the claim to ground psychology upon physiological knowledge is unrealizable ; psychological investigation has its independent task to perform, and it should rather be said that psychology is the key to this part of physiology.

1. The processes by which we pass from particular perceptions to the establishment of general propositions concerning what is given to perception, especially to the establishment of causal laws, have been traced in the preceding paragraphs only so far as referred to connections in that universe of things which is distinct from us, the perceiving and thinking subjects ; things to which, in accordance with the ordinary view, we assign a being distinct from our conscious being, and of which we assume that they may be known in the same way by all who perceive them under the same conditions. For even when we have realized, by analysing the concept of the thing, that the idea of thing has only been brought about by means of a synthesis which affirms one ground of unity for immediately given sensations, we still think of this ground as existing independently of ourselves, and it is only by this assumption that we are able to bring unity

and connection into phenomena which, from the purely subjective point of view, are irregular and accidental in their changes.

New questions arise, however, when we take up the position of the particular individual subject, and regard everything which is given in his consciousness as primarily nothing but the object of consciousness. In this way we test more closely the assumption upon which is based the whole process of the idea of an objective world, that one part of our conscious experience, sensations, are to be regarded as effects of external objects; and we investigate in particular the way in which we come to know the causal laws assumed in this view; and examine how the inductive methods, so far regarded only in their application to the external world, are applicable here; and again how the methods of induction can be applied to that total content of consciousness which includes in addition to the idea of external things all the experiences which we refer only to ourselves as conscious subjects, such as acts of thought, feelings, desires, resolutions.

We have already pointed out, when describing the elements of the psychological concepts (§ 74, 16-19, p. 137 sq.), that the general concept of causality is applied in a different meaning to those elements which make up the content of self-consciousness; and in accordance with this different meaning the methods by which we endeavour to establish psychical laws must also take a different form.

Having regard to the incomplete stage at which psychology now stands, and to the agitation in which questions of method are involved, we cannot undertake to describe more fully the methods of psychology, but only to make clear what it starts from and what it aims at, and to test the various possibilities.

2. Those connections seem to offer the simplest task which take place between a stimulus coming from the external world and the simple elementary sensations; when once we have assumed that the objects of the external world are distinct from, and independent of us, we have here a relation which, according to the concepts employed elsewhere, presents itself as one of transient causality, and it seems a matter of course that the methods by which we establish laws concerning the action of causes elsewhere can be applied easily and simply here also. The sensation of pressure or pain which is excited by a body coming into contact with the skin, the sensation of sound which follows upon the striking of a hard body, these fall naturally into that general class of connections between

successive events in different subjects, to which the relation of an efficient cause to its effect is primarily applicable in the inaccurate popular sense ; and upon the assumption that our subjective sensations are in this sense effects of external causes is based (according to § 87, 9, p. 244) all our more accurate determination of the real relations of objects, their spatial and temporal relations, their attributes, and the changes in these. In rendering this original assumption scientific what we aim at is to investigate more exactly the objective events, and more especially the intervening terms in the causal process, and this we do by tracing the vibrations which are imparted by the sounding body to the air, by the shining body to the ether, as media by means of which they reach the ear and eye ; we undertake to determine the time-relations and the amount of mechanical action, according to the general methods applicable in formulating mechanical action ; we try moreover to explain the events within the body, the function of the nerve when excited at the periphery, the process by which this peripheral excitation is transmitted to the central organs, the velocity with which this transmission takes place. Even if the exact nature of the progressive change known as transmission, and more especially the nature and extent and even the position of the change which occurs in the central organ, may be far from being exactly determined, yet we cannot doubt that such changes do take place, and that the final link in the chain, the conscious event of sensation, depends in nature and amount primarily upon these events in the central organ, and only through them upon the external causes. Where we have like external conditions—the same events between external bodies,—and a like subjective disposition—both physiological, in the nervous apparatus, and psychological, in the attention, etc.,—then sensations follow the stimulus in a definite sequence in consciousness, which we are obliged to pronounce alike, though it is true that for the most part we can judge only by comparing the present sensation with memory-images. So far as any exact control is possible here, we must also assume that for the great majority of particular individuals the consequences of the same external causes—sensations of colour, sound, etc.—are alike ; for other individuals we are able to refer the absence of certain classes of sensations to organic defects without fear of error, or at least to assert that in consequence of unknown organic dispositions anomalous sensations must occur, as in cases of colour-blindness.

All the motives which everywhere lead to the assumption of a causal

connection are given so completely and unambiguously in the actual relation between stimulus and sensation, and the dependence of the latter upon the former is so analogous to the dependence of external events upon preceding changes, that there has generally been no hesitation in seeing here also a causal relation in the same sense in which we confidently assume one between a blow and a movement of the body struck, or between the heating of a body by fire and its melting. The assumption which we make elsewhere, that by the activity of a certain real thing a change is produced in another which has not its origin in that other, can be applied here also in so far as we regard ourselves as the one permanent subject of our activities and states, and place ourselves over against external things as such ; it is these external things which act upon us, the conscious subjects, and give rise to events in consciousness.

3. If, however, we examine it more carefully, this relation presents several aspects which exclude a complete congruence between the causal laws of physics and the causal laws of psychophysics. In the former (§ 95, 17) it was necessary, before a causal law could be formulated with complete exactness, that both the amount of the acting cause and the amount of the effect should be determined, and a proportional relation established between the two. But this condition cannot be fulfilled here, inasmuch as sensations and feelings are not as such directly measurable ; and even if we had the right to take the intensities as proportional to the strength of the stimulus, yet we could establish no simple relation between qualitative differences in sensation and their corresponding differences in stimulus, which are frequently only quantitative. In the same way the "circumstances," and especially the subjective conditions of conscious sensations (attention, etc.), refuse to submit themselves to definite measurement.

These difficulties, however, would merely prevent us from formulating an accurate law, with an exact and ascertainable application to all particular cases, for the causal dependence of sensation upon stimulus ; by themselves they would be insufficient to break down the conviction that in the ordinary sense sensations are to be regarded as effects of the action of external stimuli.

4. More serious difficulties have lately been raised, which find their expression in the view that the assumption of a causal relation between a mechanical event in the external world and a conscious sensation is contradictory to the principle of the conservation of energy, and that this

principle necessitates an entirely different view of the connection between stimulus and sensation—that of psychophysical parallelism.¹ As this view would give an essentially different direction and importance to our methods in the province of psychology, it must be briefly explained here.

We may distinguish between two propositions which are contained in the assumption. There is first the *positive* statement that throughout material and physiological events there is a strict causal connection in the sense that every change in the brain substance, whether caused from without or produced by internal organic processes, must, according to the principle of the conservation of energy, have its material consequences, whether they consist in chemical changes, in a different disposition of the atoms, in the motion of fluids, or in electrical events, etc., and on the other hand that everything which takes place in the body must have its completely sufficient cause in the preceding material events. The body would then be, according to Descartes' statement (though it is not consistently carried out by him), a self-sufficing machine; everything which goes on in it would go on in just the same way from merely physical and chemical causes, if no psychical activity existed. The *negative* aspect of this proposition is that a psychical event can never be regarded as the effect of a physiological event, for the capacity for work of the brain-substance is exhausted in the equivalent physiological events, and cannot therefore produce a surplus of effect which fails to be in any way equated to molecular motions. On the other hand, a psychical event cannot be the cause of a physical event; if it were, the sum of active and potential energy which is constant in the whole material universe would experience, in defiance of the principle, an increase (or in the case of inhibition a diminution).

But experience shows us, on the one hand, that physiological events of a certain kind are regularly accompanied by phenomena of consciousness, and on the other hand, that we have every ground to believe that no psychical event goes on without corresponding physiological

¹ Cf., e.g., Hugo Münsterberg, *Ueber Aufgaben und Methoden der Psychologie* (a work which contains in detail much which is true, as well as certain principles with which I cannot agree); F. Paulsen, *Einleitung in die Philosophie*. I note expressly that I do not count Wundt amongst the supporters of this view, since, even apart from his metaphysical background, he breaks through it by the assumption of "volitional powers, which intervene in and determine the course of natural phenomena" (p. 331 of the *System*).

I have aimed above at noticing as briefly as possible the essential points, without adhering to any given formula.

changes—chemical transformation of brain-substance, change of tension in the vascular nerves, etc. ; thus the only assumption left to us is, that the two series are completely independent of each other, and yet run strictly parallel, that we are justified in looking for definite causal relations only within the series of physical events by themselves, or within the series of psychical events by themselves. Each of the two worlds presents a closed circle—at any rate so far as concerns particular causal connections, and in place of the reference of physical to psychical processes we have only the statement of the simultaneity of the corresponding series. It is the modern way of stating Spinoza's doctrine, which culminates in the proposition : *Ordo et connexio idearum idem est ac ordo et connexio rerum.*

5. Upon what this parallelism as a whole ultimately depends is a question for general metaphysics ; we may, however, draw various conclusions from it respecting our empirical knowledge of what is actually given and the methods applicable to it.

On the one hand it may be emphasized that in the physical universe the causal connection is given to us without any break ; though we may not yet have traced it in all its details, yet the material events lie before us and we may hope to resolve them into completely mechanical action. It is different in the psychical universe. Here we are quite unable to attain to a similar knowledge of necessity in the sequence of our ideas, feelings and efforts of will ; the most we can say is that the relation of an act of will to its consequences is a connection which may be known as necessary. But beyond this, psychical events are not given in unbroken connection ; the course of ideal associations, which corresponds psychologically to a general uniformity, is interrupted by sensations for which no connecting link can be found in the preceding state of consciousness. It is thus impossible to establish causal laws within the psychological region. If we want to trace an unbroken connection, we must pass to the physical side ; here there are no gaps, and certain observable laws of correspondence between cerebral states and phenomena of consciousness enable us to show that the events given in consciousness are such that in their physiological counterpart we can find a strictly causal chain, one, however, which cannot be known as between them, but only as between the changes of the organism. The co-existence and succession of psychical elements can now be explained by the co-existence and succession of physical acts ; but just because of this correspondence the necessary connections can be transferred from the physical to the psychical series. For a psychology, therefore, which aims

at understanding the causal connection of conscious events, the clue is to be found in physiology.

Over against this view there is another which assumes in principle a causal connection between psychical events also, and is obliged to take as the problem of psychological investigation the knowledge of this causal connection; as, however, an unbroken chain cannot be constructed out of the events in the consciousness of a particular individual, it is compelled to assume hypothetical elementary psychical events, which belong to subordinate centres of consciousness, but exercise an influence in the central consciousness. In this case investigation must aim first at finding regular connections within the particular psychical life, and then at extending these by means of analogy, and taking into consideration the physico-psychical parallelism, so far as is necessary to the explanation of the given. There is no doubt that the latter view alone treats the two spheres as really parallel, and that it is only in accordance with it that a psychology which goes beyond mere description is possible on this basis.

6. Without question the difficulties are serious which have led to the view that a causal connection between physiological events and psychical phenomena is inconceivable, that we are compelled to return to Spinoza's assumption that the *modi* of the two attributes are completely independent of each other, or to a mere pre-established harmony, and to relinquish the thought which is always first to suggest itself that sensation is causally dependent upon stimulus, and motion upon volitional impulse. But are the difficulties absolutely insoluble? Is the assumption of the psycho-physical parallelism called for by infallible material or methodological principles? and if not, if it is a mere hypothesis which recommends itself as a way out of certain difficulties, does it not lead to much greater difficulties which are from the nature of the case insoluble? The importance of this question for the principles of method must justify us in pointing out a few of its aspects.

First as to the meaning and application of the principle of the conservation of energy, and the justice of the inferences drawn from it. On page 112 sq. we had to distinguish between two ideas which are contained in this principle. The first follows from the relation between the concepts, and is that the effect is the measure of that which we can regard as the action of a cause; the second, which does not coincide with the first, is that within the material universe of action all changes proceed from external causes alone, and that things can exert efficient action only in so far as they stand

in relations which may give rise to a change in another thing ; and finally that within the same universe the power of a cause for efficient action is exhausted in proportion as it produces in another object an effect, which itself possesses a similar capacity for efficient action. These latter propositions are beyond doubt empirical, and extracted from a number of events in the first instance from inorganic nature ; they cannot be deduced as a necessary postulate from the general conception of causality. The principle of the conservation of energy tells us, moreover, nothing as to what effects depend upon what causes, and what the conditions are under which particular causes act ; it does not tell us that motion under certain conditions produces warmth, it refers only to quantitative relations, it says that where efficient action takes place this quantitative equality exists between the amount of capacity for work represented by the effect and the amount of capacity for work from which the effect has proceeded, between the capacity for work which one body gains and that which the other loses. By itself it tells us nothing as to the conditions under which active energy passes into potential energy, and *vice versa* ; it tells us only that when a certain motion or other change actually occurs it has been produced by active or potential energy, which must have disappeared itself in the process. What direction will be taken by the successive changes which conform to this law of quantitative equivalence must be discovered in each case empirically, as well as the equivalent numbers between the particular forms of the events. If a body thrown perpendicularly upwards should by a miracle remain suspended at the highest point for a hundred years, the principle would not be violated, any more than it would if, say, phosphorus and oxygen should subsist peacefully side by side for some time without combustion ; the sum of active and potential energy would not be altered.

But the principle of the conservation of energy, as ordinarily understood, contains the further statement that every effect actually possesses the power of producing a further and equal effect, and will produce it sooner or later, and that every energy (active or potential) points back in its turn to a previous quantitatively equal amount ; that we must refer every state of capacity for work to a previous one, and that from it states will follow *ad infinitum*, which contain the same amount of capacity for work. The principle, when thus understood, contains therefore a general statement about the actually existing causal relations of all events and states in the whole of material nature ; it contains the statement that the conditions of

capacity for work are produced by a process of constant change in such a way, that every existing capacity for work will again become actual.

That all the activities of causes which take effect are the mere continuation of previous action, and that every state which is due to an action will itself become capable of action, is, however, true only if we assume that we are dealing with absolutely permanent substances, of which the changes are conditioned only from without, and that the world is so constituted that every causally conditioned state disappears in order to bring about a further change. But the assumption is by no means the only one possible ; there is no logical law to prevent us from assuming, as effects of causes, changes in which the process comes to an end, and which need not necessarily be the sources of fresh changes in other things. The mechanical theory of heat has shown it to be the consequence of its own pre-suppositions that the course of the transformations of energy in the universe will lead to a constantly increasing amount of mechanical motion being changed into heat, which cannot be changed back again, because where there is a uniform distribution of temperature the condition is wanting under which alone heat can take effect, the presence, that is, of a colder body to which the heat of the warmer one can be imparted. Then all action, all production of change, would, as a matter of fact, come to an end ; the uniform distribution of temperature, which means the absolute cessation of all processes whatever, would be an effect only, no longer a cause. We are confirmed in what we pointed out on page 113, that the concept of energy always contains a hypothetical element ; energy belongs to the moving body only on condition that it comes into contact with another, to the warm body only on condition that it can transfer its heat to a cooler one. If these conditions cannot be realised, then capacity for action is an empty phrase ; it ceases, indeed, to exist when the conditions are absent which are necessary to action. That they always will be realized is, indeed, generally included in our meaning when we speak of the conservation of energy, but the question is made doubtful by these consequences of the mechanical theory of heat. Even if these consequences should prove to be actually incorrect, still they show that to assume them is not to contradict the physical laws ; the statement that in the course of the world no effect appears which is not itself the cause of fresh change expresses, not a necessity, but only the assumption of a matter of fact, which must be empirically demonstrated.

Limitations similar to those which we get by looking forward to the end

of the present course of the world follow also from looking back to its beginning, or more accurately to those conditions which are assumed as given in the principle of the conservation of energy. We have pointed out (App. E.) that the spatial distribution of the masses which represents potential energy must be accepted as something simply *given*, which cannot be again reduced to previous active energy; it is only in the particular case that spatial distance can be derived from active energy, and these particular motions which lead to spatial separation always presuppose an original total distribution of the masses which cannot be further explained causally. It is the same with the chemical differences between substances; they contain potential energy in so far as motions, heat, etc., are produced when they combine; but the presence of chemically different elements is pure matter of fact, it presents a condition which cannot be derived from elsewhere, and unless it were given the occurrences to which the principle of the conservation of energy is applied would not be possible.

Even if equivalence between all chemical events and mechanical motions, heat, electricity, etc., were fully established empirically, yet we could be certain of the truth of the principle only within the sphere in which its determinations were obtained, in those purely physical and chemical events of inorganic nature which we reduce to exact causal laws in such a way that every event may be calculated from its conditions. Strictly speaking, as soon as we come even to physiology we are unable to show that we are dealing only with causes which obey these laws. The fundamental physiological events, the formation and propagation of cells, do, it is true, so far as they can be directly investigated, manifest themselves as chemical combinations and divisions, and manifold mechanical and other changes connected with these; but to deduce them from known laws of the chemical combinations between the elements, and in accordance with these laws to determine by measurement and calculation the exact course to be taken by the transformation of energy from moment to moment, to state the amount of potential energy stored up in a spermatozoon or a germ, as we state the amount of potential energy represented by a kilogram of coal and the corresponding amount of oxygen, will be allowed by the most hopeful to be a problem which is at present insoluble. That it is possible is a hypothesis justified upon methodological grounds, but not a proved proposition; the statement that all that happens in the material universe, including organisms, constitutes a closed circle governed unambiguously

and without exception by the principle, is an assumption in which we extend by mere analogy what is true on certain grounds and within certain limits to a different region where an empirical proof is not possible. Or do we know enough about the chemical, electrical and vaso-motor events in all particular parts of the brain, to enable us to reduce them, even by a first rough approximation, to definite unbroken causal connections, and to weigh in a balance the gain and loss of energy? Can we show in detail what becomes of the active energy of the waves of sound which strike the ear, and how and where, within the auditory nerve, the brain, the motor or vascular nerves, it is transformed into actual or potential energy, or disengages stored potential energy? Who can determine the amount of potential energy called into play by a few words of exciting news? It can never be shown empirically that within an animated organism also all events constitute nothing but a material, closed causal connection, which conforms to the principle of the conservation of energy. The more certainly all physical causal connections and the equations for the different forms of events are established empirically, the more surely is empirical proof called for before these connections can be extended to what takes place in nerves and brain, if they are to be accepted as certainly true.

7. Nor can the possibility of regarding physiological and conscious events as mutual cause and effect, and of applying the concept of causal dependence in general to them, be refuted by means of the general concepts of cause and effect. It has been said that it is inconceivable that any equation should exist between a chemical transformation or a physical event in the brain on the one hand, and a sensation, a thought, a feeling, on the other hand; that energy should disappear from the physical side to re-appear in the wholly dissimilar form of conscious events, or that conversely a given amount of material change should correspond as effect to an immaterial conscious event. So much we must allow, that we cannot measure psychical events with any measure so as to connect by an exact causal law each amount of neural excitement with the corresponding amount of psychical process, and to reduce the different forms of psychical process to a common measure. But even in the region of natural science many causal connections have been accepted as existing beyond doubt and regarded as inductively proved before their equations were known; that friction produces heat, and that heat, through the expansion of steam, gives rise to motion, was ascertained before Mayer and Joule had found the equations which enabled them to calculate how much of the heat pro-

duced changes into motion, and how much is useless for the purposes of the steam-engine. The formulation of the causal law was incomplete, the exact limits within which it was true were as yet unknown ; but that a causal connection does exist, and that heat increases with the mechanical force of friction, and the machine's power of work with heat, was indisputable. It is much the same in psychophysical events. The intensity of pain caused by a blow cannot be exactly measured, any more than the intensity of the volitional exertion by which I contract my muscles. We are unable also to discover a common measure for physical and psychical events ; but that my pain increases with the force of the blow and the work of the muscles with the amount of exertion is so far beyond doubt, that from an unprejudiced point of view we cannot hesitate to place the two events in the relation of cause and effect. Of course, while we are so entirely ignorant of the exact physiological action of a nerve-stimulus, no one would think of stating or expressing in kilogrammes the amount of chemical transformation in a number of ganglionic cells, or the strength of the electrical current produced in them, which is due to the sensation of sunshine or of the report of a cannon ; as we know nothing about the more exact relations in the transformation of the active energy of the waves of light and sound in the nerves, we are also unable to discover which portion of it would be the proximate cause of the genesis of sensation. Nor can we ascertain how far a volitional impulse consciously exerted would, like an active force, give rise as direct cause to an excitation of the motor centres, or how far it is to be regarded merely as setting free some store of energy, like the closing of the electric current which explodes a mine.¹

But the absence of exact knowledge does not prevent us from making the general assumption that a causal relation does nevertheless exist ; and the principle of the conservation of energy is overstrained if it is taken as prohibiting this assumption. Even in the physical universe, from which it was obtained, and within which it is empirically proved, it states only that within a certain complex of material causes, which we assume to be a closed circle, and not influenced from without, the sum of active and potential energy remains constant ; and it depends essentially upon the presupposition that within this circle we are dealing only with elements of constant forces, and with conditions of their action which are contained in

¹ It might even be possible to maintain the hypothesis that the physical law of energy remains intact, and that only the conditions of the transition from active energy into potential, and *vice versa*, vary with relations to psychical events.

the external relations of position and reciprocal motion. This principle is not violated if we assume that such a system of material masses may also enter into causal relation with elements of other kinds of force, and that the effects which issue from the forces present in it may appear also outside its limits, or that it may be determined in particular parts by forces of a different nature. The principle states only that if, and in so far as, material masses act upon each other an equation will exist between the power of work of the preceding state and that of the succeeding state. In no sense, however, that can be empirically confirmed does it demand that every material change should have only material effects, or proceed only from material causes; the truth of a principle within a closed circle of constant material causes does not justify us in the inference that material things must, under all circumstances, form a circle closed on all sides.

All our assumptions of causal relations in the material world refer ultimately to relations which are contained in the nature of substances and find their expression in the concept of constant forces, but this does not mean that the material elements can stand in relations only to such elements as we can determine entirely by material attributes and forces; in addition to those relations which alone physics investigates and which it can isolate in particular cases, other relations to subjects of another kind may exist. And since more careful analysis of the physical concept of force shows us that the event which we regard as effect is due as much to the nature of the *patients* as to that of the *agens*, that the *patients* changes in the way appropriate to its own nature when it enters into a given relation with the *agens*, there is nothing in the causal concept itself to prevent us from finding between even heterogeneous substances and the events occurring in them a relation of such a kind that a given change which is appropriate to the nature of the one is responded to by the other in the way peculiar to it; a material change of some sort in the brain is responded to by the subject of consciousness with a sensation, etc. Nor is it any objection that we can form no intuitable picture of what takes place; it is a mistake to think that the principles of mechanics, for instance, are particularly recommended by being intuitable, what we can intuit is never more than the event and the linking of events, never the fact that the one is grounded by the other. For ordinary consciousness the connection between my will and the motion of my arm is just as intuitable, *i.e.*, just as firmly grounded in immediate experience and associa-

tion, as the transmission of a shock from one billiard-ball to another ; it may be, indeed, that we should find the latter even less comprehensible if we had not been previously familiar with our power of thrusting a body away by a voluntary movement of the hand.

According to the ideas which we always apply in external nature when reducing what we see happen there to causal relations, we are also compelled to interpret the external and internal events which follow in immediate temporal succession by finding the ground for the one series in the other ; by this empirical connection, which is given uninterruptedly, our thought in its search for grounds is called upon in just the same way to apprehend, at first in the popular sense, a sensation as the effect of an external event and of the change caused by it in the body, and a voluntary movement as the effect of the conscious volitional impulse directed to it, and then to analyse this connection as it first presents itself into its particular stages and to elaborate them logically. Empirical investigation must begin with these connections as we know them in particular cases. We can obtain an objective interpretation of our subjective perception, and hence the laws of nature and the principle of conservation of energy itself, only upon the presupposition that our sensations are regularly happening effects of external events, and only upon the same presupposition can we reach an intelligible explanation of those changing phenomena of consciousness which do not directly stand in any knowable connection ; nothing, therefore, but insoluble contradictions could compel us to relinquish finally this entire basis of our idea of the external world, and to look elsewhere for connections which can be found only in a purely hypothetical manner both in the physiological and the psychological universe.

8. I believe that I have shown that neither the general concept of causality nor the principle of the conservation of energy forbid us to comprehend in one causal connection the whole of that which is given to consciousness as happening, and which we break up into events within consciousness and changes in a material substratum other than ourselves. But even if we allow that the difficulties latent in a more exact formulation of psychophysical laws might make us favour the thought of a mere parallelism, could this thought be actually carried out, and would it not lead us into far greater and insuperable difficulties ? It will be enough to notice two points.

In the theory of parallelism we must assume that exactly determined

physical or chemical events correspond to all distinguishable psychical acts, and that every connection, *e.g.* of ideas, by means of which further ideas arise, is represented by a transition from one chemical or physical process to another. When I carry on the most ordinary mental operation, *e.g.* multiply or divide one number by another, the idea of the numbers, or of their digits, is represented by some states of certain ganglionic cells, the consciousness of the rules applied in calculating by others; from these states others must proceed according to general laws, which correspond to the successive stages of the operation, and, finally, the state which corresponds to the result obtained. Now there is nothing for it but to assume that the cerebral process conforms at once to two sets of laws: firstly, to the chemical laws according to which carbon, oxygen, hydrogen, nitrogen, phosphorus, etc., carry out the redistribution of their atoms in the cerebral substance, or to the physical laws according to which the forces which are disengaged or which are confined are transformed in accordance with the principle of the conservation of energy; and secondly to the logical laws of calculation, the rules of addition and subtraction, the rules of the multiplication table, etc. Thus the appropriate parts of the brain really represent a kind of calculating machine, in which, by placing the particular figures of the factors in certain positions, the figures of the product are brought together by means of mechanical motion. The same must be true for all other psychical operations; the most comprehensive combinations of elements, such as take place in far-reaching chains of thought in poetical or musical composition, have for their exact counterpart just as complicated transpositions of the cerebral substance, which take place on the one hand according to chemical and physical laws, and with strictly mechanical regularity, but also on the other hand according to the laws of logic, æsthetics, harmony, etc. Taken simply in this way, the strictest idealist might triumph over this acknowledgment that the whole mechanism of the atoms is something secondary, that its true and ultimate ground lies in conformity to logical, æsthetical and ethical law; that the arrangement of the atoms and the play of their reciprocal actions can only be really understood when they are regarded merely as consequences of higher mental laws, as the means of realizing a spiritual order: that, as Leibnitz expressed it, the kingdom of nature finds its explanation only in the kingdom of ends. If the brain is a calculating and thinking machine, then the analogy certainly leads us to regard the arrangement of its parts and of its mechanical interactions as determined by logical laws in such

a way that the mechanical laws coincide with the logical; their motions are the means of representing these logical laws.

But the supporter of parallelism now has before him the task of making conceivable the possibility of such a coincidence point for point of the claims of two codes of law; and here all detailed explanation breaks down. The calculating machine certainly seems to be a proof that purely mechanical processes may give expression to numbers and their relations, because it has to deal with equivalent units of which the numerical relations may be ultimately expressed by spatial arrangement. But can we conceive in the same way of a correlative to the consciousness of the necessity with which the product follows from the factors, and of a correlative to the consciousness of the universality of numerical concepts? In what way shall we represent the relation of the general numerical concept to the particular numbers thought of? And in a series of successive events how is the comprehensive consciousness of their course in time imaged in the states of a complex of cells? Even if we tried to make elementary parts and particular processes of the brain correspond to the particular elements of our associations and processes of thought, yet it is utterly impossible to make conceivable by any spatial arrangements, such as combinations of fibres and so on, the infinite connections which are brought about in inexhaustible variety by thought or by imaginative combination. How can all the combinations of sounds into words, of words into propositions, of propositions into comprehensive sequences of thought, proceed from the combination of elements by chemico-physical processes? Every attempt to carry this out in detail can only show the complete incomparability of the nature of the connection and sequence of material processes and the nature of the connection of images and thoughts in our consciousness, and hence the impossibility of thinking of even the simplest mental operations, of distinguishing and identifying, as represented by a system of contiguous elements external to each other; there is no conceivable correlative to the consciousness which comprehends the particular elements. The thoughts of an author are, no doubt, represented in black and white by a number of letters, but only for him who can read, and mentally reproduce from the signs, the connections which have led to their arrangement; without this unity of consciousness they show only a spatial contiguity and succession; the mental unification of the concepts of subject and predicate, the logical progress from premises to conclusion, cannot be printed with the letters. However

firmly convinced we may be that certain cerebral dispositions belong to the conditions of psychical activity, we must nevertheless, if we are wise, acknowledge that these relations cannot be so direct and so simple as these assumptions would make them, and we shall relinquish the hope of finding in the chemical processes of the brain a causal connection by which to explain the logical necessity of inference.

9. The other point which seems to me to contain an insuperable difficulty lies in the consequences which follow from the theory for the meaning of the will. That we move our limbs by means of will, that we change things in accordance with our aims, would be mere illusion; everything which goes on in the external world stands in a closed causal connection, and proceeds from physical causes; we stand in no other relation to our bodies than to the motion of the fixed stars. We cannot even secure a specific causality of will for purely psychical acts; the corresponding dependence of cerebral processes upon each other is of the same kind throughout, because it conforms to the same principle of the conservation of energy; the connection between the will and the events to which it is directed, and which, according to erroneous popular opinion, it determines, cannot differ in principle from the connection by means of which one idea involuntarily calls another into existence, the subjective phenomenon of the will is just one member in the whole series, of equal value with any other event. It is therefore quite consistent, starting from this basis, to try to eliminate the will altogether, to reduce it to the mere expectation of events about to happen, or the "feeling of innervation" which accompanies what is, physiologically regarded, a reflex transition from the sensory to the motor sphere; and this view has found its most characteristic expression in the proposition that the will is only a "complex of sensations." According to this the consciousness of exerting some force through the will is pure illusion; at the most we can only be spectators of the causal connections, which play their parts without our intervention, and more especially mere spectators of all the motions which follow at any time from the disposition of the brain.

This final consequence has already been pointed out by Occasionalism: *ubi nihil vales, nihil velis*; from this point of view the will is altogether meaningless, we cannot reasonably will where we can effect nothing; nothing remains but pure quietism; the attempt to cease willing (though this indeed is itself willing), and then to relinquish the distinction between true and false, and finally thought itself altogether. If this consequence

is impossible, if in our conscious life the will constitutes our most essential being, without which even the comprehending process of self-consciousness would be impossible, then we cannot accept psychophysical parallelism as a basis.

10. Starting from such presuppositions, moreover, how could we interpret the mutual relations of men, their spiritual community? The most obvious view is that all intercourse between man and man is mediated by external nature; the communication of thoughts and feelings always takes place through words and gestures, our influence upon the wills of others through external motions. According to that view of parallelism which allows us to trace only physical causal connections, movement and speech proceed only from physiological causes, and the processes to which they give rise in spectator and hearer are also purely physiological. When we wish to issue a communication or a command to another person our will effects nothing, either in our own body or in the body of the other; the physical causal connection gives rise upon physiological grounds to the movements of speech, the waves of sound produce in the body of the hearer those changes which, in some incomprehensible way, find their counterpart in the understanding of the hearer, and all further actions follow from the bodily changes of the hearer. Neither the visible creations of culture, nor the products of common action in the external world, nor social institutions, are the product of common thought, issuing from the interaction of individuals; we are each of us completely isolated in consciousness, thoughts and feelings are the mere reflection of our own brain and of events which are excited in it partly by other brains; that we stand in any relation of spiritual give and take is an illusion.

But if we are so far in earnest with parallelism as to maintain that psychical causal connections proceed parallel to the physical, then the physiological interaction of speaking and moving bodies must be accompanied by a corresponding interaction upon the purely psychical side; the same direct connection exists between one individual soul and another which exists between the movements of one body and the changes in another. We have immediate action of spirit upon spirit, and this would necessarily manifest itself in our spiritual life even if we should chance to have no perception and no consciousness of the external influences; we could never know whether what we think has its origin in the connection of our own consciousness, or is an inspiration from some other spirit, and this would cut off all possibility of tracing any comprehensible connection

between the ideas and thoughts which constitute my consciousness. This direct suggestion would also become the guiding principle for the explanation of historical connections; and, finally, the individual unity of our psychical life, which is said to consist only in the connection between events, would be lost in the confused stream of the totality of events within which it would be in vain to try to isolate any causal combinations, since by means of universal suggestion everything acts upon everything. Such presuppositions would ultimately make all psychology impossible, for in psychology we must start from the closed unity of the individual consciousness, and accept the fixed Ego as the centre of all relations; to continue to speak upon this basis of psychological methods by which to discover a comprehensible connection of what is given in consciousness, involves a series of inconsistencies. The objection that the concept of the soul has rendered no service to psychology applies only to the attempts of rational or metaphysical psychology to derive definite predicates from the concept of substance or of simple essence, instead of obtaining them from the given, experienced content of life; apart from that, the concept of the soul at any rate renders this service to psychology, that from the point of view of method it alone makes psychology possible.

11. If, then, we desire to proceed reasonably according to the methods which have led to constantly increased knowledge in the external world, we must refuse to ratify too hastily a hypothesis which is obtained by isolating one part of the given reality; the only justifiable way is to start from the whole complex of what is immediately experienced, and to interpret the connections we find in it by the postulate that the given is necessary and comprehensible. It is a pure fiction to say that the external connection of Nature is given as a closed circle; inasmuch as it is given to *us*, we are present with our functions of knowing, and cannot be eliminated. Quite apart from the idealistic view that this whole universe may be ultimately regarded as mere content of consciousness, even ordinary realism must allow that in every observation, every experiment, we have a complex of external and internal facts. We may, by isolating and abstracting, trace first the complex of external facts; but we must not forget that our abstraction is only provisional, adopted with the view of enabling us to analyse the whole reality; it remains as a final problem to understand according to the same methods the whole given complex. We may inquire how two bodies would be related if they were alone in space; but what we accept as true of them cannot be proclaimed as an ultimate law

of the universe, since as a matter of fact they are connected with innumerable others.

Scientific investigation always starts from that way of looking at things which first develops before philosophical reflection begins, and aims at completing it, at making it logically perfect, and, where necessary, at correcting it. But before philosophical reflection begins the total content of consciousness falls apart into two regions; into ideas which we refer to external things independent of ourselves, and into the idea of the Ego, its states and activities, and its relations to those existences which are other than it. We have already pointed out the presuppositions upon which this assumption of external things is grounded, and we have shown also that we cannot avoid assuming the existence of the one subject of our thoughts, feelings and efforts of will, which are given only in this relation to the Ego, and find in this relation their connection and possibility of combination.¹

For the beginning of the process and for the presuppositions from which it starts we do, no doubt, distinguish strictly and surely between the province of psychical occurrence and that of external things and events; there can never be any doubt as to what in the given content of consciousness I must refer to the external corporeal world, and what to myself; knowledge of myself and my internal states belongs to me alone; the external world can be perceived under certain conditions by every one in the same way as by me. The visible object before me is there for every one alike, but whether *I* see it, and how, I alone know; no one can have immediate knowledge of my seeing, and if I were to say, I see nothing here, no one could directly refute me. Another can perceive as well as I can the blow which is struck against my hand; he may also examine the contusion produced by it, the change in the tissues, the process in the nerves; but I alone feel the pain, and direct knowledge of this is inaccessible to the other; at most he can only infer its degree and nature from his own experiences, by assuming in me a psychical event similar to one which he himself has experienced.

¹ Even if we were inclined to regard that which we called "I" as only object, and as such to resolve it into mere temporally successive events, we should still be obliged, at the beginning of our investigation, to accept it at any rate as the phenomenal subject in the sense of § 91, since we are inevitably led by the processes of thought to the idea of a single subject from our connected conscious phenomena. We cannot, however, really abstract from the consciousness of a unified knowledge of these events which is inconceivable without a knowing subject.

But though the two provinces can be distinguished with certainty, yet they cannot be separated in reality, nor their connection abolished. If we isolate the external world in thought, its relation to our perception and memory still remains in the background and cannot be got rid of. If, on the other hand, we place ourselves entirely at the standpoint of the subject, and look at the content of consciousness merely as such; if we describe things as only subjective images of sight and memory, and disregard the fact that at other times we attribute to these ideas of ours a being which is independent of ourselves, yet there still remains the antithesis of thinking and what is thought; we ourselves are not what we think, any more than our dreams are ourselves; they are only the objects of our dreaming imagination, to which we know ourselves to be related, not only in the relation of thought, but also in that of joy or fright, of pleasure or of horror. This presupposition also leaves an antithesis between that which concerns the object and that which concerns ourselves as distinct from the object; even if we had only to analyse dreams, we should have to make the same distinction which is given with the fundamental fact of thought, with the division into subject and object.

If we leave these abstractions, there remains for the induction which is looking for general propositions about psychical phenomena the task of reducing to general laws those causal connections which are actually forthcoming; and, as it is generally presupposed, these connections are of two kinds; some are connections between psychical phenomena and the objective world outside us, some are connections amongst the psychical events which we refer to ourselves.

12. The way in which we may succeed in inductively establishing definite connections is prescribed by the nature of the case itself. The most important point is the psychological analysis of that which is given as co-existing in every moment of consciousness, the discrimination of the elementary events which we have to relate to each other and to external objects. In the external world things are spatially separated, and bounded by each other, and the changes which they manifest stand in definite spatial relations, by virtue of which we can distinguish and arrange them; for internal events we have no such means of discrimination and survey; all that is given is a complex whole which changes in time, and only by logical analysis, in which we compare what is immediately perceived with what is remembered, can we succeed in discriminating within the complex the qualitatively different and elementary data of consciousness.

13. It is in this way that, in the first place, the analysis of those ideas which we refer to the external world has progressed, and has shown how that which in ordinary life we call hearing and seeing—as when we say we see a man moving, or we hear a clock striking—is a very complicated process, and how the result, which is all that comes into distinct consciousness, has come about by means of a number of distinguishable acts—sensations of colour, spatial configurations, acts of discrimination and comprehension, combinations of the immediately perceived with memory-images, etc. The fact that sometimes certain of these elements appear without the others—a sound without the idea of the subject producing it, a flash of light without definite outline—may support this analysis, and it can be carried further and completed by means of experiment. On the other hand, the fact that the clearness of apprehension with which we perceive that which is similar in content varies greatly teaches us to recognise the subjective conditions which are given in attention.

This psychological analysis must precede before we can really investigate that set of causal connections which has till now attracted most attention, and has led more than any to laws which at any rate approximate to exactness, those connections, that is, in which our sensations depend upon external stimuli. Psychological analysis alone can determine the concept of sensation as that of the element which we have to regard as the immediate effect of nerve stimulation; in reality, a sensation is never given in complete isolation. All experiment in this department presupposes the possibility of this psychological analysis, in which the student must be trained before his utterances can have any scientific value.

Here we are already confronted with difficulties in formulating exact psychophysical laws. It seems easy to determine according to the general methods of induction that light of a given refrangibility will give the sensation of red, and that a string which vibrates a certain number of times when struck will give the sensation of the note A. But the actual sensation does not depend upon external causes alone, but also upon the accompanying subjective dispositions, according to which the way in which the result of the stimulus appears in consciousness varies; not only the amount of attention which is turned towards this part of the content of consciousness, but also preceding or simultaneous sensations, modify that which appears in consciousness; indeed, as a subjective phenomenon it has no definite, determinable character except in so far as it can be estimated, compared with other sensations, and identified with the

memory-images of previous impressions. If we constantly forgot our previous sensations so that at best only indistinct and uncertain traces of them remained with us, and were therefore constantly in danger of confusing different sensations all together, it would be impossible for us to ascertain that a certain subjective result corresponded to a certain external cause. Thus the only psychological result which we can study is fundamentally a judgment concerning the likeness or difference of distinct sensations; this is quite clear when we are dealing with degrees of intensity.

When therefore we say that the same sensation corresponds to the same stimulus, and that the intensity increases in a given proportion to the strength of the stimulus, we really only form a hypothesis and apply it to cases of fictitious simplicity and regularity; the hypothesis is, however, as fully justified as the law of inertia, or that of the parabolic motion of a projectile, if we can make the actual variations agree with it by introducing modifying circumstances.

A hypothetical element cannot be avoided, if only because the causal connection between stimulus and sensation is not direct, but mediated by intervening terms, of which we do not know the exact nature and mode of action. The differences of opinion as to whether the logarithmical formula of Fechner's law expresses the relation between the intensity of the external stimulus and the physiological processes of neural and cerebral excitation, while there is a direct proportion between these and the sensation; or whether the physiological processes increase in intensity in proportion to external stimulus, while in the transition to sensations the geometrical progression changes into an arithmetical, show the difficulties presented by this intervening term. The most probable solution is that which tells us to find in the conditions of our subjective estimation of differences in the strength of sensation the ground why, when a greater intensity is already present, a greater increase of stimulus is necessary before we can judge that the second sensation is stronger than the first; and this shows again that psychological analysis may help us where it is impossible to ascertain anything directly about the physiological processes.

We can only briefly mention those differences between individuals which even here make it difficult to establish exact laws which shall be true of all, or at least of the great majority of individuals. Here again the question arises whether we have to do merely with differences of the elementary events themselves—as is certainly the case in colour-blindness,

or in the incapacity to hear very high or very low notes—or with differences in estimating them; the fact that with practice our power of estimating small intervals increases points to the latter factor. The consequence is that we can formulate universal propositions for different individuals only in the form of averages, adding the limits within which the values vary.

14. When we leave this department, in which we have to deal with comparatively simple direct connections between external events and elementary phenomena of consciousness, the difficulties of investigation increase. According to analogy with our researches in the external world, the problem would be to analyse the given course of our conscious states and activities in such a way that whatever occurs at any moment should appear as the regular consequence of the preceding conditions; more exactly, as the combined result of a number of partial laws determining the influence of the different co-operative causes and circumstances. Every change in consciousness which occurs, *e.g.*, in the course of a day—and in actual perception we never have a state of rest, only an uninterrupted happening—would have to be investigated in its conditions and consequences, and the laws obtained which determine its occurrence; in much the same way as the motion of the column of quicksilver in the barometer is the combined result of variations in the pressure of the atmosphere and of the temperature, and is reduced to laws expressing the dependence of its height upon the laws of equilibrium and the laws of the expansion of matter by heat.

But if we are looking for connections which may perhaps be more accurately determined by induction, we shall find our causes falling into three main lines. First: there are those shown by psychophysics. Certain changes take place in the organism, and there is no doubt that the dependence of psychical phenomena upon the states of the organism is not confined to those elementary sensations and feelings which are found to follow external stimuli with determinable regularity, but that it holds good throughout the whole course of psychical processes of all kinds. The simple fact that by means of medicine such a total change in the state of consciousness as sleep can be introduced proves the importance of dependence in this direction.

In the second place, certain conscious events appear as dependent upon immediately preceding conscious events, as, *e.g.*, a memory-image upon the perception which awakens it, a feeling of terror upon a sensation, the

thought of a certain activity of will upon a wish that has somehow arisen. In all such cases application of the methods which serve to establish regular connections in the external world is hindered by the fact that we are not dealing with an invariable subject; the after-effects of previous events are always creating new dispositions which play their part in determining the succeeding motions of memory-images, thoughts and efforts of will, so that in the course of life the conditions upon which it depends what new activities will proceed from any given state of consciousness are literally changing from hour to hour. To this must be added the complex relations of particular forms of activity to one another: the movement of our ideas does not depend merely upon the relations of their content or upon the manner in which they were originally connected in spatial or temporal series; it depends also upon the feelings which accompany them and upon the directions taken by our interest, and this has its origin in the will. The wide differences between individuals which we are always coming across depend to a large extent upon this constitution of our mental life. The way in which our psychical activities are continued from a given moment and its content is determined by the whole history of the individual, and however similar the surroundings may be, this is certainly different for each particular individual; thus the comparable cases, which are always necessary if we are to carry out an inductive process completely, are not given, and we cannot assume that permanency in the subject which would justify us in expecting like events under the same conditions, even for one and the same individual, to say nothing of whole classes of individuals.

A third direction in which we must look for the conditions of what takes place in the individual consciousness is to its intercourse with other human beings, and the dependence of its ideas, feelings and volitions upon the ideas, feelings and volitions of others. This dependence is not, indeed, direct; all intercourse between minds passes through the external world, but in so far as we have learned to interpret by analogy with what we experience in ourselves external sense-perceived motions, words, etc., as signs of an inner life, as the expression of images, thoughts, feelings, volitions, we construct in ourselves the inner life of others, and in this way we are moved to many psychical activities which we should never have produced from ourselves or from the mere action of the external world upon our organs of sense. This is true even though the possibility of understanding what others communicate to us is always conditioned by

our being able to connect it with what we have produced ourselves, and though teaching can never do more than bring about the conditions and give the impulse for combining and elaborating ideas which are originally our own, so that to this extent the result of the external signs is altogether dependent upon our individual nature and disposition.

15. When we are fully aware of the wealth of psychical content and the manifold ways in which it acts even in the most meagre life, and realize the extent of the differences between the inner lives of individuals, we find ourselves confronted by a confused chaos of data, and the question as to how we may disentangle it and obtain even descriptive propositions which may express, as natural laws do, what really takes place, seems insoluble ; and yet our ultimate aim must be to represent the actual, *i.e.*, the whole concrete course of thoughts, feelings and volitions in the particular individual within a given time, as conforming to law.

If we look at the ways in which what we regard as relatively the most certain possession of psychology has been obtained, we shall find that they differ in many respects from the methods of natural science.

One part of the process is indeed the same. We first become aware of particular, easily noticed, regular connections between distinguishable elements, which constantly recur, and which we can therefore first pick out from the whole course by analysing and isolating them. Amongst them belongs that connection between perception and reproduction, between impression and idea, upon which Hume based his psychological analysis, and which, in a different aspect, Herbart has taken as the basis of his whole system. That perceptions which have occurred once or repeatedly will be reproduced, and that on the other hand most of our intuitable ideas which are not directly dependent upon present impressions have their ground in previous perceptions, is one of the propositions which were most easily discovered ; primarily, no doubt, it is an empirical law, but it is one which leaves no doubt as to the subsequent phenomenon being causally dependent upon the preceding one (§ 74). But even here we find a fundamental difference : the events which we thus relate as cause and effect do not for the most part succeed each other immediately in time, nor show any temporal relation which can be determined ; the memory-image emerges after a longer or shorter interval, which may sometimes be very great, and it is no easy question how we always know it to be such and distinguish it from a mere creation of imagination. There is certainly no direct and simple causal connection ; we must interpose a hypothetical middle term, in the

shape of a disposition introduced by the first impression—however we may conceive of it—and then the further question arises as to what causes this disposition to be active at the given moment and to produce a conscious image, and whether, as Herbart thinks, the removal of obstacles is sufficient, or whether a positive force is necessary to set it free.

16. The so-called laws of association, which also belong to the earliest discoveries of psychology, attempt to answer this question; they also are the result of an analysis which lays hold of a few obvious connections in the confusion of our inner life. But they have no claim to the name of laws, if only because when taken strictly they contradict each other; the one lays it down that reproductions follow the relation of contiguity in space and time, the other that they follow the quite different relation of similarity. And even apart from that—they cannot say that the corresponding ideas will always and necessarily occur whenever the occasion arises; any such statement is prevented by the fact that very many objects have entered successively into many different series of associations, of which our actual ideas follow sometimes one, sometimes another, when there is any associative reproduction at all. In most cases it seems to be entirely a matter of chance, and beyond calculation, what will occur to us upon any given occasion in the involuntary course of our thoughts; we may indeed conjecture that it was necessarily so according to some law, but we cannot point to the law. Thus the laws of association merely indicate certain directions which our reproductions may follow, or will in many cases follow, certain tendencies in the actual sequence of images, words, etc.; they cannot be represented as laws by which every actual course of ideas could be shown to be necessary, for a law it is necessary that given the same conditions the same thing always happens. Attempts to study this process of association experimentally have chiefly served to show how many are the different directions in which associations may work, without any ground being discoverable why sometimes one and sometimes another direction is taken. We cannot say even of the most habitual association between words and that which they signify, that a given word will always be certain to call up the same idea.

We cannot regard the propositions about association as psychological laws, clearly determining what happens in particular cases, if only because associations show in particular cases very different degrees of firmness in their connection, and are at one time complete, at another fragmentary. They cannot even be taken as hypothetical partial laws, yielding a definite

contribution towards the causal explanation of the particular case, since we are quite unable to determine with sufficient accuracy the conditions upon which the consequences would be dependent. And yet there is no doubt that when regarded from another point of view they are of great importance.

17. The laws of association are incomplete as an expression of psychical regularity, if we are aiming at reducing to an exact universal formula the events which proceed in time from moment to moment in the particular concrete consciousness. But the nature of our psychical life cannot be expressed by saying that conscious events follow in time upon conscious events ; it does not present such a simple course as the growth of a plant, in which cell is formed after cell, and each stage in the physiological events is present only for one moment before passing into another. The power of remembering previous impressions and experiences, the connection of new experiences with the previous store of ideas, which does not consist in mere addition but is brought about by the most varied syntheses ; the knowledge that the new is partly identical with or like to the old, partly different ; the fact that whole series of ideas and the syntheses of judgment proceeding from them can be reproduced at will—all this tends to make the moment in which the particular elements are present to consciousness unimportant in comparison with those fixed permanent connections between our ideas, which are always capable of entering into consciousness in the same way at any time. The spatial intuition of my ordinary surroundings is not present in consciousness at every moment ; nevertheless it is a fixed image which I can recall at any moment with a consciousness of its identity ; it depends not so much upon the varying occasions which remind me of it, upon the momentary causes which make me aware of it, as upon the connection in that of which I am aware.

* The numerical series exists in my memory in a fixed order ; it does not depend upon what may happen to cause me to count, nor upon the temporal succession of the subjective acts by which I think of one number after the other, or of one numerical symbol after the other ; nor does it matter whether I happen to be interrupted ; the important point is my comprehensive consciousness of a certain order, which I am able to think objectively as a sequence of numbers, and not merely of my acts of thought, and which I can therefore run through backwards when I like. Thus the ideas show an association not merely in the sense that I can observe and watch the passage of consciousness from one to the other ;

this is always the case when one occurs and the next follows ; but in the sense that I comprehend their own objective connection, and thus survey the whole series as with one glance. Association in the purely subjective causal sense as the law of the actual sequence of my ideas would be present, even if I should forget every term at once, if one idea should draw another into consciousness only, as it were, for an outside observer. The possibility of thinking the connected terms simultaneously is indeed really limited to a certain number ; the essential point is that I should myself be conscious of the series and of the law according to which its terms succeed each other, so that I may run through the whole connection with a certain consciousness of its order.

Thus what the so-called laws of association are meant to express is not merely laws for events in time, but also the much more important fact that my ideas present themselves in various connections which have become fixed for my thought, and are constantly repeated independently of differences in time and with the consciousness of their identity. And this objective and persisting relation is much more accessible to observation than fleeting, changing particular phenomena, or than the way in which these associations have formed themselves in the course of time ; their formation began at a time when there was no question of clear consciousness, and we can only infer hypothetically and from what we observe in particular cases, the course which it has taken.

These fixed and permanent connections are to our present conscious life as the permanent background, upon which the changing light of momentary consciousness flits to and fro ; in them we can find rules by which the transition of actual consciousness from one term to another is governed, more or less completely, according as certain disturbances are present or not. We cannot, indeed, calculate from them the actual life of ideas, since that depends upon many other factors as well, but we can at any rate subsume a great number of actual sequences under them, and so far explain them.

In so far as they are the result of a gradual development they serve as sign-posts telling us in what directions observation and analysis of actual events must proceed and what they must notice. They raise questions for us to answer in our attempts to proceed, it may be by experimental psychology, and lead us to enquire how under given conditions new connections of the same kind may arise, and they also serve to control our procedure ; our hypotheses about the gradual development of these

fixed connections can be satisfactory only if they explain the state of things which actually exists in them.

In taking this for our starting point there is the further advantage that we get a fundamental analysis, and that the varying influences of other forms of psychical activity are eliminated; we are here dealing with connections which belong only to the sphere of ideas, and which present themselves first as relations between the objects of ideas. The subjective accompanying phenomena, the manifold feeling, etc., do indeed determine the direction which reproduction will follow in particular cases, but they do not affect the spatial or temporal relations of ideas, the similarity or dissimilarity of their content.

Finally, in starting from this fixed portion of our ideal world, we gain the important advantage that in this way the differences between particular individuals may be more easily overcome, inasmuch as we obtain a basis which in its essential characteristics is the same for all individuals.

How the idea of the surrounding world forms itself in each individual from the varied play of sensations, we are unable to trace even approximately and for ourselves; and of directly perceiving psychical processes in other people there is not even a possibility. Nevertheless, that this idea is essentially the same for all, that every one comprehends and distinguishes the same things in the same space and in the same spatial relations, that they agree in the way in which they connect their experiences in time, and recognise the same similarities and differences, we know with sufficient certainty from the agreement in their statements and behaviour; we are even able to ascertain certain differences, as in the estimation of distance, etc. That such an image of the surrounding world should take the same shape in every one in spite of the innumerable differences in the succession of particular sensations, points to a psychological necessity acting in the same way in every one, by which particular sensations are connected in certain ways; a necessity which could never be discovered by observing particular events. Analysis will aim at discovering this necessity and at determining the particular distinguishable functions which co-operate in this result. For this purpose it will make use of the events which we actually observe in ourselves in particular cases, in forming and verifying its hypotheses. From this point of view we can at once state the law that the particular elements combine, partly according to similarity, partly in spatial and temporal series, because we find them regularly combined in these two ways in our image of the world. At the same time, if we take

this view, we shall guard against building too much upon the analysis of particular events, which is not to be relied upon for completeness, and against believing that such ideas as those of thing and causality are reducible to the mere associations of sense impressions, or that the idea of space is based merely upon the formation of qualitatively different sense impressions into series. Kant's procedure, although not intended as psychological, gives us the right clue to psychological analysis; what we have to do is to discover the different forms of synthesis by which that has been combined, which we find combined in our present content of consciousness.

18. A second instance of the connections which confront us clearly and obviously throughout the whole course of life is to be found in the sphere of the will, in the connection between the end and the means, in the mutual dependence between willing the end and willing the means, and in the order which reference to conscious aims imposes upon our various activities. As we grow older the fortuitous and involuntary element in consciousness becomes continually less important in comparison with that arrangement of our activities which is guided by conscious ends. This arrangement directs most of the functions, both theoretical and practical, and into this main stream all greater or smaller tributaries, which come from other sources, flow. It is on the one hand intellectual, and guided by our knowledge of the causal connections between end and means; on the other hand it is the consequence of and determined by the nature of the will itself, by virtue of which a resolution, especially when directed towards a universal end, can determine whole series of activities which work under its permanent sway. This connection, again, is different from any natural connection, in that no causal chain of conscious events which is continuous in time is created by it, any more than by the relation between perception and memory; on the contrary, the willing of an end maintains itself throughout all possible interruptions, and often combines remote activities into one connection. In a present which is independent of change in moments of time, the system of ends rules over the particular activities, which are due to external occasions, or proceed from the system according to a self-created arrangement of change of work; and the willing of these ends shows its power further in the fact that it governs even involuntary associations, and gives rise to habits which, in their turn, act like original laws in determining the production of one kind of activity after another. We are thus able to discover within the varied play of particular events a fixed connection which combines a great part of them within

itself; and even though, from the variety of human aims, we do not find the same agreement as in the image of the external world, the form of the connection is nevertheless comprehensible and everywhere the same. It is indeed fundamentally different from the causal connection of nature, for it is no unconscious uniformity ruling over conscious life and determining it once for all which has given rise to its orderliness, but only our will itself which maintains the end and directs particular activities towards it.¹ In this way we may disregard the casual disturbances of the particular life, and by starting from a given universal end construct a normal course of purposive action. In asking: how would a certain will, together with a given knowledge of causal connection, produce and arrange particular activities? we are again taking for our basis a case of fictitious simplicity, in which we can calculate everything; comparison with what actually takes place then shows us the many other influences which supervene to bring about variations or disturbances.

The further analysis to which we submit the process of willing enquires as to the conditions and presuppositions of will; and here the fact that particular individuals agree in the general directions taken by their will leads us to certain fundamental tendencies of human nature, which are to some extent more or less closely bound up with organic dispositions, and which, though they are in different individuals combined in very different proportions, yet give us a basis from which to determine the forces which are active in the empirically given course of our inner life.

19. If understanding of the particular can only be gained by first seeing clearly the larger and more widespread connections which are present in the comprehending consciousness, if the only safe starting point for psychological investigation is a general survey of the whole constitution of developed consciousness, it follows that the direct observation of particular events, and more especially the experimental methods, however much they may contribute to accuracy and precision, can yet have only a subordinate importance in this department. They can never give us more than fragments, nor directly determine the conditions given in the particu-

¹ In stating that the connection between willing and what is willed always appears as necessary, and is therefore a counterpart to the mechanical necessity in nature (*Aufgaben und Methoden der Psychologie*, p. 107 sq.), Münsterberg overlooks the fundamental opposition between the principle of inertia and the consequence of willing. The latter does not follow of itself when once a resolution has been formed, but only by means of continued willing; and it is for this reason that we do not feel ourselves subjected to constraint, but feel ourselves to be free.

lar case, since amongst them are included habits of thought and will which do not enter as such into consciousness: they presuppose, if they are to be of any service at all, these comprehensive views, and their results so far have done, perhaps, more to show the extent of differences between individuals than to teach us any general uniformity. This is itself a service; and beyond this they are important as enabling us to support or verify hypotheses which, starting from the whole of our present life, we form concerning its gradual growth. But from the fragmentary elements which alone they can give us, we are unable to construct the whole.

20. But the whole state of our knowledge and opinions, and of the complex of ends which we pursue by applying known means, has not arisen in us without manifold co-operation on the part of others, without chance or intentional teaching, guidance, and education. It is just this which gives to the psychical life of man its peculiar character, by which it is divided by a wide gulf from that of animals; for in the latter, influences from without must be extremely small compared to what each individual acquires independently of others. Here there is no doubt that we have to do with causal relations between individual and individual; but here also we are confronted by an insoluble problem if we aim at establishing exact laws for this influence upon individuals, which are directly applicable to the particular case, and which would make of education and government an art which could be practised with as much certainty of success as billiards or photography. The experience of every schoolmaster who finds that his teaching and educating has very different results with different individuals, shows that it would be in vain to attempt to derive general laws from particular observations and experiments. The very way in which this influence is exerted, and the means by which it is carried out, prohibits such general laws; as we have already pointed out, we can never have a simple direct creation of thoughts or resolutions; all we can do is to present the conditions and the stimulus by which the pupil is incited to exert, according to psychological laws, his own activities as contained in his nature, and prepared for by previous development.

No doubt the practice and theory of pedagogy has developed a number of rules of method, which are based upon certain assumptions concerning the causal relations between the action of the teacher and educator and the progress of the pupil, and, to go further back, upon certain psychological assumptions concerning conditions which are contained in general laws of psychical development, and of the further elaboration of received stimu-

lus and incitement. The history of pedagogy shows clearly the influence of certain psychological theories, such as those of Rousseau or Herbart, upon the forms which methods have taken ; the onesidedness involved in these theories has always been corrected, at any rate to some extent, by practice. It would be unjust also not to recognise that in the specially didactic sphere, where we have to do with the imparting of knowledge and insight into logical connections, the correctness of the methods employed, and of their psychological presuppositions has generally been justified by their average results. There is, moreover, this great advantage in the school, that by its examinations, which are ultimately nothing less than psychological experiments, it is able to ascertain how far the intended effects of instruction have taken place. Thus, though the experience of the school cannot give us exact laws determining each particular case, yet we can obtain from it general views concerning the psychological conditions of educational action, and the average course and result of educational influence.

But this same experience of the school shows also how complicated the conditions are. Didactic methods must attempt to isolate the intellectual factor ; in reality the attention of the scholar and his inclination to follow the impulses given to him to exert his memory and understanding, depends partly upon his individual nature, partly upon conditions which act on his will, upon the skill of the teacher in interesting him, and upon his personal authority, which may itself be acquired in very different ways. Even the most obvious assumption, that the will may be influenced by rewards and punishments, cannot appeal to any law which would be true without exception ; and the opinions of teachers as to the advisability of applying these means are well known to be widely different.

Thus even where we have the most favourable opportunity and the strongest inducement to establish general causal laws, we are at best dealing with propositions which state that the application of certain means will, in the great majority of cases, have an average, though within certain limits a varying result. We could speak of strict laws only by assuming typical scholars, in whom we disregarded actual deviations and disturbances.

The most modern school of historical literary investigation has attempted to trace in a particular direction the influences acting upon poetical creation ; and this attempt would triumph if it could show from whom Goethe borrowed every expression which he uses, and could discover the model

from which he copied every figure which he depicts. Detailed proof might be given of the exact coincidences ; yet after all it would only be proved that in the long run it does not depend upon these particular incitements and reminiscences, but upon what the poet makes of them, and that for this reason we can obtain no general propositions from these particular observations, unless perhaps that the raw material of our thought and combinations is not produced from within, but makes its way into consciousness from the most varied sources.

What we can learn from particular departments of our experience concerning the influence of individuals upon each other, cannot do more than explain certain aspects of the relations existing there ; if we want to get really general and comprehensive conclusions, the basis of more accurate knowledge must here again be the whole of the facts which we have before us concerning the mutual relation between individuals.

The phenomenon that notwithstanding the innumerable differences between individuals, each person lives in a surrounding atmosphere of views and opinions concerning all possible things, which he has not produced from within, but has accepted from society ; the fact that he understands and speaks the language of his circle—though it may be with personal discrimination—and thus makes known that his thought agrees with that of others not merely in its material of particular elements, but also in its references and combinations ; the fact that manners and customs agree down to the smallest details—this whole historical state of things is what we first find. It shows in broad outlines the directions in which the mutual influence of individuals acts, and first presents definite problems for analysis ; it has to be discovered how these different uniformities have come about and are maintained, and our analysis could never lead to any end if we should start from the particular observation of a given instance in which I communicate something to another person, or influence him by a command. The particular example can never be simply accepted as representative of a general concept, nor is it analysable into its conditions ; the complication of every present moment with the whole psychical past prohibits here again the simple application of the methods of natural science, which can start from the particular instance, to discover the law in it. Only when we have the whole result before us can we set to work to seek the elementary processes first in our own experience and then in what we know of the life of others, to lay bare the motives from which one conforms to the other, and the ways in which uniformity of ideas, manners,

etc., grows up ; and in this way we shall prove that what we first guessed from the larger connections is a *vera causa*, inasmuch as we can show it in our own consciousness and in the experiences of particular individuals.

We may find an instructive example of this method of psychological investigation in the development of the science of language. Scientific study turns first to the common element in language which seems to have an existence of its own apart from individuals, it describes the store of words and the rules of grammar on the assumption that these actually govern the particular ; they are descriptive empirical laws of the total phenomenon of a particular language. By tracing the history of particular languages we find certain transformations, *e.g.*, in the sounds ; we are still discovering general laws according to which the change of sounds takes place—the Latin *c* and *g* soften, etc. These general phenomena show a far-reaching conformity to law. Knowledge of this sort had to precede before the psychological aspect of the matter could be successfully attacked and analysis pushed forward in this province. It is really a very simple conception, and yet one which has been slow to make its importance felt, that what we call language consists finally only in the uniform habits of a number of people having intercourse with each other, and that it depends entirely upon intellectual processes, upon memory and adroitness in certain movements of our organs of speech, and that all the phenomena of language lead us back to the psychology of the individual. What then has to be done is to find in the particular experience, and examine, what it is which goes on in the appropriation of language, both of the sounds and of the meanings ; to trace in the fragmentary perception of the individual those processes which we find on a larger scale in the history of language. In proportion as our use of language is unaccompanied by express consciousness because we have already become fixed in the use of all its connected activities by the time we begin to reflect, and in proportion as everything connected with our first learning to speak is more remote, the more difficult would it be to discover by mere observation that tendency of the individual to vary customary forms of speech, or to make new applications of familiar expressions, etc., by which changes in language must be ultimately explained. With respect to many phenomena the actual process can only be hypothetically constructed according to analogy ; it can only be inferred from general historical facts that it has taken place ; but these facts can make other processes visible to us which would have escaped our attention unless it had been guided by some such interest.

21. If, after these considerations, we finally turn again to those causal connections which we mentioned first (p. 398), to the dependence of psychical activities upon physiological events, we find that what we have to do is clearly to trace in detail this dependence which we find to be so extensive, and to establish the most exact laws possible as to what causal relations exist between certain changes in organic life and changes in psychical life even beyond the limits of psychophysics in the proper sense. In face of the hopes, however, which lead some to expect ultimately to raise psychology to an exact science by following this line of investigation, it is especially needful to bear in mind what the data are which can be given as the basis for our inferences. The process must consist in observing, on the one hand, variations in the state of the brain, which alone can be accepted as immediate causes, on the other hand, variations in conscious events; in establishing by means of these observations what are at first only empirical laws of relation, and in raising these, where possible, by means of further elaboration and precision, to exact causal laws. Now to every individual his own conscious states are given, and we may at first disregard the difficulties by which we are met in perceiving and describing them more exactly; but there is no possibility of comparing with them the preceding or simultaneous states of one's own brain. Only vague conjectures are possible as to what chemical and other changes go on in the particular cells and fibres of the brain from moment to moment; that they are partly conditioned by the circulation of the blood and by the substances contained in the blood, is certainly beyond doubt, but we have no accurate knowledge of the processes nor of the changes produced by stimulus to the senses, etc. In the normal course of life the changes in the cerebral substance are not only withdrawn from our direct knowledge, they cannot even be in any way accurately established from more remote causes. Thus there is no direct process by which we might resolve the general conviction that the state of consciousness depends upon the state of the brain, into special causal laws.

The basis from which we can at first work consists in a comparison of psychical disturbances which occur during life, with the results of subsequent dissection, or in the case of wounds with a knowledge—which is always incomplete—of the lesion; and, further, in experiments upon animals. In the former case we compare what an observer infers from the behaviour and utterances of a patient as to his psychical state, with what

he knows, or only infers by all sorts of combination, about the changes in the brain ; but such inferences as to the psychical life of others are more uncertain in proportion as the disturbance is deep-seated and easily recognised anatomically, because it generally happens in this case that the ability of the patient to give an account of his own states is more or less injured ; in any case these inferences, if they are to have any value, presuppose a large amount of psychological knowledge. Inferences as to the psychical life of animals are from the nature of the case still more hazardous, and the changes which can be observed in their behaviour still more difficult to interpret.

22. What we must first attempt in this direction, as involving the condition of all further determination, is the so-called localization of psychical events, *i.e.* the description of those spatially limited parts of the brain with the wounding, or disease, or even the stimulation of which there is connected the disappearance or the intensification of certain psychical activities, of which, therefore, we can assume that their normal function conditions the corresponding psychical activity. But this localization has not, as yet, been unanimously established, even for the simplest events, the perception of sensations and the voluntary movements of particular limbs—it is unnecessary here to enter more fully into the differences of opinion among particular investigators. But in all the more complicated psychical events the difficulties of localizing them naturally increase ; on the one hand the situation of the particular elements, on the other hand that of their connections, has to be established. The much discussed question of disturbances affecting speech is an instance of how defective a psychological analysis was employed until Kussmaul showed that the various psychical functions which co-operate in speech (or in understanding, reading and writing) must first be distinguished and observed in their relation to one another, before we can think of localizing disturbances of speech. At best, therefore, psychological analysis must precede, to enable us to interpret the anatomical discovery ; upon the correctness of this analysis everything depends, and only by referring to it can we frame conjectures as to the significance of circumscribed parts of the brain. But no one has ever yet got so far as to predict from clinical observation the exact results of dissection, or to diagnose from the result of dissection alone the exact nature of the disturbance.

But even if we have succeeded in our localization, all that we have so far established is that the normal function of certain parts of the brain is

one of the conditions of the normal course of the psychical activities ; that is very different from the proposition that the physiological function is unconditionally the cause (or even the exact correlative) of the corresponding psychical function. From the fact that when a wheel breaks the watch stops it does not follow that this wheel was the cause of the watch going correctly.

23. The theory which aims at explaining the psychic life by physiological events, would have to show what the processes are in the particular parts of the brain, which condition particular and specific psychical acts ; where the brain cells are situated, and what goes on in them, when I remember a previous impression, or when I count from 1 to 100. To fully realize the problem (cf. p. 389) is sufficient to show us how impossible it is to solve it with the means at our disposal. As a matter of fact all these events are really only translated from psychology into the physiology of the brain. With a certain amount of imagination we may indeed attribute all sorts of meanings to the millions of ganglionic cells and fibres, in order to explain by them the course of ideas, recollections and associations. But hardly since her first beginnings has any school of philosophy been guilty of such rash and airy speculations, and trifled so with difficulties, as in hoping to reduce the whole complex of thought and will to chemical and physical events.

24. But even supposing that this object had been realized, what should we really have gained? Should we have obtained any insight into the necessity of the particular event, any possibility of calculating it beforehand, any means of practically utilizing this calculation? For this it would be necessary to survey at any moment the whole of any brain, with all its organic dispositions, and—for practical purposes—to have the means of modifying it in such a way as to directly introduce certain changes, to produce certain ideas, combinations of thought and resolutions. But such definite dispositions of the brain are unknown to us in the particular case ; all we can do is to infer back to them from the psychical facts, which are alone accessible, and possibly from accompanying bodily symptoms. Nor have we any laws according to which one state of the brain passes into another ; at best we could only guess from the course of ideas, etc., how the chemistry of the cells is proceeding. That they are governed by a uniformity which is important for psychical life, we can merely infer from the comprehensible connections of conscious life, and when the normal course has been destroyed it is made known to us only

by manifestations of psychical disturbance. Where certain types of psychical disease, such as are described in psychopathology, are found to agree, we infer that similar changes have taken place in the brain, though in many cases they cannot be directly ascertained. Here again psychological knowledge is the basis from which we make inferences about cerebral states. It has long been known in therapeutics that by modifications in nutrition, by physical and medical intervention, psychical effects may be attained; we know, not indeed the immediate, but at any rate the more remote causes, which have an influence upon the course of psychical activities. But can we in this way produce definite ideas and combinations of thought which might not be explained by known psychological connection? Can we bring about any definite content in men, instil into them any knowledge, instead of imparting it to them in the ordinary way? Even the fancies of the maniac are psychologically grounded in the mental store which he has acquired; they arise according to general laws, under abnormal conditions; it is not their particular content, but only the fact that they are checked, or intensified, or confused, which can be referred to organic disturbance. The treatment of the body may establish the conditions of a course of the organic functions which will be favourable to the normal psychical life; but where we are concerned with the events of psychical life as such, psychical influence is as important as treatment of the body.

Even if we had the knowledge desired, it would not be superfluous to study scientifically the uniformity in the course of psychical processes by themselves; it is the connection of these results only which is accessible to our observation, so that psychology would still remain the key to all physiological knowledge. We may be most firmly convinced that all our psychical activities are conditioned by organic states, and yet see that accurate knowledge of this dependence rests entirely upon direct psychological investigation and can never be substituted for it.

25. If we take up the empirical standpoint, and begin by regarding the two spheres only as separate, we must consider also the other side of the question; at first sight we shall find just as extensive a reaction of events, for which we know only psychical causes, upon the organic functions. Even those physiological processes which are withdrawn from the direct influence of the will, are extensively influenced by ideas which can be explained only by communication from without, or association, by moods for which we know no ground but intellectual or æsthetic excite-

ment, and the relation between our experiences and our wishes or aims ; must we necessarily convert this causal connection which first presents itself, and upon what ground must we do so ?

Here, again, the right method must be to start from that which is given by immediate perception in both spheres, and this shows us, in the first place, a connection between volition and motion which we cannot refuse to regard as a real causal connection unless we are already involved in certain hypotheses ; and, secondly, changes in the physiological sphere—at first in the form of empirical laws of relation—which accompany activities having a psychical origin and a psychical motive. To extend our knowledge of these relations, and to formulate them more precisely, is again a problem for inductive investigation. We have here to guard against a danger which is not very remote ; it may happen that from tracing this organic resonance of our psychical activities a tendency will arise to substitute for the main phenomenon those sensations and feelings which make us aware of these organic reactions, to substitute the sensation of palpitation and of trembling knees for the feeling of anxiety, and the sensations of commencing tension in the muscles for the consciousness of energetic will, thus taking the sign for the real thing. As naturally perceived, the two are most closely connected, and as a result of correct observation the affections are often denoted in language by their external manifestation, but panting and clenching the fist is not anger itself, nor does shame or confusion consist in blushing.

In that part of psychological work which is concerned with finding a basis for all the sciences which treat of human life as an historical whole, the physiological side of psychological investigation has so far proved, on the whole, rather sterile and barren, except where questions arise—as in the physiology of speech—which specially belong to the psychophysical sphere. To explain the state and justice, science and religion is still left to that psychology, of which the first task is to point out the general connection between psychical phenomena themselves.

We do not mean by this that the investigation of connections between psychical life and its organic conditions is not a work of the greatest importance ; from it we shall obtain an increasing store of material towards the fundamental question as to the relation between phenomena of the body and the mind. We would only protest against proclaiming knowledge which is not yet found as the certain basis of an investigation which is, on the contrary, bound to confine itself at first to immediately

certain experience, to the facts of consciousness, and to test and correct its hypotheses constantly by this standard.

The process of induction with which we have so far been more particularly engaged, aimed at establishing between perceptible attributes and events universally valid relations which might be accepted as the expression of a necessity. The propositions realizing this aim took the form : if the conditions a, b, c are given, then d is connected with them ; if certain subjects are given in certain states, and in certain relations to each other, then there are necessarily connected with them certain states or changes in one or more of these subjects. Such propositions were to be called laws in the strict sense when the predicate was absolutely definite, and exactly determined for every modification of the condition.

Some of these laws were causal laws, properly so-called, expressing the action of perceptible things upon others, and they, when logically completed, were entitled to be the expression of a real necessity ; and we found the most conclusive confirmation of a causal law in the ability to make the result by bringing about the conditions, and so to show that nature obeys the thought which we possess in the form of the universal proposition ; these laws form the true centre and starting point of the understanding. Other laws, similar to these in their determinateness, could nevertheless not be accepted as laws of efficient action, but merely as descriptive expressions of uniform happening, or as expressions of actually given uniform relations.

In other departments, finally, no strict laws could be obtained, partly because of the differences between individuals which prevent us from stating any definite amount of the particular phenomenon as universally valid, and at best allow us to state only average values, partly because the nature of the objects did not admit of any measurement of the events, and partly because the complication of conditions was too great to be completely analysed, and the contribution of each one to be distinguished. In this case inductive analysis can only lead to a statement of the general forms and directions in which particular events are connected, and which we regard as tendencies ; these govern what happens under simple conditions, but are counteracted and modified in many ways in the particular case, and are in consequence only to be recognised, by help of comparison, in the average agreement of many cases.

These results must be presupposed before we can proceed with the task to which we were led in the last section but one in the form of generalization, and in the last section by the consideration of the psychological methods, the task of explaining what actually happens, *i.e.*, of finding grounds, either internal or external, for the Given.

VII. THE EXPLANATION OF THE GIVEN.

Starting from the general postulate that we must regard the Given as necessary, the inductive process had first to aim at establishing in general propositions the effects of certain causes and the influences of the circumstances modifying their effects; but in stating the problems involved in this more carefully, we were led on certain points to the question, how are we to discover the conditions from which given phenomena necessarily proceed?

If, in accordance with the terminology of § 82, 1, p. 203, we call the derivation of an actually established proposition gained from immediate experience, from a universally valid major premise, an explanation, then every given co-existence of attributes and every event is *explained* when it can be derived, according to a valid proposition, from a ground which is actually forthcoming.

Thus all EXPLANATION is in its essence deduction. But the problems combined under this term may be separated into three, which are essentially different.

1. To explain a phenomenon is to present it as the necessary consequence of another phenomenon, according to a known proposition, or according to a proposition deducible from known propositions. Either the ground of the explanation is present from the first, and what we have to do is to apply the law according to which it produces the phenomenon, or else it is the causal law of which we are first conscious, and the ground demanded by it has to be shown to be really forthcoming. The rainbow is thus explained as the necessary consequence of the actually forthcoming sunshine falling upon the raindrops, according to the laws of refraction and reflection; and an illness is explained by finding the bacteria which produce it.

2. To explain a phenomenon in another sense is to infer on the ground of known causal connections the cause which must have given rise to it, but which is not directly perceptible. I explain the dampness of

the ground, which I perceive in the morning, by the fact that it must have rained in the night, although I did not perceive the rain. Thus explanation in this sense seeks to establish the existence of a certain state of things on the ground of known causal connections.

3. Explanation in the third sense tries to find the grounds for the laws of action of causes, and of empirical uniformities, in the nature and relations of substances, in accordance with the logical completion of the causal concept, which is described in § 73. Kepler's laws are explained in this way by gravitation and inertia as universal qualities of matter. Explanation in this sense is final ; it endeavours to complete the essential concepts both of material substances and of the subjects of conscious processes ; but it can only do this hypothetically, inasmuch as the grounds which it assumes for the immediately Given can from their nature never be directly shown.

§ 98.

The CAUSAL EXPLANATION of an event, or of a chain of events, of which all the stages are perceptible, takes place in simple syllogisms, which present the given case as consequence of a known law ; or in a combination of known laws in a syllogistic chain. The processes are the same as those employed in finding a proof for a given proposition.

If this derivation from known laws cannot be carried out, then we must either establish a probable connection directly by applying the methods of agreement and difference, and follow up the conjecture thus arrived at, or find an explanation by extending known laws by means of analogy.

The causal explanation of an event will never, strictly speaking, enable us to point to a single agent as its sole ground.

1. The simplest form of explanation is the subsumption of a perceived connection of changes under a known law. That lacmus paper will turn red when dipped in acid is explained when I know that acids always bring about this change in colour ; whether I know beforehand that the fluid is an acid, or only ascertain it subsequently, makes no essential difference ; the process is a simple syllogism.

2. The next cases are those in which I have to combine different laws in the form of a syllogistic chain. I may, perhaps, be unable to subsume under a known special law the fact that a bottle will break when water freezes in it ; but if I know that water, when it freezes, expands with great

force, and that the glass is too brittle to admit of any extension, then by combining these two rules I get the conclusion that the glass must necessarily break. The logical processes which take place here are throughout similar to those which we employed in discovering proofs for a given proposition (§ 81, 3, p. 194), except that here the major premises introduced are only inductively established. For this reason every such explanation is also a new confirmation of the proposition by which we explain; the consequences exacted by the law are fulfilled. If any discrepancy should appear between the Given and that which follows as conclusion from the assumed laws, this would point to the fact either that the law is not correct or that some circumstance which modifies the result has escaped our notice.

3. Where we fail to subsume a given event under laws of which the conditions are perceptible, we shall be tempted at first to proceed to a direct comparison of different cases in which the same event takes place, and to look around for an antecedent or a circumstance which is common to the different cases in which the phenomenon to be explained occurs. If we notice that intermittent fever is only found in places where there are marshes, but which differ in every other possible respect, that cases of intermittent fever multiply in places east of a marsh when a west wind blows, and west of it when an east wind blows, then we have sufficient ground for assuming that the marsh is causally connected with the intermittent fever, and that the air above it is, or contains, the cause of the fever. If, wherever we find oak-apples, we also notice an insect which pierces the oak-leaves, then this insect is to be regarded as the cause of the formation of oak-apples.

A proposition obtained in this way can, however, be quite certain only if we can reverse the process, and show experimentally that men always fall ill with intermittent fever when they breathe marsh-air, and that oak-apples appear on the leaves upon which the insects are placed; when, that is, the conjecture obtained by comparison can be made the basis for the reverse process of observing the results which arise under given conditions.

4. In other instances favourable circumstances give us the opportunity of applying the method of difference. If we try to establish the conditions under which typhus arises by observing a great number of cases and finding the circumstances which always preceded, we shall indeed derive little benefit from the instructions to eliminate the different antecedents in the different cases and to retain those which are common to all; even

if uncertainty as to whether the perceptible conditions are ultimately always the same, or whether the same form of illness may not proceed from different causes, did not make us hesitate from the first, yet it would be absolutely impossible to enumerate all the antecedents which ought perhaps to be taken into consideration, and to be certain that none was overlooked.

But when we find an epidemic of typhus in a town,¹ and one part of the town contains all those attacked and another part none; if all the infected houses draw their water from one reservoir *A*, while no case is found among those supplied from the second reservoir *B*; and if, in addition, the same facts should occur in a second town at a considerable distance, then we should have an indication that the conditions of the illness lay in the first reservoir of water. For all the cases of illness, however different their circumstances may be otherwise, would then agree in the fact that they have for their antecedent the use of water from source *A*; and where under similar circumstances of locality, climate, etc., the result is absent, the antecedent is also absent. If it also happened that when the first reservoir of water was cut off no more cases of illness occurred, then the first result would be confirmed in another way. If it should then be found that the waters from the two sources differ in the fact that the first is defiled by refuse, then we have singled out a definite circumstance connected with that condition; and if in the second town, the other circumstances being quite different, the partial epidemic should again attack a district supplied from a source similarly defiled, then the proof that such defiled water is, or contains, the cause of the illness would be sufficiently complete. The fact that all do not fall ill who use the water can be explained by the general observation that certain individuals are not liable to such infections.

But it would still only be proved that in these cases the impure water is, or contains, the cause of the typhus; we have not proved the universal proposition that wherever typhus appears impure water has been drunk. The method of difference does indeed prove (according to § 95, 8, p. 340) causal connection for the particular case; it cannot give us a universal proposition to the effect that a given phenomenon is always produced by the same cause.

5. It is obvious that a comparison under such selected circumstances can only be made in favourable instances. In the great majority of cases

¹ Cf. *Deutsches Archiv für klinische Medizin*, vii., p. 155 sq.

it is not possible to find causes for given phenomena in this more direct way ; it can only be done by means of a deductive process making use of analogy as a guiding principle.

Suppose, taking the instance employed by Mill to illustrate his methods, that we had to say merely upon the ground of observation, without experiments or other presuppositions, upon what conditions the phenomenon of dew depends ; we might watch the phenomenon for a long time without getting any clue, and the attempt to gather from the antecedents observed in the different instances the whole of the conditions upon which the phenomenon itself, and the degree to which it is present, depends, would certainly fail. It would first occur to us that night and a clear sky constitute the condition, for this is always present when dew falls ; but the proof breaks down because there are clear nights in which no dew falls, so that dew cannot be necessarily connected with a clear sky. Moreover, we find that during the same night certain objects will be more strongly bedewed than others, so that night cannot be the sole condition ; we are however quite at a loss to say how those which are more strongly bedewed differ from the others until we are led by some conjecture to examine them as to their capacity for conducting and radiating heat. Neither the absolute temperature of the night, nor its difference from the temperature of the day, gives us any definite measure for the amount of dew ; in short, we are confronted by a confusion of varying phenomena which refuse to conform to any comprehensible rule, and which we should never succeed in disentangling without applying rules which we have learned elsewhere and established experimentally.

The explanation of dew has, on the contrary, like the explanation of most events which we cannot establish immediately as produced by perceptible causes, taken an entirely different course ; it has proceeded deductively by the subsumption of the phenomenon under a law which is known as a ground for a similar result, and by showing that the conditions of this law are present whenever dew falls.

The point we start from is the regularly observed fact that water settles in drops upon bodies which are colder than the surrounding atmosphere, *e.g.* upon a bottle of cold water when brought into a warm room. Dew is not only similar in form to this deposit ; it also agrees with it in the fact that it appears without any visible water being present and falling. The attempt now follows to extend this special rule, and to subsume under it the formation of dew ; and this is done by showing that similar

conditions are present where dew forms, that the bedewed bodies are actually colder, at any rate were for a time during the night colder, than the surrounding atmosphere, and that under circumstances alike in other respects bodies with more dew upon them are colder than those with less. This may be shown either directly by measurement of temperatures, or deductively from known laws of the conduction and radiation of heat.

In a similar manner we refer all variations in the formation of dew to their conditions by subsuming them under known connections, and showing that the requisite conditions are actually forthcoming; and this process confirms in its turn the validity of the rule upon which it is based by showing its efficacy in new combinations.

In the same way animal heat is explained by showing that a known cause of heat, combustion (and other chemical processes), takes place in animals, although in a peculiar form; and the explanation would be complete if the amount of heat produced by an animal body could be shown to be the result of a given quantity of chemically combining substances, of burnt carbon, etc.

6. The inference may be represented in the following form :

There is given a phenomenon *E*.

It is known that *E* occurs under conditions *A B C*.

Now in the given case *A B C* are present;

Therefore they must have *E* for consequence;

Therefore the given consequence *E* can be due to no other condition.

For the inference to be conclusive, it is of course necessary that the given *E* shall be just the same in quantity also as the *E* which follows according to known laws from the conditions *A B C*.

Explanations in this sense take place sometimes by subsumption under causal laws, properly so called, sometimes by subsumption under merely empirical uniformities; the process is the same in either case, and consists in showing that the requisite conditions are present, and that, therefore, what actually occurs is known to be identical with what is required by the rule.

By means of these processes, a connection is found between observable phenomena which it would be difficult to find in a direct way, but which can be established by simple subsumption under known laws, or by generalization of those laws. It is here that we are frequently guided by the principle of analogy, according to which we expect to find similar conditions for similar results, and which, when we do find them, is a ground for

the conjecture that the same connection exists between A and B which exists between an A_1 and B_1 which are similar to them, and which fall under the same higher concept. When it has been shown that the oak-apple is due to the bite of an insect, we are led by analogy to refer similar formations in other leaves to a similar ground: if the conjecture is confirmed, we have the beginning of a generalizing induction. The modern development of the theory of infectious diseases has been guided entirely by such inferences from analogy.

The main condition upon which success in this method of deduction from inductive propositions depends consists, therefore, in our being able to subsume a given phenomenon under another of which the conditions are known, or to discover the element common to both. Without quickness of combination, by which we can call up a number of possible analogies, and apply them to the unexplained case; without a happy power of divination, which is guided by unanalysable associations to discover that analogy which embraces most aspects of the event; finally, without imagination to construct connections for which the only ground may be a hidden similarity, our thoughts, if compelled to proceed strictly according to method, would frequently be condemned by the impossibility of discovering in this way a sufficiently grounded connection to complete stagnation.

But this fact is in no way contrary to the nature of induction; it is a necessary consequence of it. We cannot even begin the process of inference without making general assumptions; and the general proposition which we get by summing up a number of instances is really a hypothesis to which, if it is true, we are led clearly and certainly in this case. But between these most general presuppositions, upon which all induction is grounded, and the simplest cases to which they can be applied, there is a wide region within which the hypotheses which are always necessary for induction can only be formed tentatively, in order to give some definite direction to investigation, to serve in our analysis of phenomena into their elements as a means of breaking up complete phenomena on certain lines, and to invent the experiments which will make it possible to confirm or refute an opinion.

It is therefore inevitable that we should always have wider and more comprehensive ideas about the connections in nature than we can verify in detail; anticipations of nature, as Bacon calls them, are no doubt the sources of numerous errors, but they are also the indispensable conditions of progress, and there can never be any question of wishing to prohibit

them, but only of applying the strictest test to their empirical proof. The proof is itself of such a nature that it presupposes a hypothesis, and its chief characteristic, which is that it can refute beyond doubt but can never strictly confirm, causes the course of investigation to be such that every definite and irreversible step in advance must consist in refuting errors.

7. Since events and chains of events are explained as the effects of certain causes by the application of known causal laws, an instance will, strictly speaking, be explained according to the nature of the causal laws upon which the actual event depends, only when all the conditions upon which it depends, efficient causes, circumstances, and negative conditions, are enumerated; and, as a rule, if we are to speak quite strictly, it will not be possible to denote any particular thing as the sole and complete cause of a given event. More especially where the effect is mediated by a number of intervening links is it an inaccurate abbreviation to describe a cause from which there issues a chain of effects as the cause of the whole course. It produces the final effect only if the given circumstances are present and counteracting conditions are absent; the presence of these circumstances, the absence of counteracting causes, have themselves many other causes, and every particular event is thus determined by an incalculable number of preceding conditions, which are to a large extent independent of each other.

The problem of finding a single cause as the complete ground of an event could only be really solved if it could be shown that the action of a thing must by itself produce the effect under all circumstances, and could be prevented by no obstacle. But this can never be shown. If I strike a dagger into a man's heart, I have, of course, caused his death, and no one will hesitate to describe my action as the whole and sole cause of his death, because my action produced a state in the wounded man which made death inevitable. Accurately speaking, however, all which I directly do is to impart a certain velocity in a certain direction to my arm and to the weapon I am holding; whether the blow is fatal or not depends upon the position or movement of the victim at the time; if he swerve out of the way, the blow will miss, but the position and motion of the victim are conditioned by a number of circumstances which may be quite independent of my action. The reason why in such cases we describe the blow with the dagger as the whole and sole cause of death is ultimately the fact that the blow was intended, and that its direction was determined by the different circum-

stances co-operating *idealiter* in the thought of the person aiming it; to this extent the whole cause is represented only in the preconceived intention, and is completely represented in proportion as all circumstances are taken into account. Apart from this subjective element it must be impossible to find a formula which would distinguish between the cases in which an agent and his deed might be fairly described as whole and sole cause of an event, and those in which he is only the co-operating cause. If I injure a man by a wound which is made fatal only by the circumstances, and would under other circumstances be curable, the case differs from the preceding one only by the fact that a greater number of conditions have co-operated to bring about the result. If I had taken all these circumstances into consideration, and foreseen the result with certainty, then the whole cause would here also have been ideally represented in me, and there would have been the same reason for saying that I had killed the man; but if these circumstances had no determining influence upon my will, it cannot be said in the same sense that I have killed the man, although I constituted an integral part of the cause, and without my action the remaining circumstances would not have taken effect.

Our every action, even the most unimportant, gives rise to circumstances which may be connected in such a way as to co-operate towards a remote result; every action of ours upon our surroundings has incalculable consequences, and co-operates as cause in an endless series of effects. Whether I take part in causing this or that event by an accident for which I am not accountable, or whether it is to be attributed to me as merit or guilt, cannot be decided by the purely objective relation of causality, according to which a constantly diminishing part of the cause is ascribed to me, but only by the relation of the consequence to my conscious intention and purpose.¹

8. We can only refer briefly to the relation between those deductive processes which determine the causal connections of given events and those by which we seek means for our ends. Here, again, we have to find the major premises, according to which a desired result follows from known conditions, and then to bring about these conditions; or, if they do not lie directly in our power, to go back to other laws until we

¹ For the juridical application of the causal concept cf. my article: *Der Begriff des Wollens und sein Verhältniss zum Begriff der Ursache*. *Kleine Schriften*, II., 115 sq.

find conditions which we can directly create. It is the same process as in finding a proof, and it is also similar to the finding of a proof in that the result may be attainable in various ways.

We must, therefore, observe one precaution. If we represent the process thus :

E is desired,

If *A, B, C* is, *E* will follow ;

Therefore, *A B C* must be realized, in order that *E* may follow, then this *must* is objectively justified only if we know that *E* can proceed from no other conditions, that *A, B, C* is the only real ground of *E*. If, on the other hand, we merely do not know any other real ground for *E*, the *must* has a merely subjective significance ; if *we* will that *E* is to be, then *we* must will *A, B, C*, because our limited knowledge makes us unable to will anything else.

§ 99.

The explanation of a fact, of which we have not been able to perceive the genesis, from actual conditions of which we can only infer the existence, presupposes the knowledge of propositions according to which it could only depend upon one or a limited number of conditions. Such propositions, however, can only be made probable by increasing our knowledge to the greatest possible extent.

Even where such propositions are valid, the condition cannot be inferred in its complete determination, but only those aspects of it which are indicated by the given phenomenon. The reconstruction of the conditions succeeds more completely in proportion as the fact to be explained presents a combination of many elements, and so makes it more probable that those conditions in relation to which this combination would be fortuitous should be excluded.

The test of every such assumption consists in the deductive development of all its consequences, which must not conflict with any known fact ; this process strengthens the probability of the hypothesis if it also explains facts from which it was not derived.

The general rule is, to presuppose, if possible, only conditions which are known to be actually forthcoming elsewhere (*causæ veræ*).

The most important application of inferences from a given fact to the reality of the causes which have produced it takes place in HISTORICAL INVESTIGATION.

The problem here is to infer back from given external products of human activity—it may be writings or creations of human art—to the definite psychical processes which have produced them; and to explain these processes themselves by preceding internal or external conditions. It is chiefly psychological knowledge which is presupposed in this investigation.

Upon such psychological knowledge even the preliminary stages, literary criticism, philological exegesis, investigation as to the credibility of authorities, depend; as well as the reconstruction of the continuous and actual course of events from the fragmentary facts which alone we are at first able to infer.

In consequence of the comprehensive interaction between the individual and the community in which he lives, the ideal of complete historical knowledge would require that the life of each individual should be known as interwoven with that of all others, as depending upon tradition and as influencing contemporary and successive generations; and that in this way every historical phenomenon should be traced back to its ultimate sources in the vital activities of individuals, and represented as the collective result of a complicated psychical causal connection.

This unattainable knowledge can be in part replaced by the investigation of those vital phenomena which are shared in essentially the same way by all the members of a nation, such as language, morality, religion, technical methods, social and national ordinances and legislation. These form the background from which proceeds the particular mode of action of eminent and well-known personalities.

Within the common life it is possible to analyse particular tendencies which manifest a connection which can be investigated; this analysis is most complete when it can start from the general aims which control the action of the individual in a particular direction.

The gradual remodelling of common thoughts and volitions may be derived from general laws in so far as we can find in it a logical necessity.

Historical investigation points beyond the limits which are imposed by its means of knowledge in two directions. In the first place, it points to a first state which can only be hypothetically constructed; and in the second place, to certain assumptions concerning the nature of man in which empirical psychology also concludes.

1.. Explanation takes another direction when direct observation of the

conditions under which a phenomenon occurs is impossible, and the question takes the form of asking what must have been if the Given is to be understood as necessarily following from it ; when, therefore, it has to be proved that certain conditions have actually taken place.

Thus I explain a wound found in a man's body by a shot having been fired at him, the rising of the thermometer by the increased heat of the atmosphere, the polished surface of a stone by the action of a glacier ; I infer from the presence of a certain state that a certain cause has acted. By far the most important and comprehensive application of this class of inferences takes place in the interpretation from words, gestures and actions of the thoughts, feelings and resolutions of other people, which can indeed be known in no other way ; we must, therefore, be especially careful to make clear the basis and justification of this kind of inference.

When it is understood that those propositions about causal relations of which we can be most certain state the dependence of a consequence upon certain conditions, then it is clear, in the first place, that it is not possible to infer strictly according to general rules from the existence of the effect to the existence of the efficient cause upon which the law makes it depend. For even if the major premise : If *A* is, then *B* is, were absolutely certain, yet the hypothetical judgment does not allow us to infer from the validity of the consequent to the validity of the antecedent ; from the fact that the consequent is true we can only infer that *possibly* the antecedent is true.

For such an inference to be formally correct, it would be necessary for the proposition to be convertible : If *B* is, then *A* is ; or, in other words, it must be known that *B* is *only* when *A* is.

2. But it is just this *only* which is the difficult point. We found that the principle that like consequences follow from like grounds was indeed a regulative principle, but not an unconditionally safe guide ; and even if we were fully justified in acting upon it, if we could assume that difference in the grounds would somehow make itself known in difference in the consequences, yet this assumption would only apply when the whole result of a presupposed cause was known to us, and we could watch its development at all points. What we are concerned with, however, is only the perceptible part of the effect, generally only one state, which we cannot observe in its growth ; it may even be a state mediated by several intervening causes, by the action of which the characteristics peculiar to the remote cause have been obliterated. When I find a coin lying upon the

hard ground, I know that it has certainly not grown out of the ground or fallen from the sky, but the most careful investigation will not reveal to me whether it has been placed there, or has fallen from some height, or has been thrown there, and there may be no conceivable trace about it of who has brought it there or lost it.

Moreover, what we have to do is to construct what has really taken place from its effect. But what has really taken place is perfectly concrete and determinate. We know, however, that the attributes of the cause and of the circumstances under which it acts do not all of them contribute to the result, but that the circumstances conditioning the result are connected with many others which are indifferent, and therefore that, even if I had not to choose among different kinds of conditions, the condition assumed would be subject to a certain generality and indeterminateness. If I find a round hole in a window, I may be justified by my knowledge in believing that a circular body has forced itself through the pane with great velocity; but whether it was a conical or a round bullet, whether it was made of lead, or iron, or some other material, the hole does not tell me, although any difference in material and form might have betrayed itself by certain differences in what actually took place, if only I could have observed it.

3. If we disregard this difficulty for the present, and ask upon what we can base our conviction that a result could only proceed from one kind of condition, though it might admit of subordinate differences, we find that it ultimately depends only upon the extent of our knowledge. If it has been observed in a great number of cases that *B* occurred after *A*, and if *A* was the preceding condition in all cases in which *B* was found, and in which there was any possibility of perceiving the accompanying and preceding circumstances, then we generally assume that *B* can be produced in no other way, that *A* is not merely the cause, but also the *conditio sine qua non* of *B*.

We are in this position with respect to the genesis of organisms. We know from hundreds of thousands of observations of the most different plants and animals that organisms arise by propagation, that a germ springing from a parent organism develops under certain further conditions to a new organism; and whenever we have been able to observe carefully the growth of organic individuals there has always been such a germ. When we add the experience that more careful investigation shows that the exceptions previously believed in are not forthcoming; that we have succeeded in subsuming under the general law the previously unexplained

origin of infusoria by finding the germ which had escaped observation ; that no case has been ascertained beyond doubt in which organic life has arisen in any other way, then the great extent of this concurrent experience gives us the right to assume that organisms can arise in no other way, that the presence of a germ springing from a parental organism is the *conditio sine qua non* of their existence.

4. Nevertheless, the certainty of this proposition is essentially different from the certainty which we have to the opposite effect, that all men die. For since death occurs under the most varied circumstances, even those which are most favourable to the preservation of life, it must be that the nature of man, together with the general conditions of his life, makes death necessary ; otherwise it would not occur so inevitably ; that a man should continue to live for ever appears as an impossibility. But from the fact that we never see organisms arise without previous organisms, it does not follow that it is impossible for them to arise otherwise ; it only follows that within the range of our experience the conditions are not given ; we may therefore believe that it is impossible within the sphere in which we move, and within circumstances which are like those under which we observe, but only that it is relatively, not absolutely possible. A negative proposition can never be established upon the ground of a limited experience ; this may give us the constant consequences of certain conditions ; but no proposition which says that if *A* is *B* is, can by itself justify the proposition if *A* is not *B* is not.

For this reason, if we can be sufficiently certain of finding the conditions, a much narrower experience will suffice to prove that a consequence is inseparable from its conditions, than will show that a certain condition or a certain kind of condition is inseparable from a certain consequence. It was for a long time believed that urea could only be produced in animal organisms, and there was as much ground for the belief as for any statement concerning the exclusiveness of certain conditions ; yet the proposition was false, while the reverse proposition that such and such animals produce urea could be established as a universal proposition after only a few observations.

Thus we are somewhat at a loss to justify the assumption that certain consequences depend only upon certain conditions, and yet there are many cases in which we do not hesitate to make use of such assumptions. Every snail-shell and every piece of coral, every bird's nest and every spider's web, points unmistakably to a certain origin, and we do not

hesitate for a moment as to what cause to assume for these things; if it were objected that inference from a given effect to a certain cause is uncertain, we should regard it as logical chicanery. But by what are we ultimately guided in our inference? Not only by never having observed such products made by other kinds of animals than those known to us, or forming themselves by any natural force, but also by finding in them a certain character which is common to a great number of similar products of whole classes of animals; and further by the fact that it is improbable that anything capable of forming snail-shells or bird's-nests in this way should have escaped our notice, and finally because there is no ground to prevent us from assuming the existence of a snail or of a bird from which the object in question may have come, at any spot on the earth. The assumption that the snail-shell is due to a snail agrees with all that we know; the assumption that it is due to something else would force us to assume a cause as yet entirely unknown. This is in accordance with the condition laid down by Newton in the first *Regula philosophandi*: *causas rerum naturalium non plures admitti debere, quam quæ veræ sint*. A *causa vera* can only be one of which we know that it does exist, and can produce the effect in question.

5. To this we must add another consideration. It is inferred in the analysis of the spectrum that the dark and light lines in the spectrum of the protuberances in the sun or the fixed stars, or of the nebulae, are due to the fact that the same substances are present in these bodies, which manifest the same lines upon the earth. The fact that all the different substances observed upon the earth produce different lines proves, because of our approximately exhaustive knowledge of terrestrial substances, that where we find the same phenomenon upon the earth it will have the same condition for its ground. It does not of course follow from this that there are not other substances in the universe, which, while differing in other respects from those of earth, agree with them in emitting light of the same refrangibility; and we should be justified only in the inference from analogy that there also the same lines indicated like substances, and different lines different substances. But the nature of the phenomenon strengthens our certainty; the positions which might be occupied by the particular lines in the spectrum being so many, while the possible number of combinations of the particular lines is quite incalculable, it would be the most extraordinary coincidence if so many lines as appear, e.g., in terrestrial iron, should be produced in exactly

the same combination by a different substance, and if of the innumerable possible combinations it should always be just this and no other which was realized. It is therefore the combination of a great number of different attributes which, so far as we know, are independent of each other, which forces us to the conclusion that things which agree in so many particulars must be the same. Thus the process is the same as when a chemist investigates a substance; if it reacts like a known substance, we are not indeed justified as yet *in abstracto* in the conclusion that it is the same; but if it manifests a large number of phenomena which are like and none which are different, then the improbability increases that different bodies should act in the same way and that their difference should betray itself in no single point. It is the same in a case of crime. If a large number of facts which are independent of each other are explained by a certain person having committed the deed, while in order to acquit him we should have to assume that each particular item of incriminating evidence had a special and independent ground, and that they were all brought together by chance alone, then no one would have any doubt as to how to decide.

Thus it is not merely a question of inferring sameness of conditions from sameness of results, but of inferring from the sameness of the *combination* of a great number of specific determinations in a result, that its condition is that from which just this combination would necessarily follow. Each particular characteristic may in itself admit of different interpretations, but of these interpretations of each particular characteristic there is always one which leads to the same point to which one of the interpretations of each other characteristic leads, whilst the remaining interpretations all diverge; and that gives us the right to refer it to that one point, and not to the improbable chance coincidence of a number of independent conditions.

6. Having investigated the conditions under which we can affirm the proposition that a result is produced exclusively by a certain condition, and having found that they consist partly in a comprehensive knowledge of the ways in which actually existing conditions act and in the great differences in the results of large classes of conditions, partly in the specific combinations of many attributes which we can expect only from one kind of condition and not from many: we now proceed to establish the reality of a certain condition by a simple hypothetical inference of which the major premise states that if *B* is, *A* was its condition.

7. In doing this we have to keep in view two points. In the first place the particular value of *B* requires a particular value of *A*, and this can only be established if the relations between *A* and *B* can be expressed in all their variations by comprehensive formulæ; from a certain height of the thermometer we infer a certain degree of heat in the surrounding medium, and not merely heat in general, and our *A* must therefore be such that *B* appears as its necessary consequence according to the known relations between them.

In the second place, the inference needs to be tested in proportion as its major premise is not absolutely certain; and this test consists in trying whether consequences must not follow from the assumed *A* which would contradict some actually existing state of things. From this it follows that the certainty of the inference from a given fact to its conditions always depends ultimately upon the certainty with which we can reverse the process and calculate the results of assumed or known conditions.

8. In proportion as we lack established laws showing the dependence of quantitatively determined effects upon quantitatively determined conditions, it will be more difficult to infer back to causes. In many cases we have no proposition affirming the exclusive dependence of an event upon a single species of cause, and a simple hypothetical inference will be impossible.

There is a way out of the difficulty if we can succeed in establishing a disjunctive judgment, stating that if *B* was then its condition was either *A* or *A'* or *A''*. We need not point out that the possibility of finding an exhaustive disjunction depends upon the same conditions as the finding of a simple proposition; upon the most comprehensive knowledge of the conditions which always produce the consequence *B*, and upon the knowledge that no other condition within the range of our experience has this consequence. This disjunction may refer to different classes of conditions, or only to different modifications of similar conditions; the process is the same in either case.

The first use we make of the disjunction is to develop each of the disjunct conditions into its results, and to inquire whether it has not additional consequences which contradict a fact, or others which agree with a given fact which would be otherwise inexplicable; by means of the former we should refute one term and proceed by exclusion; by means of the latter we should strengthen the assumption in favour of a given term. The alternation of day and night may have for its condition either the rotation of the

earth or the revolution of the sun ; direct perception does not enable us to ascertain what actually takes place. But if the earth rotates, the centrifugal force will be greatest at the equator, and this explains why the seconds pendulum is shorter there, a fact for which we should otherwise know no ground ; and this consequence is a presumption in favour of the first alternative, especially since it agrees exactly in amount with what may be calculated from the assumption.

If it is impossible to come to such a decision, we may find help in a consideration of probabilities. If we find two books with different dates on the title-page, both being exactly alike, even to errors in printing, then either the whole book has been re-printed, or only the title has been altered. Either assumption is in itself possible, but we know from experience in the manipulation of type that few pages issue from the press which exactly correspond to the copy, and that it is therefore most improbable that in a second edition there would not be differences from the first, even if it was meant to be exactly the same, especially in printer's errors, which are easily overlooked. The first assumption is therefore most improbable, and we shall not hesitate to decide for the second. The same probability serves to show that two copies of a text which deviate from the original in the same passage have not been made independently of each other ; where they do depend upon each other it is most probable that one will have copied the error of the other ; where they do not, it is most improbable that it should contain exactly the same error as the other.

We prefer, therefore, that assumption from which the result would most probably proceed ; and where direct means of proof are wanting we must always have recourse to this criterion, though it is only in extreme cases that it can be determined and estimated with any certainty. Nor can it be determined in any rules or numbers where the probability is to be regarded as equal to certainty ; here, again, everything depends upon the degree of certainty with which we know the causal connections.

9. If our knowledge is insufficient even for such disjunctions, there is open to us an unlimited range of conjectures, by which to construct the possible antecedents ; and here we can have no other rule but that we have no right to any assumption which is not based upon a connection known to be in accordance with experience. What we always have to do is to construct the circumstances from which the result to be explained may have proceeded according to known rules. But any form of deduction ceases to be possible where there is no limit to the number of cases, nor even an

estimation of probabilities in the sense that the condition most frequently found would be preferred, for where it is not possible to exhaust the instances there is no ground for even this estimation.

10. The whole problem would take another and far more favourable form if it were possible to construct the required conditions, not backwards from the result, but forwards from more remote conditions which are known to us, and to show that it is the necessary result of existing relations. The fact that the mammoth and rhinoceros have lived in far higher latitudes than their relations now inhabit presupposes either that the climate was then warmer, or that the constitution of the animals has changed; the warmer climate might have been due to the fact that the cooling of the earth by radiation had not then proceeded as far as now, or to a more intense heat from the sun, or to the fact that the heat was then differently distributed upon the earth. If it could be shown, say, that owing to the existing relations of our solar system, to variations in the eccentricity of the earth's orbit, and to the precession of the equinoxes, there must be a periodical change in the distribution of heat upon the earth, and that the amount of these variations is exactly enough to bring about sufficient heat in the northern latitudes for the existence of these pachydermata, then we should decide in favour of this alternative, and the other would be superfluous. In the same way we may always make a rule of deciding even merely hypothetical assumptions by way of deduction in the first place, and of dealing with the circumstances introduced for the sake of explanation just as we do with the phenomena to be explained; that is, by finding out the conditions from which they proceed of necessity. In this way they would take their place in a wider connection of causes and effects.

11. By far the most important, but also the most difficult application of such inferences from a given fact to the reality of a cause which produced it, is called for by the problem of inferring from perceptible manifestations, words, gestures, actions, and their results, the reality of inner events in other people, of finding out their thoughts, feelings, intentions, and resolutions.

In daily intercourse, it is true, this process of inference goes on extensively in such a way that we are conscious neither of special operations of thought and their conditions, nor of any difficulties. We understand each other, and we are frequently quite unconscious of adding anything in thought to the words we hear, or of regarding them as signs of inner events in another person. When communications of a purely theoretical nature

are made to us, we generally think only of the subject matter to which they refer, of the event which we are told about, or of the content of the proposition which is being explained to us; by means of the accustomed association the words bring about directly the idea which they are intended to call up, and it frequently happens that we do not think at all of the person speaking, or consider that it is his ideas which are revealed in the first instance. To whom would it occur, in reading the newspaper, with its miscellaneous information, to think at each item of the unknown reporter, and to realize that the printed words must be referred in the first instance to the content in the consciousness of the reporter? It is enough for us that we learn what has happened, and are entertained by it and our curiosity satisfied. It is only when a doubt arises as to whether some one can know what he is relating, or a question as to why he is making the communication, or when purely inner events, mere conjectures, doubts, feelings, intentions, are expressly imparted to us as such, when the speaker speaks of himself, that we are called upon to construct his mind from his words, and to think of certain psychical events as the sources of his words. But even this generally takes place without consciousness of any mediating process; we remember the events which occurred in ourselves upon similar occasions, and this recollection being reproduced by the familiar word and its connected associations, we are able to portray within ourselves the experiences of the other person; we can "imagine how he feels," can estimate his joy or his sorrow. It never occurs to us that in so doing we need or employ any general presuppositions; we transfer without any conscious process of inference what we ourselves feel into the mind of the other person. It is similar with the interpretation of actions; in the majority of simple cases, we recognise at once from the movement what the intention is. If some one is striking a nail on the head with a hammer, he wants to drive it into the wall, and if he is arranging his paper and dipping a pen into the ink, he wants to write; the immediate ground of his movements is at once made obvious to us by the ground of similar actions in ourselves. We do not indeed think that we possess a complete picture of his mind, of his whole conscious state at the time; the visible movements do not tell us why he is driving the nail in, and what he is going to write, and all that we can at first know is that fragment of his mind from which his mode of action immediately issues; what is more remote can at best be revealed to us by combinations. Moreover, we know that much goes on in us which we do not reveal or make known by

any external sign. The most we can do, therefore, is to conjecture, according to the psychological connections familiar to our own consciousness, what intentions or ultimate thoughts not directly to be known determine the nearest aim of an action or spoken word which can be known.

The expression of emotions in face and gesture is again similar, although here the process is somewhat more complicated. We have no direct or clear idea of our own countenances and looks ; we do not, like the actor, study the change of our facial muscles before the looking-glass ; and because this change takes place for the most part involuntarily, we do not generally notice the accompanying motor sensations, nor could we translate them into any intuitable image. We learn principally from other people what expressions belong to certain emotions ; we first construct the state of mind from other signs, from the occasion which we know to have been present, from words and actions, and we see that it is accompanied by a certain mimetic expression, which then for the first time becomes a comprehensible sign. This is particularly obvious when we consider how individuals differ in these expressive movements ; we have first to learn by a process of combination what it means when a person assumes a certain expression, but long intercourse enables us to carry out this process with sufficient certainty. Still, the reconstruction of what goes on in some one else, especially in complicated cases of lively emotions, always remains somewhat vague and indefinite ; we can only attribute to others what we have experienced in ourselves, or what we at least find analogies for in ourselves. The limits to this possibility of mental reproduction must not, however, be drawn too close ; we have many stirrings within us of which we repress the consequences, but which we have nevertheless experienced, and which we can therefore understand when they find expression in others ; and imagination enables us to think of our own experiences as heightened and extended.

Thus our power of referring given manifestations even to their immediate psychical sources varies between a maximum of subjective certainty and objective truth, and a limit where we are conscious of the impossibility of understanding another person, where we cannot reproduce within ourselves a state which would explain the external perceptible signs according to known laws. The ground of this impossibility lies either in the fact that what is unknown to us is only the particular connection between external sign and psychical cause, or that we can form no idea at all of the inner event because analogies are wanting in our own experience. The

former is the case when any one speaks to us in a strange language. But when we come across an incomprehensible sentence in a known language, we may indeed be able to think of the ideas corresponding to the elements, but the power of combining them in a comprehensible whole is wanting ; we cannot even form a probable conjecture as to the thought which has led to this expression. It is the same with actions of which we cannot guess the meaning and purpose ; we assume at once that they are voluntary, that the ground of the particular movement lay in an impulse directed towards it ; but we are unable to construct the one end of the various activities.

12. But to infer the inner connection, the more remote psychical causes and conditions of particular manifestations, is just the problem which the knowledge of our fellow-men, so desirable in our intercourse, requires to be solved. We are impelled by our own experience to regard the particular manifestations of others as parts of a larger connection, and to penetrate into this so far as to be able to foresee and calculate as far as possible the future of their psychical life. Such a knowledge might be grounded partly upon general and universally valid laws concerning the causal dependence of psychical events upon each other, partly upon special knowledge of the way in which the psychical activities are determined in each particular individual by his special nature. If our inference is to be reliable, there must always be the certainty that certain events are produced only in one way.

Such a special knowledge we do not possess, but only an idea of the most general forms of the connection between ends and means, between states of feeling and their results, between premises and their consequences ; and the calculation is made still more uncertain by the knowledge drawn from our own experience, *e.g.*, that the effects of certain emotions and the impulse to utter our thoughts may be suppressed by self-control for the sake of other ends. Even where we see regular connections comparatively clearly, we should consider any one who should employ his psychological knowledge without further precautions to refer particular manifestations to corresponding actual inner events according to general rules, to infer the opinion or the intention from the words, as very simple-minded ; for we know that our speech and action is often calculated to create illusion and to lead astray, the visible actions being really motivated, not by the events which would first be inferred from them, but only by the purpose of exciting the belief that these events are there.

This is true not only of the simple lie ; it would also be foolish to take all social courtesy literally, all oratorical pathos for the genuine expression of real inspiration or indignation, all the promises of a candidate for election as signs of his serious will and intention. That such an illusion can be brought about is due, of course, to the fact that certain relations have been abstracted from psychological experience as generally valid, otherwise we could create no illusion ; but as soon as the intention of deceit is conjectured, such inferences lose their validity, and we are always confronted by the question whether we can apply our psychological knowledge in its original meaning, *i.e.* whether we may assume frankness and straightforwardness in another person, or whether we must mistrust him.

It is not necessary to analyse these phenomena of daily life any further in order to see that inferences as to the inner events of others as causes of their manifestations, if they are to be in any way probable, cannot be based upon simple psychological laws easily applied in the particular case, but only upon comprehensive combinations ; that the particular and fragmentary words or actions of others frequently admit of the most different interpretations ; that here it is almost the rule for the same event to have different causes. It is only by assuming that for every one there are certain ends which determine his actions, that he pursues these ends by employing suitable means, that he is preoccupied by certain views, and guided by certain principles and habits, that we are gradually able to construct the inner connection ; only from the harmony between his different manifestations and modes of action can we infer that he is candid or consistent, only from contradictions in his behaviour that he is hypocritical or inconsistent. In our ordinary estimation of men we are quite familiar with the fact that we must know the whole man before we can correctly refer his actions and words to their ground ; in this way there are formed images of certain types of individualities, such as Theophrastus attempted to portray in his characters.¹

We are here confronted by a circle like that we found in a much simpler form in the most elementary problem of finding the causal relation between two external events in certain things : we cannot establish the particular instance as an instance of causation unless we have a general proposition, and we can only infer the general proposition from the particular facts. Nevertheless we succeeded, at any rate, in obtaining definite laws, which were gradually confirmed ; but here, in the psychological

¹ Cf. my *Kleine Schriften*, II., 211 sq., "Die Unterschiede der Individualitäten."

region, analogous laws which might serve as a safe guide in inferring from the effect to the cause are excluded by the nature of the subject. Each particular manifestation admits of various interpretations; only by comparing and combining many of them together do we find that amongst the various interpretations of each and all there is always one which refers us to the same point, and so accept the hypothesis by which all may be harmoniously explained.

We find especially obvious in this department a relation between the particular case and general conditions, which enables us to "understand" the particular event before us, without being able to show its necessity in the strict sense, or even to calculate it beforehand from given conditions. We may explain an error, a hasty action, an outburst of emotion, a crime, in so far as the production of the effect by what is known to us of the facts is in accordance with our knowledge of the general forms of connection between 'psychical events; we cannot, as with a physical event, show that it *necessarily* happened so, because we know neither the actual antecedents in their completeness, nor do we possess strict laws according to which definite consequences must follow from these antecedents; the differences between individuals will cause the same stimulus to have different consequences. Strictly speaking, the only clearly determined laws are the normal laws for our different functions, in which is implied the question as to how far they will be actually obeyed. It is true that the result of a calculation follows clearly and of necessity from the problem set, a conclusion from its premises, the sentence of a judge from the code of the law and the facts falling under it, a determination of the will from moral principles; but this necessity is logical and not psychological, and it determines the actual event only when the thinking and acting man lets himself be guided by this objective necessity and not by chance impulses; otherwise error and unreasonable action would be impossible. We can explain a breach of faith by pressure of need, a fatal blow by the passion of anger, because we know how temptation to crime may arise in this way, but that the crime was necessary follows from no general law; the actual facts included also the insufficient feeling of duty or self-command which was present, and this—apart from the question of freedom—can only be constructed for the individual and not from general laws. Our "explanation" is satisfied with showing the presence of a part of the conditions, from which experience shows us the action might proceed, if other conditions were added or counteracting influences were absent; whether this is

so we infer only from the fact itself, without being able to ascertain it in any other way, or even to show that it is necessary.

13. We have been obliged to treat somewhat fully of these relations, which are familiar to us from our own practice in daily intercourse with others, because they form the basis upon which to estimate the methods which can be applied in the scientific treatment of history in the widest sense.

There can be no doubt that the psychical events which take place in men form the real kernel of history; whatever appears in external nature belongs to history only in so far as it has its origin in, and reacts upon, human spiritual life; and the facts which belong to nature gain historical importance only in so far as they contain conditions for spiritual action and exercise an influence in determining its direction.

It is here that natural science and history diverge. Within the visible world shown to us by our senses an invisible one actually exists which can be directly perceived only by the self-consciousness of particular individuals. For this reason the conditions of knowledge make this invisible world accessible to us only in so far as effects proceed from it into the external world, which are attainable by direct observation, or can be indirectly ascertained; what we have to do is to recognise these effects as the products of spiritual forces, thus distinguishing them from merely natural results, and to determine their causes as exactly as possible. But history has its highest interest in the fact that our own individual and social life depends upon these causes by means of their external effects; hence, from the point of view of this interest, we regard as historical in the narrower sense only those events which we can regard as continuing to exert an influence.

Nor can there be any doubt that what we aim at in all historical investigations, in the first place, is to inquire into the particular concrete event which has taken place at a particular time in a particular place; from the material before us we infer definite events and try to determine their data and sequence; we do not attempt to obtain general concepts and general propositions, but a picture of what actually has happened represented in narrative judgments about the particular. Historical investigation may be supplemented by attempts to derive from these judgments general laws, but this presupposes that the first part of the work has been completed, since we cannot begin by anticipating a uniformity like that in the province of nature.

14. The problem of discovering from the external events which we can

perceive the reality and concrete nature of their spiritual causes recurs in a different form at the various stages of historical investigation.

First, there is the task of discovering what has taken place externally, *i.e.*, of ascertaining the reality of those external events which are withdrawn from our observation, and which we have to regard as the effect of human mental activity. In natural history also there is historical investigation in an analogous sense ; it attempts to construct those earlier states and events in the material universe which have produced the present state according to natural laws ; and in order to mark off the two departments of work from each other, we assume that we are able to distinguish with certainty between what is independent of conscious human action and what is due to this alone in the external world. In particular instances we may be in doubt ; we may be unable to decide whether a ruin which is discovered points to human destructiveness or an earthquake, whether a skeleton should arouse a suspicion of murder or of a natural accident. But on the whole we distinguish with complete certainty by an inference from the nature of the given what is to be referred to human activity. We regard every potsherd which is discovered as an infallible proof of the presence of a human hand which formed it, and in so doing are guided by the consideration that we find men constantly making pots, and have never found them produced by animals, or forming themselves by chance in the course of nature ; and by the further consideration that they imply the idea of a purpose and an amount of observation and skill which we can attribute to man alone of all known animals, and that this product agrees in its general characteristics with numerous other human manufactures. Thus what we first see in many tangible objects is an action of man, an activity of his intentionally exercised mechanical power ; every building points in the first place and immediately to the work of the builders, and only in the second place to the purpose and inner conception of the architect.

15. But the most important material for historical investigation does not consist in potsherds, stone knives, implements, ornaments, or buildings ; important as these are for our knowledge of particular branches of human activity, of technical skill, of methods of satisfying human needs, of artistic products and of the ideas expressed in them, and many as are the results to be obtained from them by a comprehensive comparison, yet they fall far short of the written evidences of human mental activity which are handed down to us, and which are either immediate, intended to

make permanent the thoughts and opinions of the writer, or mediate, such as records and narratives of the action of others.

But what do all these documents prove? Directly and beyond doubt they prove only that they have been produced by a human hand; they prove further, with very great probability, that the written symbols point ultimately to ideas, thoughts, and opinions of a person writing or dictating, expressed in a certain language. It is when we attempt to infer beyond this that uncertainty and the work of mere conjecture begins.

There is, first, the merely literary question of the age and origin of the documents. We can here only briefly remind the reader upon how many assumptions the inferences frequently depend by which the date is first fixed for a document handed down, either in the original or in a copy; how comprehensive is the knowledge required concerning the history of the written characters, of the language, of the material, etc., and concerning the dates of the historical events recorded or presupposed in the document; knowledge which can itself only be obtained by a comprehensive comparison of the documents, so that we stand again before a circle in which hypotheses must move before they can attain a higher degree of probability.¹ Particular points can only be established

¹ To give only one example: the presuppositions upon which the investigations as to the genuineness and order of the writings handed down under Plato's name are based are taken from external evidences which can only be fully accepted when their origin has been investigated and established in a similar way, and from considerations concerning the language used. But how do we know what language Plato used? Even if we start only from those writings which are undoubtedly genuine, the Egyptian discovery of the manuscript of the *Phædo* suggests the question whether the text handed down is really that of Plato, or whether all sorts of changes have not found their way into the copies and made the language more like, or, perhaps, more unlike, than it originally was. The attempt to construct their order from statistics about certain forms of speech is based upon psychological considerations concerning habits of speaking and writing which cannot be proved with certainty from such comparatively limited material. But even if we had a number of documents in their original form, how wide a scope must be allowed for the variations of expression which are possible in the same writer, and what is the amount of deviation which compels us to assume another author? We cannot draw any certain conclusion from the presence of a word or a phrase which is absent elsewhere; not everything has been written which has been spoken. It is the same with the content. The psychological assumption that a writer must always be consistent is obviously false; but how far deviations may go cannot be ascertained with universal validity. To give any certain foundation to the various critical presuppositions which have often been employed, we should have to begin by dealing with some author whose authentic and dated works extend over decades, from the various points of view, in order to see how far the presuppositions hold good; the statistics of phraseology would have to be obtained, and agreement in style and in thought tested with regard to such specimens. If, for instance, acting upon the principle of starting from the known to infer from it the

gradually by their greater probability, and they form a basis from which to work further, always reserving the right to correct first assumptions.

16. When the author and date of a document have been ascertained, and the textual questions—which to some extent move in a similar circle—have been decided, we still know only that the author desired either to register something for his own remembrance, or in the majority of cases to communicate something to other people. What he wants to communicate, what ideas and thoughts he desires to excite, and what rules he desires to impress upon the reader's notice, must be discovered by philological exegesis; and here, again, we can only briefly indicate the logical processes called into use, and the limits of what is attainable. Even if the language were as familiar to us as a whole as our own, still the word, even if it has only one meaning, is not a completely exhaustive and accurate expression of what the writer means to denote by it; its value is determined for it by combination with others in a proposition or in a larger connection; we are called upon to reconstruct the whole from its elements, and what we need for this is partly knowledge of the subject matter, partly logical and psychological presuppositions concerning the mutual dependence and determination of ideas. Where the word has more than one meaning, or where, as in a more remote language, its meaning has to be determined by comparisons, then the process is a hypothetical one; several possibilities lie open, amongst which we must choose, and the meaning to be assumed here is either obtained directly from the fact that it is required by the context, or by way of exclusion, because all the other possibilities contradict the context. The presupposition on which all these attempts are based is that psychological and logical agreement in the combinations of ideas and judgments by virtue of which alone we can comprehend and repeat in our own thought what the writer meant to express.

But the combinations of the individual depend partly upon the whole store of concepts and intuitions, and where we are dealing with remote times we can reconstruct this only incompletely. Who could entirely transport himself into the sphere of thought in which Heraclitus moved?

That we cannot succeed in making our reproduction exactly agree with the original matters less, however, where we are not dealing with scientific or poetical representation, but with the narration of facts, of

problematical, we should try whether these presuppositions hold good when applied to Schelling's works, we should obtain very remarkable results.

definite external and internal events; here we must always be satisfied with placing them under general concepts, since the actual can never be completely expressed in terms of a general significance, but these concepts may be taken as coinciding with them in so far as they serve to state prominent characteristics. When it is recorded that Alexander died of fever, the physician may be doubtful as to the nature of the illness, and we cannot ascertain what exact ideas the recorder connects with the term "die"; but all that is important is contained in the general concept of death, and the essential content of this is independent of these subsidiary circumstances, and unambiguous. The most detailed description can only proceed in this way by subsuming the Given under general concepts, and leaving it to the reader to reconstruct therefrom, by means of his imagination, the concrete reality.

17. When the exegesis, from which we learn what the author desired to say, is completed, there begins historical criticism (in the narrower sense) of that part of the literature which does not merely record the thoughts and subjective views of the writer, but records facts and events. This criticism consists in investigating the trustworthiness of the author, both as to whether he really wished to say what he regarded as truth, and whether what he regarded as truth really is truth, whether he did not deceive himself or let himself be deceived by his informants; in considering, where contradictions arise between different recorders, how they may be solved, and which statement is to be preferred; and where they agree, whether they do not depend upon each other or upon a common source, in which case the agreement would lose that power of proof which it would otherwise have. Our inferences here will be grounded upon considerations partly of the whole mental constitution of the recorder and his attitude to the events, partly of the connection possible between the recorded events, and this possibility will again be estimated to a large extent from a psychological point of view.¹

18. The events which can frequently be established in this way with a certainty beyond doubt, and dated in sequence, are actions which manifest themselves in the external world, visible deeds; they are the *res gestæ*, the wanderings of tribes, the campaigns of armies, the battles won, revolts, plebiscites, the promulgation of laws, the taking office or

¹ It is scarcely necessary to note specially that in practice the investigation of origin, philological exegesis, and historical criticism are closely connected and mutually dependent.

resignation of magistrates, the first appearance and death of prominent men, and all such facts as find their place in the school chronologies as the most tangible framework of history. But however accurately we may determine all attainable facts, they represent only particular prominent points or fragments of a continuous course. This course must be interpolated before we can get even a connected picture of external events, as with the route taken by an army of which only the night quarters are noted.

But there is a more important supplementation and interpolation to be effected, and this lies within the province of psychology. Here alone, in the consciousness of the persons acting, is there any continuous connection of events which can be apprehended in its causal significance; actions are only the expressions of the thoughts, motives, and calculations upon which they are grounded, and from which they necessarily proceed. The problem is, therefore, to infer this mental activity for the particular person acting; and here the insufficiency of data introduces, in an intensified form, all the difficulties by which we are met when we attempt, in daily intercourse, really to understand other people, and to fathom what goes on in their minds. We cannot here enter more fully into the processes of comprehensive combination which have to be employed, and which demand from the historian a large amount of creative imagination in suggesting possibilities, and testing them by historical traditions; the inexhaustible variety of the particular makes it impossible to give brief instruction as to method. In so far as there is any safe process it consists in simple inferences from major premises of the most various kinds, which contain knowledge about reality. But work of this kind, which is mainly biographical, indicates a wider background.

19. The individual, with his knowledge, his range of ideas, his aims, and the means of attaining them at his disposal, stands within a certain community; that which is active within him has not its origin in himself alone, but is conditioned on all sides, both by the men around him, and by the past of the community. It is this which constitutes the historical continuity and the distinguishing characteristic of the psychical life of men, that the content of every individual consciousness points to a long series of preceding conditions, which were actual in preceding generations, and is quite incomprehensible without this history. Every word which we speak, every letter which we write, the way in which we clothe ourselves and behave, in which we manifest joy and sorrow—all this may, to some

extent, be traced back through thousands of years; the individual, in his most ordinary functions, repeats with only slow modifications the habits of long-vanished generations. And in the present also the action of the individual is constantly influenced by the living, by the authority of their opinion, by their advice or commands; his actions are guided by the calculation as to what impression he will make upon others, to what thoughts and resolutions he will determine them; at every step he is entangled in this complicated net of relations. Every word which I address to another on the one hand presupposes an idea of his mental habit, and, on the other hand, may be regarded as a kind of experiment, in so far as I cannot be quite certain of the effect intended.

The import of that which we can attain, and for which alone, therefore, there is any sense in striving, is most easily obvious when we compare it with the ideal of an exhaustive and all-including knowledge of human psychical life. In this complete knowledge of the actual causes of historical events, the whole of the psychical movements of all the individuals in any way socially connected would be open to us, like the motions of the planets and moons of our solar system; all direct and indirect relations between individual and individual would also be knowable; no one is quite without influence upon others, each contributes in different ways his share towards the social life of the whole, and it is the result of the complicated interaction of all these individual unities which the science of history endeavours to know in the first place. Spiritual content is actual in the consciousness of the individual alone; there are no thoughts which think themselves, no language which has existed except in the speech of the individual, no belief and no science which has shone of itself like a universal sun above the heads of individuals, no constitution which has existed elsewhere than in the consciousness, the will, the feeling of duty or fear, of the particular citizen. The connection between individuals, their influence upon each other, are also brought about only by their conscious activities; each individual is, as conscious self, an independent unit, and moves apart from others as in an empty space; no influence passes immediately from one soul into the other, no cohesion, acting with unconscious necessity, binds individuals together; the ground of the bond lies within each individual, in the relation of the thoughts, feelings, and volitions, which he is constrained by mere signs to produce in himself. To speak of a common will, or common consciousness, as if we meant a universal spiritual substance, of which particular souls are only

the modi or parts, is merely to hypostatize a general or collective concept, and we are always in danger of confusing a mere sum of similar phenomena, which exists only for the spectator, and forms a homogeneous aggregate only for him, with the consciousness of community which individuals feel. We can only speak of the common consciousness, or common will of a people, in the proper and useful sense of the term, when the individual includes others in his consciousness, is aware of his agreement with them, wills that which he knows that others will also, when he consciously and voluntarily harmonizes his action with the action of others. Otherwise we might ascribe a common consciousness and common will even to prisoners in solitary confinement. There are, of course, many phenomena occurring in many people which are in harmony and are recognised by us as being in harmony, and which are, nevertheless, independent of any conscious relation between the individuals. The sum total of these phenomena, which primarily exists only for us, may itself be regarded as a whole if they happen to exercise a collective influence. But we must distinguish carefully between this merely objective similarity and that community which exists for the consciousness of the individual. It is upon the latter alone that historical continuity and human interaction between individuals depend; it is only by recognising others as like myself, by inferring their thoughts from their utterances and taking them into myself, by letting myself be determined by their example or their will, that I bring about the relation which frees me from the complete isolation in which I live primarily in my consciousness as an individual. The only ground of real connection lies in the consciousness of the individual.

But the activities of each individual proceed for the most part not from particular isolated conditions, but from the whole of his opinions, knowledge, and volitions; the source from which they spring is the whole personality. Every sentence which we utter, every resolution which we carry out, points to a multitude of conditions, which, by means of the interaction of our ideas, feelings, and desires, have produced a certain psychical complex, particular aspects of which co-operate in each of our activities. *Si duo faciunt idem, non est idem.* The store of knowledge, the habits of thought, the ruling interests of each person, combine in a manner peculiar to him to give a certain tendency to his consideration of any question, and to his practical decision on any occasion.

Thus a complete knowledge of historical phenomena would include all mutually connected events in all individual centres, as well as the whole

of their causal determination, whether by individual dispositions and tendencies, or by relations with the external world, or by intercourse with others and the tradition of previous generations. Every historical state of a permanent nature, manners, habits, forms of society, political constitutions, religious forms of life, these are all the common product and the common expression of activities which continually recur in the same way amongst individuals who are combined in society; their stability is due to the memory and constant volitions of individuals; every historical event which produces a change in them is the product of impulses which have first become active in a single individual or in several together, and have determined them to a fresh activity.

20. When we have realized in its full concrete determination and completeness what really takes place in the life of a people or of humanity, what we should have to know in order to have a complete knowledge of the whole of the life of which only fragmentary manifestations can be established from historical documents, then we see at once how far removed that which our knowledge can really attain is from an exhaustive insight into the whole of what actually takes place, an insight combining the description of its course with knowledge of the causes which determine it according to universal laws.

So much is obvious and cannot be seriously disputed, that the fundamental *laws* of all historical events, from which their necessity might be deduced, can only be psychological laws determining events in particular individuals; these are the basis of all that happens, the essential part of all history unfolds itself in them and through them. We should have a satisfactory science if we could so determine the forces of individual souls and the laws of their action that everything could be known to follow of necessity from the nature of the individuals given, and from the relations in which they stand to each other and to the external world,—if we could explain historical phenomena as the collective action of these individuals, as we calculate the direction and velocity of a stream from the nature and interaction of the atoms of water in a certain bed of a certain fall. We find here that the whole province of the natural sciences is distinguished from that of the mental sciences by the fact that, in the former, the ultimate unities to which analysis leads us as the subjects of the fundamental forces are purely hypothetical, while in the latter their nature is immediately known to us, because we experience immediately in ourselves the elementary modes in which they work. The difficulty of the

problem consists only in establishing the conditions of the particular event on the one hand, and on the other in synthesizing in particular individuals the impulses from different sources, and in combining together the activities which issue from individuals and summing them up into collective action.

But of the great majority of these units we know at best only their approximate number and the general physical condition under which they live, their habitat and its climate, etc. We know nothing of their special existence, of their individual peculiarities, or of their development in detail; they form a confused background for the appearance of the prominent individuals of whom historians primarily relate; we cannot, therefore, refer directly to them in order to explain known facts from their thought and action.

21. But these individuals are combined in families, tribes and states; and there is handed down to us, or inferred from tradition, besides the fates and deeds of individuals, that which is *common* to the individuals of such a group, and that which we must regard as their collective action, whether it is directly the sum of particular actions, or brought about by means of a definite organization of their activity. This common element we may assume to be present in all, though not in an absolutely identical form; we may at any rate construct from the innumerable individual phenomena an average type, in which differences cease to be important. The most important among these common possessions of a large group is language. The dead languages, however, have been preserved in only a few monuments, meagre in amount as compared with what has actually been spoken (what is the whole Greek literature, in comparison with all that has actually been said in every house and every market-place since Homer's time?). Neither the store of words, nor their different applications, nor the variations of dialect have been completely preserved for us; nevertheless language represents an important part of the common spiritual life, and we may infer that all who have used the language have been more or less alike in their ideas and in their manner of employing them; it represents a common intellectual turn of mind.

It is the same with religious ideas of every kind; we cannot say what form they have taken in the mind of each individual, and what moods and feelings were in each case connected with them, or how far they acted as motives to action; but from analogy with what we see more clearly and completely around us, we assume that in what has been handed down to

us we are dealing with a common background of life. And here we have again to take into consideration that relation between leaders or rulers and followers or dependants which we find in every community of whatever kind it may be; the few think for the many, and in order that we may regard a whole of religious ideas as a common possession it is not requisite that each particular member should possess this whole completely, nor that its various consequences should take effect in each; it is sufficient that the leaders and rulers should be filled with them, they serve as representatives of the others. And where we find gradual changes in such ideas, it is in itself possible that, in consequence of general associations, certain new ideas should appear in many people at the same time; but those cases in which we are able to trace the process more accurately make it seem probable that the impulse started from individual points, and only that became common to all which most easily won assent in accordance with general logical and psychological laws.

With this is connected our knowledge of manners and customs, habits of life, mode of agriculture, and construction of implements. We know from experience of our own immediate neighbourhood that in these matters there is generally community and persistency, that invention plays a part subordinate to that of imitation, that custom and habit exercise a constraint which is often tyrannical. Hence a few examples enable us to infer that in essential points all will agree. The deviations of individuals, however, are generally without effect; and for changes it is again true that they are most easily explained by assuming that they start from particular points.

The forms of government, by which the will of the individual is confined within certain limits, and the ends which he is to aim at are dictated by an authoritative will, generally possess the chief interest for historical investigation. Here again we find the fundamental fact that more or less close forms of social combination have arisen everywhere, of which the really constitutive element is the power by which individual wills may be combined for common ends, and their divergent tendencies restrained. However much disputed the ultimate ground of the formation of states may be, it is unquestionable that here again we have a total phenomenon before us in which the great majority of individual persons agree, and which so far serves as a substitute for the unknown movements of individuals inasmuch as it is just this organization which causes the forces of individuals to subserve the one will and combines them for collective

action. And however different the motives may be from which the individual obeys authority and the laws, the collective result is a certain one and accessible to our knowledge. The most certain means of bringing all activities into uniformity, and guiding them into one direction, is not spontaneous unanimity, but the constraint of force.

These uniform and comparatively permanent habits form the background to the action of particular prominent personalities, the starting point for all particular events.

As in considering the particular life the fixed and constant connections of ideas and volitions are more important than their accidental manifestations, so also in the life of the community the uniform tendencies of thought and will form the constant factors which determine what will happen upon varying occasions ; they represent in one general picture the unknown multiplicity of the individual life in such a way that, while individual peculiarities are blotted out, the common elements which have the strongest and most constant effects stand out in clearer relief. •

22. These collective phenomena cannot, indeed, put an end to causal explanation ; on the contrary, they constitute the chief problem for final psychological explanation, they give rise to the questions : how must individuals be constituted, and to what laws must their development and interaction conform, to enable us to explain the particular forms and particular content of the life of the community ?

The view that from the course of history as a whole certain laws might be obtained which would refer, not to individuals, but to masses as such, laws easy of application and governing the collective life of people, even though they should only be laws in the sense in which we formulate empirical uniformities such as the alternation of the seasons in the province of nature—this view has indeed been repeatedly expressed (most decidedly by Buckle), but always rightly disputed and rejected. The expression “law” must be deprived of all its logical characteristics, if the vague similarities, or parallel cases, which may be discovered by a historical bird's-eye view are to be proclaimed as historical laws. The same combinations never recur, and it is therefore quite useless to attempt to discover by simple comparison, by enumerating similar sequences of conditions, general uniformities showing themselves in the complicated course of history ; unless, indeed, we give the title of historical laws to such statements as that bad harvests create famine, and weak governments discontent. Although the comparison of analogous relations is an im-

portant means of interpreting and supplementing what we know in the particular instance, and of connecting fragmentary items into a comprehensible whole, yet any attempt to obtain in this way general laws in the strict sense would be stifled by the number of exceptions.

Nevertheless we must first start from collective phenomena, since these are more easily seen than their elementary factors, which are for the most part unknown; but our only chance of success will lie in an analytical process, distinguishing within the whole complex of the vital phenomenon, e.g., of a people over a certain period of time, certain features which show a connection within themselves. We find here again on a larger scale what we have already found for psychology: actual action from moment to moment proceeds from the unity of the individual soul; the interaction of the particular elements, of ideas, feelings, intentions, is so complete that every given activity is the resultant of manifoldly complicated conditions. Nevertheless we can isolate and bring to view particular connections, we can distinguish particular threads interwoven in the fabric, which show a causal connection, and which may therefore take the place of partial laws, not in the sense that such forms of connection can by themselves determine what happens, but still in the sense that they express ruling tendencies which work in one direction within manifold complications.¹

We can proceed analogously with these historical collective phenomena; they can be analysed—chiefly, according to psychological categories—into component parts, as it were; such a part is, for instance, language. What we say, and how we say it, depends upon the whole psychical life in all its aspects, upon the development of ideas, upon the greater or less wealth of imagination, upon excitability of temper, upon the ends which are pursued; but by disregarding in actual speech the varying content of the moment, and by considering only the constant elements, the methods in which thoughts are expressed by words, and the habitual ways of using the organs of speech in uttering the traditional symbols, we find uniformities which we cannot indeed call laws in the strict sense of the word, since they are broken through by many exceptions, but which show nevertheless in the great majority of cases a general similarity in what takes place, which constitute a general tendency. The tendency to make the utterance of words more easy and convenient which is guided by physiological conditions in the way in which the organs act, the tendency

¹ See the careful distinctions drawn by F. J. Neumann in *Naturgesetz* and *Wirtschaftsgesetz*, 1892.

to construct new forms of speech in analogy with others, the tendency to extend the ordinary meanings of words according to certain associations so as to denote new turns of thought by them, such as appears in our innumerable metaphors—all these are uniformities which appear alike in different languages, and which, if we use the term in a somewhat wide and lax sense, may well be called empirical laws. Although they may be frequently contradicted by particular cases, they do nevertheless express tendencies which can be recognised as determining in a great number of cases.

Particular connections can be traced in this analytical way still more clearly where we are not dealing, as in language, with events in which conscious intention plays only a subordinate part, but with the conscious pursuit of ends. The life of the grown man is determined in its main aspects by the ends which he endeavours to realize; they may be regarded as constant causes, from which the successive particular activities proceed. But what a man will set before him as an end is generally determined by his nature, which on the one hand constrains him imperatively to the satisfaction of certain needs, and on the other hand causes him, by the wide receptibility of his disposition for pleasure of all kinds, to imagine attainable results which tempt him to bring them about; how he will satisfy these needs and wishes depends upon the means at his disposal and the organization of society. The satisfaction of hunger and thirst, the guarding against cold and heat, physical comfort and the gaining of the means to it, all the vital feelings of sense and emotion appertaining to the difference of the sexes, the satisfaction derived from the approbation and appreciation of others, or from the consciousness of power and dominion (to say nothing of the highest moral and religious objects)—all these are ends which we set before ourselves with more or less distinct consciousness, and endeavour to attain with more or less energy. In reality the particular action is generally determined by different motives at once; the artizan does not necessarily think only of earning his bread; he finds pleasure in his skill, and in what he produces; he desires to reap honour from it, to excel his rivals, to fulfil an engagement he has entered into, he regards his work as a vocation which he has to fulfil before God and in the service of the community—all this may determine the way in which he works, and the amount of energy which he devotes to the work. But we are able to isolate these particular ends in thought, and by a consideration of the causal connections which are

known to us and the means which are within our reach we may construct the process required by each of these ends, or find in what respects the same modes of action are determined by the one or the other consideration; we proceed as in natural science by imagining a process which would occur as the result of a given cause if the conditions were as simple as possible. Of course we cannot assume a process here which must *necessarily* follow, for it is a part of the assumption that men act in conformity with a *purpose* and reasonably, and that they have a correct knowledge of the means: what we therefore construct is the normal mode of action to which an end gives rise, and in doing so we assume that we have here a connection which can be known, and which will be at least approximately realized in the course of phenomena as a whole. For that which follows from the given conditions with logical necessity and certainty will ultimately maintain itself against the many divergent cases; errors correct themselves by their consequences, and cannot yield uniform results tending in the same direction.

In the sphere of political economy, the question as to the right method of regarding and investigating economic phenomena has been actively discussed. For the most part this has been with reference to the theory propounded by Adam Smith, that economic phenomena are determined by the principle of egoism, and that, starting from this basis, general laws may be found as to the regulation of price by supply and demand, variations in wages, etc. On the other hand, the historical school have rightly insisted that economic egoism is far from being the only factor to be taken into consideration, that the concrete form of every economic system is also influenced by the whole state of culture, and by religious and ethical motives, that the doctrine involves an untrue abstraction, and that given economic phenomena can only be causally connected with other aspects of the national life, and laws established amongst them, by the historical method and by observation of all the phenomena in actual co-operation.

But though these objections are fully justified, and though credit is indisputably due to Knies, in the first place, for emphasizing the comprehensive historical point of view, yet we cannot dispense with the other view when we are not merely tracing causal connections which are historical in the narrower sense, *i.e.*, essentially narrative and particular concrete causal connections, but are attempting to analyse particular factors and to establish partial laws, the combined fulfilment of which is historical reality,

and so to obtain an "exact" theory of phenomena showing that certain consequences must follow from certain causes regarded as isolated. But though this attempt also is justified, it does seem to me that the method employed under the influence of the hypothesis of egoism has not been that which corresponds best to the nature of things. It is, of course, allowable in the interests of science to start from such a fiction as that a number of persons producing and exchanging commodities are ruled only by egoism, and from the further fiction that the means of satisfying this egoism in the best way are seen and applied by them reasonably and consistently. But because a reality corresponding to these hypotheses can be nowhere ascertained—the nearest approach is found in the methods of international trade in money and commodities, and even these, as a matter of fact, cannot proceed entirely from egoistic motives—the deductions of this exact process cannot be confronted with reality. Fictitious simple cases are assumed in physics also, but the subjects, to which the elementary laws are applicable, are assumed to be real; here the subjects are fictitious, or are at least subjects whose existence cannot be ascertained, for particular motives can never be directly shown, they can only be surmised. It seems a more fruitful way, and one more easily compared with reality, to start from ends. To obtain the greatest possible value with the least possible outlay of labour and capital is an end which is not only quite comprehensible, but is extensively present and acknowledged; from what motives individuals place this end before them is a further question which it is not necessary to take into immediate consideration for the consequences of the end, particularly as the production of commodities is not an *ultimate* end. Commodities have value only as the means to the various ultimate ends; to a large extent they are sought for the sake of the power which they bestow, and as the condition of satisfaction of every sort, and it is this satisfaction which is primarily important, and the real motive to action. And we certainly cannot say that the motives must be egoistic which make us wish to receive the highest possible reward for our work, and to earn a competency; it is an end which may be accepted from the most unassailable moral motives. The economic end is so universal only because it includes the conditions of existence, and, therefore, of all other purposive activity. It is this proximate end which must logically determine the behaviour of men to a large extent; whatever follows from this end under given actual conditions can be constructed, and can be directly compared with

reality where the end either acts alone or predominantly. In this way we can judge also whether the action of the individual, or the forms of exchange, etc., are adapted to the end or not; we can distinguish between those who really follow this end and those in whom it is limited or entirely suppressed by other ends. In construction from motives we must work from the fictitious normal man, who is not merely possessed entirely by the principle of economic egoism, but is also free from inertia, carelessness, and extravagance, and we must operate with these fictitious elements; if we construct from the end, that in itself includes the determination of a normal behaviour, and the possibility of various deviations from it without our being compelled to think of reality as other than it is. It is only necessary to isolate one of the ends which actually determine the action of men in the economic world, and to trace its consequences; to ask what modes of action are called for by it under the conditions given at any moment, and how far reality corresponds to the course of action so constructed.¹

The credit of having given a leading place to the analysis of historical reality by distinguishing between different systems of ends, and of having based upon it the methods of the mental sciences, belongs to Dilthey.²

23. Both in our common intuitions and in our ends and the rules of action dependent upon them, we find a continual change going on, which constitutes what we are accustomed to call the historical development of intuitions, institutions, etc.; no generation simply repeats the actions of the previous one; more or less slowly change takes place in what we may call the common spiritual possession of a people, or of a complex of peoples. These changes themselves take place ultimately in particular individuals; on the one hand, the individual progresses to new thoughts, new knowledge, new ideals, new means of realizing them; on the other hand, what we receive by mere tradition undergoes, in consequence of the processes in which the appropriation of tradition consists, more or less substantial changes from one generation to another.³ And just because there is uninterrupted interaction in

¹ In the literature of this question with which I am acquainted, I find that H. Dietzel ("Beiträge zur Methodik der Wirthschafts-Wissenschaft," *Jahrb. für Nationalökonomie und Statistik*, N. F., vol. 9, p. 34) definitely emphasizes this point of view of the end as the starting-point for economic theories, as opposed to the reference to motives; for what he calls (in agreement with A. Wagner) "economic principle" is, according to his own representation, nothing but a rule of action following from a general end.

² See his *Einleitung in die Geisteswissenschaften*, I., 1883.

³ This has been shown, e.g., with reference to the gradual changes in language, completely and convincingly by Paul, *Prinzipien der Sprachgeschichte*.

the particular individual between the different sides of his psychical life, there must also be in the greater whole a connection between the different spheres of science, of religious intuitions, of morals and law, and of art.

Such developments become comprehensible, and can be recognised as essentially connected, in proportion as we can refer to psychological laws which are true in the same way of every one, and to which it is due that what is first thought of and striven after by individuals is assented to and imitated by others. Such progress can be most easily traced where we are dealing with pure theory, in the history of science. For our knowledge of the objective world the other psychological factors are less important; that which is logically correct exerts through its self-evidence a constraint upon us which cannot be withstood permanently by merely subjective and fortuitous combinations; it can be accepted universally while errors diverge. It is, therefore, to be expected that when an opinion is universally accepted its necessary consequences will also hold good; while, on the other hand, every deficiency or contradiction in a theory will afford an impulse to its supplementation or correction. It is thus that we are justified in attempting to introduce the logical necessity of the process from ground to consequence, and of the solution of contradictions as the motive power in historical development, to connect its particular stages by this bond, and so to explain its sequences; we shall be driven to this expedient especially where we have before us only the results of scientific research and not the whole mental processes of the particular co-operators. But here, again, we cannot do more than retrospectively arrange what has actually happened; here again it is true that logical necessity is not a self-subsisting force, which, acting, as it were, independently of individuals, determines what happens. The correctness of a conclusion is no guarantee that it has been actually inferred, and neither the individual nor communities have so painful a sensibility for contradictions in their opinions or in their particular commissions and omissions, that it would be impossible for them to believe contradictions and give expression to them in their actions without being especially troubled by it, or even without noticing it. The strict legislation of logic applies only to an ideal consciousness containing at one and the same time all the elements of its ideal world in all their relations, and thoughts do not think themselves; amongst all the thoughts which occur involuntarily to the more active

and intelligent a critical selection must be made according to logical rules. If it were not so, it would be incomprehensible how things which now seem quite evident should have been so slow to be recognised. For this reason we can never assert with reference to reality that any scientific progress has been necessary; from all that has been produced we can subsequently select that which is to be described as progress, but we cannot calculate what will follow; for all progress is of the nature of invention.

That factor of development which is peculiar to the individual, and cannot be calculated, naturally plays a much greater part in all those departments which are closely connected with the feelings and with practical interests. Here again the dependence of individuals upon each other, and the force of habit and of existing institutions, provides for a certain stability; the judgments as to what is beautiful and becoming, what is just and worthy of endeavour, change but slowly within a community. Nevertheless the changes within these departments also can be explained by general psychological motives; whatever would, or at any rate, might follow in an individual consciousness from the presence of a certain content, we are justified in transferring to the whole of the intuitions and endeavours which manifest themselves within a people, just because all progress and all development can take place only in the individual. We must, however, always bear in mind that for the most part what we can directly prove are only the derived consequences of events which we cannot know in detail, and frequently cannot even construct hypothetically because we have not sufficient grounds to go upon.

24. That general way of regarding historical events which determines the particular direction which will be taken by the causal explanation of facts will always oscillate between two extremes. On the one hand, it will lay stress on the importance of individuals, the influence of prominent and original personalities which, by the superiority of their intellect and the strength of their will, give new content to the life of the people and new directions to their common action; on the other hand, it will emphasize the common element by regarding the individuals as only the expression and organ of the universal spirit, and explaining their thoughts and action by the total life of their time and their people. The two lines of thought must supplement each other. From the second point of view the kernel of history consists, not in particular deeds,

but in the ideas which are realized in the common life of the nations ; but ideas do not grow of themselves ; they are originally produced by individuals ; they cannot be known directly but can only be inferred from their consequences, and their importance is ultimately due to the concrete actions which spring from them and which alone are capable of producing further effects. There is a similar relation between the tendency to explain everything by conscious purpose and calculation and the tendency to refer to a necessity, concealed from the particular consciousness, by virtue of which individuals produce more than they know or will, and are subservient to the law of development of the race. This opposition is reconciled by the view that in our conscious voluntary action we can only set before us ends which are inherent in the nature of man and which first appear in the form of an involuntary impulse ; nor again can we refer to conscious purpose the fact that thoughts arise which, by the satisfaction which they promise, stimulate the will to seek the means. We find here what in other departments we call instinct, a form of action which is objectively directed towards an end, but which cannot be explained by conscious ends.

Our historical treatment, which began by merely seeking the reality of concrete causes by which to explain the sequence of events, here takes us beyond itself. We come upon a limit in two directions. First, in following our original inquiry, and penetrating the past, we come upon a point where historical records fail us, a primary state which we can only construct by hypothesis. How human society has founded itself, how language arose, how the simplest forms of skill developed, can only be conjectured. Such conjectures must be partly supported by analogies which, taken from the known course of history, serve for the construction of previous stages ; but for the most part they depend upon certain assumptions concerning that general nature of man of which the course of history is the explication. Here, therefore, we are again brought to the third and last of the problems of explanation distinguished on p. 418, explanation by the essential concepts of substances.

§ 100.

In order that the causal concept (§ 73) may be logically complete we have to find the *ULTIMATE GROUND* of all perceptible events in substances which, by virtue of their nature and their essential relations to

other substances, necessarily give rise to the activities appearing in them ; and at the same time we complete the ESSENTIAL CONCEPTS OF SUBSTANCES.

The simplest, and, from a logical point of view, most obvious form in which we can carry out this reference to substances is that of a mechanism ; *i.e.* of a relation between a limited number of invariable substances to each other, such that they vary in their relations to each other according to invariable laws. Here the PRINCIPLE OF THE CONSERVATION OF ENERGY serves as a clue in referring the action of perceptible causes to the forces essentially inherent in substances.

But we cannot infer that this hypothesis is the only one to be relied upon ; we must distinguish between leading principles of the methods of investigation and metaphysical axioms.

In the organic world the expression of facts requires the concept of DEVELOPMENT. This can be applied in two ways ; either we may—according to the mechanical theory of life—assume that the ultimate subjects are invariable atoms, or the concept may be employed for the essential determination of subjects which contain in themselves the ground of a change and increased efficiency which is determined by their own activity.

Psychical phenomena require as the ultimate ground of explanation single subjects for that which is contained in one consciousness and the concept of inner development. The most obvious assumption to employ in the methods of investigation is that of determinism ; but it is neither the only one possible, nor is it a conclusion confirmed on empirical grounds.

1. In so far as the inductive methods we have as yet discussed were directed to the investigation of causal connections in external nature, the concept of cause upon which they were based was that which approached most nearly to the popular meaning of the term. The causal connection which we were looking for existed between a perceptible activity of a thing and a connected perceptible change in another thing ; the action of the cause consisted in an event in time. The effect came about in the same time and must therefore be completed after the beginning of the action. That the effect must be subsequent to the cause is still more certain where we have chains of effects, where a change in one thing transfers itself successively to other things. The certainty of causal connections always depends primarily upon the knowledge of such causal laws, which only give us a real explanation when definite things can be.

pointed out as efficient causes. In the principle that the effect must always be the measure of the action we found a means of determining where we might assume a complete and definitely established causal law of this kind.

2. The logical elaboration of this popular concept which we developed in § 73 leads us to new problems. We have shown (pp. 102-116) that it follows from the concept of efficient action itself that the action and the genesis of the effect must be contemporaneous, if we regard them not as units, but as an event passing continuously through a series of moments. It was shown further that when the event is referred to the nature of the substances entering into a causal relation as its ultimate ground, the effect must be derived both from the nature of the agent and that of the patient, *i.e.* it must be grounded upon a relation between these two which finds its expression in the concept of force. Finally, it was shown that in every effect we must distinguish two things: first, that consequence of the state brought about by the agent in the patient which continues from moment to moment, and which is grounded only in the patient itself; and secondly, that amount of change (which again is introduced from moment to moment) which must be referred to the continued action of the cause. Galileo's law of inertia, and its importance in the explanation of motions, was an illustration of the last point; the continuance of a motion, when once brought about, no longer appeared as the immediate action of the cause which first produced it, but the necessary consequence of the nature of the moved body itself by virtue of its attribute of inertia.

It is similar in other departments. In ordinary language the illness produced by injury to an organ, and the consequent death, are the effect of the injury; but by more exact analysis we find that the proximate effect, which the injurious cause produces, is only the mechanical separation of certain tissues, or the introduction of a strange body. The ground of all which follows is found only in the organization of the injured person, and develops therein from one stage to another according to physiological laws.

3. The first problem arising out of this analysis is to break up every event induced by a cause into that part which follows merely from the nature of the patient, and that part which is due to the agent; to determine, therefore, the rules according to which, when a state has once been brought about in a thing from without, its subsequent states follow of themselves, and the rules according to which these sequences are altered

by changing relations to other things. The latter rules must be grounded on the relation between substances themselves.

To carry out this process directly would be possible only if we could isolate particular things, so as to observe how they would behave when placed in a certain state and then left entirely to themselves; but this isolation is not possible. We cannot observe any body which is able to continue its motion without being acted upon by any causes, nor any development in which, from a given moment, the inner succession of the states which follow from the nature of the organism itself would unfold itself without being influenced from without. Thus the division of the ground of a perceptible effect into the nature of the patient by itself, and its relation to the agent, is always hypothetical; the analysis must be carried out in thought. Comparisons may indeed give us a clue to a hypothesis; by observing that a body prolongs its motion in proportion as obstacles are diminished we may be brought to think that if there were no obstacle at all it would continue to move indefinitely; but the conjecture cannot be confirmed by any direct experiment. It is only the possibility of inferring from such assumed propositions what really happens with absolute accuracy which raises the hypotheses to that degree of probability which can be bestowed upon them by the inference from the truth of the consequence to the truth of the premises.

4. The principle of the conservation of energy plays a part here similar to that taken by the principle of the permanence of substance, when in § 91, 4, we were dealing with the reduction of the phenomenal subjects of our judgments of perception to real subjects persisting throughout change. In the first place, the principle establishes an equation from which can be determined whether successive changes are related entirely and purely as cause and effect; and it further affords in the equivalent terms a common measure for efficient action, just as the principle that weight is the measure of the quantity of matter affords a common measure for substances.

In so far as this principle is unassailable and can be empirically verified it serves as a clue, not only in determining whether we are justified in regarding a given effect as the whole and sole action of a cause, but also in deciding the question how a composite effect is to be referred to the substances taking part in it. When the drawing of the trigger is followed by the explosion of the powder, and this by the flight of the ball, or when the blasting of a rock follows the connection of the conductors in a

galvanic battery, it is evident that there is no equality between the mechanical energy of the hand and the energy of the movements which result from it. We cannot therefore refer the whole of the succeeding events directly to the action of the cause ; on the contrary, the greater part of it must be referred to another ground, and for this we cannot assume the same form of causal action as in our first discovered causal laws, but are forced to postulate causal relations which make possible the transition from a state of rest to one of motion, powers of action which are always present, and are disengaged under certain conditions.

5. The simplest example of such a causal relation is gravity. It was noticed on p. 369 that direct observation leaves us completely at a loss when we try to say *why* bodies fall ; all that we see are negative conditions, the absence of circumstances which hinder the fall, not active causes. Gravity itself as an attribute is primarily only another expression for the general phenomenon, but as a relational concept this predicate is not alone sufficient to determine where the ground of the phenomenon is to be looked for. All definite assumptions about the ground of gravity and the tendency to fall are hypothetical, and by means of them we place in the nature of certain substances the ground of the changes which occur under certain conditions. But the more comprehensive the combinations upon which such hypotheses rest, the less possible it is to state any elementary method for them ; the history of the concept of gravity and of the theory of falling bodies shows sufficiently how remote from analysable processes, such as could be learned by rule, were the inspired conceptions of Galileo and Newton.

6. Newton himself gives an example of the double meaning of such hypotheses. In guarding himself against being supposed to wish to determine the physical causes of gravity, he explains the expressions "attractive force" and so on, as merely paraphrases of the law manifested in the way in which bodies behave ; motions occur *as if* there were an active force in the centre of the sun which attracts bodies in the inverse ratio of the square of the distance. For him, therefore, the force of attraction is merely a concept which helps him in the mathematical formulation of the laws, but which has no significance for reality ; it contains no statement as to a universal essential attribute of matter.

This purely formal use of hypothetical ideas as to the ultimate grounds of actual phenomena is opposed to that which we may call the metaphysical use. This believes itself justified in assuming that whatever

enables us to explain phenomena harmoniously constitutes the actual nature of the substances and completes their essential concepts. Notwithstanding Newton's precautions, his immediate successors regarded the force of attraction as such an essential determination of matter, and included it in the concept of matter; the only justification for this lies in the fact that our thought, which moves in the syntheses of the categories of substance and causality, finds it necessary, in order to reduce observed events to general propositions, to frame somehow or other concepts of substances and of their modes of interacting. The hypothetical character of all concepts thus constructed continues in their metaphysical significance; they can only be confirmed by the fact that the observed event can be deduced from them in its whole extent; and those cases in which previously inexplicable phenomena, from which the hypothesis was not derived, can be explained by it, or in which it enables us to discover the facts which it requires, occupy the position of confirmations of a proposition inductively obtained by experiment. The explanation of the oblateness of the earth and the tides of the sea by an assumption grounded only upon the fall of terrestrial bodies, and the motions of the planets, and the discovery of Neptune, are *exempla illustria* representing the highest attainable degree of confirmation of a hypothetical assumption which cannot be directly affirmed.

In a similar way chemical processes are referred to fundamental relations of the particular substances, by virtue of which certain motions and positions of the atoms occur under certain conditions. And even if the attempt to refer all the events in the universe, including the phenomena of gravity, to the mere communication of motion between the ultimate elements should succeed, it must still be part of the permanent nature of these elements that under certain spatial conditions they fall into motion determined both in direction and velocity; all actual events would be the consequence of this permanent nature, which includes a definite relation of each particular element to all the others.

7. If the end at which we aim in this stage of explaining phenomena by their grounds were attained with the highest degree of probability, then we should have the concepts of all the substances active in the material world, complete and self-contained, in a form which would enable us to infer from the definition of every substance how it would act towards every other in every relation into which it might enter with it; what consequences would proceed for all from every change in their

relations, and how every modification introduced in every substance would have consequent modifications in itself; we should know, further, by the laws of the composition of effects what must proceed from every combination of substances which are in simultaneous but different relations to each other.

In the science of the mechanism of the heavens we have a grand and simple illustration of such construction, though indeed it is itself, strictly speaking, only possible by means of an abstraction; only the movements of the bodies in space are regarded, and other relations, such as those of the radiation of light and heat, are unnoticed; only two invariable attributes common to all bodies are taken as the ground of their motions; and then we can say that the orbits and velocities of the planets follow according to simple laws from their masses, distances and previous motion, and the chief difficulty lies in the methods of calculation, by which we have to derive from the continually changing relations of a number of elements further changes always following the same laws. The popular concept of cause and effect has completely disappeared here; the corresponding acceleration or retardation of motion is absolutely simultaneous with every change of distance; the force, as the attribute of the masses, remains the same, but in different relations it gives rise to different acceleration; and however different the form which these relations may take, still from every collocation of invariable masses endowed with force their motions proceed according to the same laws. All distinction between particular intervals of time disappears; the whole course from the immemorial past to the boundless future may be regarded as the one consequence of one ground, which, unaffected by time, governs what happens in a continual present; or this course may be broken up into immeasurably small intervals, of which each contains the whole of the conditions from which the subsequent one necessarily proceeds. And in so far as all which happens is caused only by these elements we can calculate from any moment backwards to the past as well as forwards to the future.

8. It is easy to understand how the logical clearness with which concepts of active substances, of their forces and laws of action, are here constructed, and the exactness with which calculations made from them agree with our observations, might lead to the ideal of an all-embracing formula, in which the whole series of states of all things could be similarly explained by relations varying according to constant laws.

But attractive as this ideal is, it is necessary to distinguish between methodological aspirations and propositions for which a proof is presented, or is at any rate possible, according to the general principles of induction. We must not forget that here we have to determine, from the given phenomena and their phenomenal laws, two unknowns: first, the concepts of the single active substances, and, secondly, the laws of the action of their forces; and that no proposition can be directly verified as where we are dealing with laws referring to events in phenomenal unities. In the latter case the elements with which we are dealing are directly measurable magnitudes; in the former they are constructed concepts, the existence of which cannot be demonstrated to the senses. Behind the perceptible universe there is being built up an intelligible universe of mere concepts, which can reveal its presence in its influence upon our senses, but can never immediately enter into the series of phenomena. From the point of view of method there is no difference between the universe of Platonic ideas and Aristotelian forms, which determine a matter which is similarly imperceptible and only to be apprehended in thought, and the atoms of modern science, which, themselves imperceptible, are taken as the ultimate active subjects in all phenomena. All that can be demanded of such constructions is that they should suffice for the explanation of the immediately given,—ultimately, therefore, of those affections of consciousness which are given to the senses, and should enable us to deduce them in their whole concrete determination, with all their quantitative attributes; the more particular nature of the hypotheses is determined not by general logical considerations, but by the nature of what is given.

9. All that we can do from the point of view of a general methodology is to show what formal attributes are most desirable in the hypotheses from a logical point of view, and offer the simplest and most convenient means to attaining the ends of science, *in case they can be carried out*; what attributes, therefore, it is reasonable to look for and trace to their results first, before proceeding to other assumptions. From the nature of the universal propositions which we endeavour to attain, and from the nature of the concepts of substance and causality, we may derive some rules of method, which must guide us at first in the construction of these hypothetical assumptions, but which must not be elevated from their position as mere methodological principles of investigation to the dignity of metaphysical axioms; it does not follow that they are *true*, because they

would give us the consistent explanation of the given on the easiest conditions.

The following may be taken as instances of these methodological guiding principles :

1. It is a logical requirement to break up the continuous into the discrete, to assume as the true subjects of all phenomena actual enumerable unities, and to think of these unities as *simple*, and not themselves consisting of heterogeneous or homogeneous parts, so that every predicate of these subjects is true absolutely, without limitation or distinction.

2. It is a logical requirement to think of these unities as determined in such a way as to fall under a number of *concepts* which can be surveyed, or which is at any rate finite, and to think of them as being exhaustively expressed by these concepts ; to assume, therefore, that the *infima species* represent a number of beings which are absolutely indistinguishable, thus making it possible to have strictly universal judgments with fully determined predicates.

3. It is a logical requirement to think of these unities as absolutely *invariable* in time, so that every proposition which refers to them may be true without limitation of time, that they can enter into every relation without changing, and always have the same predicates under the same conditions ; or, to express it differently, that every change of predicate has its ground in external relation alone.

We have here nothing but the points of view by which we were guided in the logical elaboration of the concept of substance (§ 72),—hypotheses in which we assume such substances would best answer our logical needs. Atomism satisfies these demands, and it therefore recommends itself from the logical point of view as the most obvious assumption. Atoms are these discrete homogeneous unities, which exclude all plurality of distinguishable parts, are invariable in time, and fall under comparatively few fixed concepts ; and for this reason they are so constituted that one atom can be substituted for every one of the same sort without any difference, that under like conditions they always act in the same way, that no change takes place within them, and that every general proposition about them is true without limitation of time.

10. There is a second series of logical demands having reference to the nature of the predicates which are to be expressed in the universal judgments we try to find. Where we have absolutely single and invariable subjects such predicates can only be the relations which exist between

them ; all that remains to be known about them is their relation to our sensation and to each other under certain conditions. But their relation to our sensation depends upon their spatial relations to each other, so that ultimately the only predicates which are the results of their essence, *i.e.* of their invariable forces, are their situation and motion with respect to each other. Here we have obtained the most favourable conditions for the determinateness of the predicates. The continuum of the purely mathematical predicates enables us to express the smallest differences in conceptual exactitude by means of measurement, so that every universal judgment assumes the character of a mathematical formula which represents spatial and temporal relations as the function of spatial and temporal relations.

If we are to be able to construct and use these formulæ, the most favourable assumption we can make is that the fundamental relations are as simple as possible, in order that the art of mathematical analysis may be equal to them, that is, that the functions, which make the motion depend upon the distance, are as simple as possible.¹

11. We can understand from this how it is that the conception of a mechanism of the universe which attempts to represent all perceptible events as the motion of invariable atoms according to invariable laws is the form of explanation which would most easily and completely satisfy our logical requirements. According to it no other data than different kinds of atoms and space would be needed to enable us to calculate what would be the results of any number of atoms and any distribution of them in space ; and if it should ultimately prove that even the hetero-

¹ I cannot convince myself that this general postulate includes also the particular assumptions which are sometimes derived from it ; *i.e.* Galileo's law of inertia and Newton's law of the diminution of efficient action according to the square of the distance. Forces which should be modified with the velocity of a body, if the modifications occurred according to a constant law, would also be invariable forces in the sense needed by the theory ; Galileo's law of inertia is only the simplest case conceivable in which the forces exercised by a body are independent of its own motion. In the same way Newton's law does not seem to be given by the condition that a force is invariable in space ; the invariability requisite for general propositions can merely mean that the force exercises the same effect at every position in space, hence that the propositions undergo no change when applied to different regions in space. That the sum of the actions remains the same when, diminishing indirectly as the square of the distance, they extend over more and more remote spherical surfaces, is obvious only for the case in which an action transmits itself within a uniformly filled space, as in the motion of waves ; where we are dealing with an *actio in distans* the demand for an invariable force would be satisfied also by a force which should diminish in simple or cubic proportion to the distance.

geneity of the chemical atoms could be explained by combinations of homogeneous atoms, then the whole manifold of the universe would be reduced to the smallest number conceivable of principles of a purely logical and mathematical and therefore completely comprehensible nature. We can understand the charm with which this view attracts the most scientific spirits, and the zeal with which this hypothesis is proclaimed as the only scientific one.

But there is a great difference between laying down rules for the construction of logically desirable hypotheses and carrying them out; we may willingly acknowledge the justification for trying such assumptions first, and yet deny that the nature of given external and internal facts affords any sure prospect of realizing this ideal formula of the universe, or that there is any proof that there is no need of other principles, or that no others are admissible. There is no logical law to prevent us from assuming substances which do not possess that invariability which causes them always to act according to the same laws, but which on the contrary have an inward development according to the law of which their activities give rise to each other, and each stage manifests a different reaction externally; there is no logical law which requires us to find the ground of every event in external relations alone. Nor is it anywhere written that substances are impossible which, within the same general concept, differ from one individual to the other in such a way that no one can be simply substituted for another; the proposition that no two things in the universe are exactly alike is not a metaphysical axiom, nor yet the proposition that for every thing there must be an unlimited number of absolutely similar things under the same concept. It is not even impossible to think of substances as being generated in time; the principle of the permanence of substance is an empirical one, and can only be accepted as true within that sphere in which its truth has been proved; it does not refute the possibility that in some other sphere a genesis may take place. We must of course inquire into the ground of the existence of every substance to which we attribute any real being, so as to apply to them the postulate that the Given is necessary, especially if in their existence and action they stand in causal relations to the rest of the universe. But the nature of this ground and the law according to which it works could only be inferred from the most comprehensive consideration of the whole connection of phenomena.

12. The only decisive justification for any hypothesis is to show that

the Given can be completely explained from it ; that from the nature of the elements which it assumes, the whole course of the universe is made as comprehensible as the motion of the planets and the satellites from the inertia and gravitation of masses.

In the province of inorganic nature this end has not indeed been yet attained ; no one can say that the concepts of the atoms and of their original forces and laws of action are already so completely and unanimously determined, that all observable events could be unambiguously derived from them. The relation of ponderable atoms to those of ether has not yet been established, nor has the conflict of different hypotheses led to any certain result with reference to the laws of motion and the forces to which they refer ; the wide sphere of electricity and magnetism still awaits the realization of definite mechanical ideas, in comprehensible relation to the hypotheses which apply in other spheres. But notwithstanding this incomplete state of the physical hypotheses we are justified, partly by the character of physical phenomena in general, partly by the results which have been obtained in particular departments, in assuming that we shall succeed here in determining the concepts of the ultimate elements of the material universe in such a way that all phenomena may be derived from the invariable laws of the relations between them. The strictness with which the physical laws hold good, and the exactitude with which, under the same conditions, the same phenomena always recur in all parts of the same kinds of substance, indicate that substances are of such a nature as atomism assumes ; the main outlines of the theory are prescribed in this way, and all we have to do is to determine more accurately the constant attributes and forces to be attributed to the various kinds of material elements. There is no occasion to include more in the concepts of the elements than is required by the phenomena.

13. But whether these assumptions can be carried out with reference to the organic world is doubtful. In so far as organic bodies are built up, directly or indirectly, from substances which belong to inorganic nature, we are indeed justified methodologically in attempting to reduce all which goes on in them to known physical and chemical laws, and in assuming as the ultimate bases of events just those simple and invariable substances to which we are led in investigating lifeless nature, only in more complicated connections and more varied interaction. Should the attempt succeed, the aim of knowledge would be attained in the simplest and most comprehensible way. But these rules of method do not justify

us in asserting that this hypothesis is the only one scientifically possible, and that every other is excluded by principle ; only the result can decide as to the correctness of this as of every other conjecture. If the attempts to deduce as recognisable consequences from the general attributes of the substances constituting the organic body their combination in this particular form, the processes of the formation and division of cells, the course of development of individual unities, the processes of propagation, etc., should leave an insoluble remainder,—if experiment fails to produce artificially a cell, then it is not only allowable but necessary to introduce other hypotheses. The specific empirical laws which are obtained from observation of organic life require subjects of another sort, acting according to other laws. However insufficient the attempts have hitherto been to find any conceivable subject for the specific vital forces, and however easy it has therefore been for the mechanical theory to wage war against these vague concepts, still there has been a perceptible diminution in the confidence that this theory can be carried out with complete success, and its supporters begin to complain of the heresy of the “Neovitalists.”

If we leave aside for the present the attempt to explain organisms deductively from mechanical principles, and turn ourselves to the task of finding the fundamental concepts which complete our knowledge of the living as such, these would seem to be beyond doubt the concepts of the individual and of development. The former gives us the subject to which the predicates belong ; the latter expresses the fundamental mode of being by which the nature of this subject is determined in distinction from the invariability of atoms and their forces.

The concept of the individual was explained in § 78, starting from the question what it is which determines us to speak of the unity of a thing. We found that our ground for applying to a system of parts the concept of the individual, in distinction from that of the merely externally limited thing or piece, consisted partly in causal considerations, as with a crystal, partly in teleological, as with an organism (or even with a clock).

This point of view leaves it still open to which ultimate ground the unity of the individual is to be referred. The real point at issue in the view to be taken of organic individuals is, whether the organic individual is only a collective unity of atoms which interact by virtue of their forces, and to which the concept of finality can only be applied in a formal sense ; or whether it includes a real indivisible unity, by which alone this arrangement of different parts is made possible and explained,—whether, therefore,

we have to assume an ultimate, really individual subject, a central monad as the ground of the unity of the individual.

14. This brings us to a difference of meaning in the concept of development.

We can speak of development even where we are dealing with purely mechanical atomism. In the first place, all the actions of a given atom, and all the states into which it falls, may be regarded as the development of all which is grounded in those constant forces which are unaffected by differences of time ; for these forces only become active under conditions which are brought about in time by its changing relations to others. When the chemist prepares a synthesis of different substances which has never been known before, and which perhaps has never yet existed, then the atoms manifest for the first time actions which were from the first included in their invariable nature, but had never found the necessary condition of actuality ; what had been only possible now becomes real. But what is remarkable is, that this reality is only brought about by actual relations to other atoms, the atom itself does not change its nature ; it remains the same, and it is as the same that it enters into each new combination.

The concept of development is here applied to the relation between the constant force and the temporal manifestation of this force in one and the same atom, but in another sense it may be applied to a larger or smaller system of atoms, ultimately to the whole universe. The transformation of Kant's nebulous globe into the solar system, the breaking up of the masses into separate bodies, the processes upon the particular planets by which their surfaces take shape, etc., may all be described as development of an original total state, in so far as the states implied in the forces of the elements and in their spatial distributions become successively real. The whole development merely represents that collective sum of the successive motions of the particular atoms, which, under the given conditions, is grounded in their forces.

For the mechanical theory of life the development of every organic individual is of this kind. All changes, all chemical transformations and building up of form, proceed from the collocation of the different atoms in the germ and their relation to their surroundings ; each state thus developed leads according to the same laws to a new one, and the ground of the whole process lies in the forces and laws of motion of the particular invariable elements, just as from the present distribution and motion of the sun and the planets there proceeds by virtue of gravitation and inertia

their collocation in each subsequent moment. The cessation of the vital processes, the death of the organic individual, is the continuation of the same series,—is in the same sense the development of the preceding relations of its elements according to invariable laws.

Essentially different is that concept of development which is the first result of our reflection upon the organic world. The expression is primarily meant only to express descriptively the particular nature of what happens, not to offer an explanation, and this meaning is immediately connected with the etymology of the term and its synonyms.

The original meaning of the word implies that something previously folded up expands spatially, thus revealing an extension or plurality of parts which was not previously perceived. It is in this way that we unfold a rolled-up scroll, and that a bud unfolds when it breaks through the covering around it and spreads its petals out. What now appears was previously there, but it was so arranged as not to be visible, the parts were not separated by intervening spaces, and therefore not distinguishable. To this is added the next enlargement of the concept, which may be illustrated by the same process ; the particular petals do not merely unfold, they grow at the same time, they change their size and form, differences appear which were not there before. From a small bud there grows a whole bough, from a grain of corn a whole plant. Growth and development now belong together ; when we speak of the growth of plants, we do not mean a simple increase of volume, but also a differentiation, a genesis of new forms from the present, which are more complicated in shape and manifest new attributes and functions.

Primarily development is a general concept, by means of which a series of successive and continuous changes of this kind is expressed as having a nature of its own, but it also contains the thought that all the particular stages are referred to the same individual subject which develops, and, when we have occasion to enclose the series of changes within definite limits, the antithesis of the beginning and the final state of this subject. The beginning is that *from* which, the end that *towards* which, it develops ; and the more closely the processes resemble each other in similar subjects, the nearer we approach to finding laws of development in the sense of general descriptive propositions, which are true of individuals of the same species (§ 96, p. 366).

The next step is to the thought that the subject in its first state already contains the ground of the whole subsequent series, that in its particular nature

is contained the necessity of passing through these stages, that in virtue of this nature each stage produces the subsequent one ; that therefore the subject itself changes itself, and attains to new capacities of action. This involves also that only the end of the development reveals what the beginning contains ; it first contains as disposition what ultimately comes to pass through the successive activities. Thus the whole process is brought under the teleological point of view, which, starting from the final result, explains the preceding stages as conditions or means, through which the *τέλος* is realized, but is also forced to include in the concept of the subject the full ground of that which it becomes.

Those who, in dealing with the organic world, abstain from hypotheses concerning ultimate subjects, will begin by taking individuals as the unities in reference to which they formulate their concept of development, and the formula of the concept will contain the law of development of the particular species ; in this way the essence is described of individuals of the same species. From this point generalization leads us to more general laws and conceptual formulæ.

15. The concept of development extends further beyond particular individuals to the whole range of the organic universe, and thus gains a new meaning. We are told to find in the temporal sequence of organic forms a development, which is so far analogous to the development of the individual that it leads to a constantly progressive change of form, to a constantly greater differentiation ; and here again we come upon teleological consideration, in so far as the later forms are judged to be the more perfect, to represent a higher stage of organic life. The difficulty in applying this concept consists in describing the subject to which this universal development is to be ascribed.

Only in a logical sense we can attribute development to general concepts, such as the organic in general, which differentiates itself into genera and species, as Schelling and Hegel attempted to do in their philosophy of Nature ; when we regard development as an actual process, it requires a concrete subject existing in time. But the only subjects which actually exist are the particular organisms which succeed and produce each other ; hence this concept of development in its application to the totality of organisms requires as its necessary supplement that of propagation and hereditary transmission, in the sense that in the subsequent generation not only is the previous one repeated, to pass through the same course of development, but that the results of the development of the elder genera-

tion pass into the germs of the younger one in such a way that they are capable of making further progress. However we may think of it in detail, the conditions of that which the particular germ becomes are now contained in the whole history of its ancestors. Here again it is true, however, that the disposition to all which becomes actual in the successive generations of its descendants must be present in the nature of the first germ ; here again it is true that only the end of the development reveals what the beginning contains. Thus any earlier form, from which higher ones develop, is not exhausted in what it visibly presents ; it also has a latent significance by virtue of that which proceeds from it by way of heredity in later generations, in its own life it manifests only a part of its nature.

The thought of development has sometimes been treated like a logical charm by means of which we may explain without difficulty hitherto inexplicable phenomena ; all the puzzles of the wonderfully purposive complication of forms and functions in the higher organisms are to be solved by regarding them as the result of an incalculably long development, in which the forms have had time to change quite gradually by imperceptible degrees. Sometimes it is also implicitly thought that if this long series were broken up into as many and as small parts as possible it would be no longer necessary to seek a cause for such a minimum of progress as each part would represent—Darwin is helped by the general concept of variation, which he introduces simply as a fact. It is as if we should say, that though force is necessary to lift a weight a given height perpendicularly, yet if it is brought to an inclined surface, and this surface is made very long, so that over small intervals the weight will rise imperceptibly, it might be able to lift itself ; as if we should say that because the force may diminish in proportion as the time in which it is exerted increases, therefore the force may be dispensed with altogether if only we take a very long time. In opposition to such hazy applications of the concept of development, we must insist that the question as to the ultimate ground of the successive stages must be answered, whether the time be long or short, and the change great or small. If we trace the unfolding of a germ through all its stages, we get indeed a picture of its growth, but no explanation, unless we can show from general laws that the preceding state is the whole cause of the succeeding one. But it is just that which we cannot do ; if we could, the concept of development in its specific sense would be superfluous. This concept presupposes a

ground which must be entirely present at the beginning of the process, if it is to explain anything ; for even if we should try to refer the transformations which occur in the course of time to external causes, to the external conditions and mutual relations of individuals, these causes could only take effect if it were part of the nature of organic beings to respond to the stimulus by a change in form and function ; it is only the direction of the development which is determined from without ; that development takes place at all, and not merely a repetition of the same processes, must be due to the original disposition. The law of development does not determine everything even in the particular organic individual ; its complete expression would be that under these conditions this course takes place, under other conditions another course, that these causes act favourably, while others act unfavourably or destructively.

If the concept of development is to be not merely a general formula for the description of facts, if it is to be a ground for their explanation, then it requires a subject of such a nature that by its successive changes new conditions for further changes in a fixed series are continually introduced, and that this series leads ultimately to an end in which the original disposition is completely realized. It is not by itself, but by virtue of the permanent nature of the developing subject, that each particular stage is the cause of the next (§ 73, 26, p. 116).

If we regard the organic world merely in its material aspect, as is most natural in the vegetable kingdom, then the attempt to pass from the organic individuals which are the phenomenal subjects of development to the combining atoms as constituting the ultimate grounds, and to reduce the specific concept of development to the more general one, which is applicable also to mechanism, is not merely possible, but is justified methodologically. But for the ultimate subjects, in so far as they are invariable, there is no development in the proper and narrower meaning.

16. Turning to the last problem, that of determining the fundamental concepts to which psychical phenomena lead, we find that the nature of the facts leaves us no choice. In constructing our concepts here, we must start from our own consciousness, and can only apply what we find here to the animal world ; to start from an assumed elementary psychical life in the lower animals, to construct from it our own, and to lay down rules as to how we must regard it, would be to explain the known by the unknown, to illuminate light by darkness. Even if our consciousness had gradually developed from that of animals, it would be only this mature and

highest form which would show us what we have to presuppose as original disposition, and what significance attaches to the lower stages; our hypotheses as to the course of the development must be guided by what we find as a result in our own consciousness. If we had before us only the acorn, we could not deduce from its form and chemical constitution what it can and must become; only the oak tree reveals to us what is contained but cannot be perceived in the acorn; and in the same way, only human consciousness can give us the key to the lower stages, only from it can we obtain the fundamental concepts in which, as in ultimate presuppositions, the explanation of the given comes to an end.

We have already shown that the facts of consciousness require a subject which is a unity; the assumption that they may have no subject at all is as impossible as the assumption that a mere collection, a plurality of connected ultimate unities, can be thought of as their subject. The simplest and most elementary psychical event, which we do not hesitate to assume even in the lowest animal, a feeling of pain, contains the consciousness of a state, a reference of a being to itself; it is impossible to distribute this feeling amongst different unities,—each could then only feel its own pain, and for this reason it is also impossible to regard a feeling of pain, which for our consciousness is a single state of only intensive magnitude, as a collective sum of the feelings of a plurality. Such a plurality could only combine to the unity of one feeling if each element of the plurality shared in the pains of the other, and so increased its own pain. To regard a pain as a sum is as impossible as to represent the velocity of a body as the sum of smaller velocities of its parts.

If we must start from the unity of the subject, the next problem is to determine the nature of this unity in such a way as to show that the given facts are grounded in it, and to explain the sequence and connection of the functions which actually occur. For this purpose, as we showed in § 74, 17, p. 140, the concept of faculty is quite appropriate, for it gives expression to the thought which forces itself upon us everywhere, that the activities of the psychical subject are not merely changes due to external influences in some way proportioned to them, but that new elements come into being as through some creative power, for which external influences are only the occasional causes or stimuli, and that from preceding activities there proceed by virtue of the nature of the subject alone further and higher activities. If stress is laid upon this spontaneity, we shall determine the concept of faculty more closely by that of tendency, which represents

the faculty not merely as a passive force awaiting its stimulus, but as striving of itself towards activity : thus it is quite correct to speak of a faculty of feeling, and of a tendency to activity. All psychological laws will then express how the subject is in part excited from without by virtue of its receptivity, and in part develops new activities by virtue of its spontaneous tendencies.

The most difficult questions arise out of the problem of determining the relation of the psychical unity to the plurality of the elements which constitute the body. The history of psychology contains very different attempts in this direction. From the point of view of methodology no definite way can be prescribed for this problem, which is only to be solved by the most comprehensive combination ; all we can do is to guard against looking only to the dependence of the psychical life upon that organic life which is supposed to be more easily seen and comprehended. It is just as possible to begin by explaining the peculiarities of the latter by its connection with the psychical life, and Wundt deserves great credit for having emphasized this point of view. The closer the connection between the two aspects of man which is made comprehensible by our way of regarding the fundamental concepts, the more capable these concepts will be of expressing reality.

17. Comparative psychology, together with the doctrine of evolution, is confronted by the question whether any definite limited concept of man can be formulated in such a way as to enable us to assume that, so far as our historical knowledge extends, the essential concept of man is constant. It may be safely asserted that the empirically known facts not only allow, but require a definite demarcation from the animal world ; the intermediate terms required by the theory are only hypothetical. It is another question whether within these limits we must assume a development in the sense that we must think of the man of to-day, especially the man of culture, as furnished with faculties and tendencies which are at any rate quantitatively different from those of previous generations ; whether the nature of man changes by heredity, whether we must assume natural differences between men of to-day and the most ancient who are indicated by historical remains, which are greater than those which we now find to be due to individual peculiarities. We are tempted to attribute great importance to the inheritance of capabilities which are gained in the course of culture ; it will, however, be more prudent to employ as far as possible known and demonstrable grounds of explanation. The most important form of heredity to

which we can point in the psycho-historical sphere is tradition, and it will be methodologically right to employ so far as possible for the explanation of historical development this concept, which can be explained by general psychological laws ; in this way the mutual support of historical investigation, and of that psychological investigation which starts from present perception, may be extended over the whole sphere.

Finally, induction in its various forms, throughout psychology and historical investigation, always assumes determinism, simply because we cannot have a scientific explanation unless we can refer the particular phenomena of the psychical life to laws, partly evolutionary, partly causal, according to which certain phenomena of consciousness occur under certain conditions. But here again we must not be in a hurry to regard what is a regulative principle of investigation as the only possible result, especially when we recollect that the general presuppositions by which we are guided, and which are necessary to an inductive method as providing major premises, are themselves the expressions of a will, and derive their justification from the energy of this will, which sets before it as an end the knowledge of the universe. Our thought can neither really break down the unity of self-consciousness which forms its ultimate and irreversible presupposition, nor deny that independence of the will through which alone it is active. The only empirical confirmation of determinism would be the conceptual construction of individual peculiarities, and the finding of laws from which the will of every individual might be calculated.

VIII. METHODS AUXILIARY TO INDUCTION.

The Statistical Process, and the Calculation of Probabilities as Grounded upon it.

§ 101.

Where the differences between the individual objects falling under our concepts, and the impossibility of isolating the conditions of given predicates, make it impossible to establish laws, properly so called, the comparison of statistical numbers leads first to empirical uniformities. These uniformities, which are expressed in constantly recurring averages, refer partly to the differentiation of a general concept, partly to the distribution of things of a certain kind in space, or of events of a certain kind in time.

These uniformities are primarily of a merely descriptive nature, and incapable of expressing any necessity, unless the assumption is made that the conditions, from which in any department the particular varying cases proceed, are constant in their totality.

Causal connections can be inferred only by the method of difference, which, by comparing the partial averages in which cases of a distinguishing nature are grouped together with the general average, justifies us in assuming that the differences of the partial averages from the general average are conditioned through that factor by which the specially numerated cases are distinguished from the others.

The same considerations apply to the statistical methods of numeration which guided us in the description and characterizing comparison of collective wholes in their constitution and temporal changes.

1. The possibility of establishing strict uniformities among events, whether as empirical laws or as causal laws, and of explaining by them particular phenomena in their concrete determination, is limited to the region within which events depend upon conditions which can be accurately expressed in general concepts and separated from each other, either experimentally or at any rate by logical analysis; within which, moreover, it is possible to have a knowledge of all the facts influencing the course of events, or, if this is not the case, where the unknown facts are perfectly constant. But within wide regions we cannot attain to the logical ideal of strict laws and adequate explanation of phenomena; neither our knowledge of the facts upon which the phenomena depend, nor of the laws according to which they depend upon them, can be expressed in those logically and sharply determined propositions, which are to be found, *e.g.*, in physics and chemistry. In meteorology, for example, the vastness of the atmosphere and the inaccessibility of its higher levels prevent us from directly observing the whole of the circumstances to which change of weather is due, and every attempt to establish uniform sequences between known antecedents and consequents breaks down before the irregularity with which phenomena, wherever they may be observed, succeed each other. Throughout the whole of physiology the complication of conditions and the irreconcilable differences between individual living beings make it impossible at any rate to discover exactly all particular conditions and to establish certain relations of magnitude. We cannot determine what result will be produced by any interference in an organism with the confidence with which we can determine what will be the rate

of oscillation of a given pendulum, or at what angle a ray of light will be refracted when it passes from air into water.

2. These two examples suffice to show that we do not stand in quite the same position towards all instances of this kind. In meteorology we know a large number of particular connections upon which changes depend: the weight of the atmosphere, its rarefaction by heat and densification by cold, the aerostatic laws according to which the more rarefied air rises and the denser must occupy the space beneath it, the evaporation of water through heat and its condensation through cold, the different absorbing powers of the air at different temperatures, and so on; in this way we possess no doubt a large part of the elementary connections which concur in the meteorological processes, but we cannot construct the weather from them because it is impossible to obtain even their magnitudes apart from each other—we do not know, for example, whether the heat radiating from the sun to the earth is a constant or a variable magnitude, and if the latter, according to what law it varies—and because the constantly changing distribution of the operative elements and their relations to each other never allows us to ascertain the whole circle of conditions at any given point of time.

In physiology we are often still far from having any knowledge of even the elementary laws. We lack not only the magnitudes of the conditions of the vital phenomena and their combination at any time, but also the mathematically accurate rule according to which any change depends upon its conditions; and to establish such a rule is difficult if only because particular individuals are not exactly alike either in the totality of their conditions or in their causal relations, while their differences cannot be simply formulated. The whole complex of conditions is no doubt contained in the organic constitution of the living beings and their external circumstances; but according to what rule each particular circumstance varies in its effect for each different individual and in what way their results combine is unknown. No one knows, *e.g.*, what determines whether in any particular instance a boy or a girl will be born; in spite of millions of experiences the sex of every child remains an incalculable chance. We must assume that certain conditions are present which determine the sex of the foetus, and are justified in supposing that the proximate causes at least are entirely contained within the parental organisms; but what they consist in no induction has yet been able to discover.

Are we then left quite without resource in the face of such problems, obliged to wait for what may happen, and unable to obtain any approach to a general rule?

3. Scientific practice shows that even in these cases there are means of establishing definite empirical uniformities, and even causal connections; and where the analysis and comparison of particular instances gives us no result the last expedient lies in the statistical numeration of particular cases, and in the attempt to obtain from these by inference results which may serve as a substitute for the unattainable laws in the true sense.

We have explained in § 92, 5-8, pp. 284-7, the significance attaching to the numeration of particular things under certain categories, as a comprehensive representation of particular perceptions. With reference to particular objects at rest, it serves partly to describe the distribution in space of the things falling under a concept, partly to represent the numerical relations in which the differences of a general concept are realized, whether the differences are specific or merely quantitative; with reference to variable objects, it serves primarily as a description of the particular by stating the number of given events which take place in them, and their distribution in time. Numeration becomes especially important as a means to exact description of collective wholes, of their composition from different unities, and their change in the course of time.

In proportion as we are unable to reduce the particular event to rules and laws, the numeration of particular objects becomes the only means of obtaining comprehensive propositions about that which is for our knowledge fortuitous; as soon as the laws are found statistical numeration ceases to be of interest. There was some interest in counting how many eclipses of the moon and sun took place year by year, so long as they occurred unexpectedly and inexplicably; since the rule has been found according to which they occur and can be calculated for centuries past and to come, that interest has vanished. But we still count how many thunderstorms and hailstorms occur at a given place or within a given district, how many persons die, and how many bushels of fruit a given area produces, because we are not in a position to calculate these events from their conditions.

4. The next treatment of the results so obtained aims at representing the relations of the numbers in such a way that they may be surveyed as easily as possible, thus enabling us to compare different regions, whether of space and time, or of concepts.

Where we are dealing with the division of a genus into its species we have an easy means of making the numbers comprehensible, we state the relations of the numbers which have been found for the particular differences either in the smallest and most convenient numbers possible, or in percentages of the whole number or of the number of a part. When we say that so far as has been counted the number of male to female births is approximately as 17 to 16, or that of 1,000 born 515 are male, 485 female, or that for every 100 girls there are 106 boys, we have a more comprehensible relation than if we should compare millions with millions. The same reduction enables us to compare the spatial or temporal regions within which the numeration takes place ; in the one region, the ratio may be 100 to 105, in the other 100 to 107 ; we then see at once the greater preponderance of the male sex in the second.

When we are dealing with the distribution of a number of objects in space and time, we can only compare their density or frequency by reducing them to a common measure. The population of Europe and Asia cannot be compared until the magnitudes of the continents are taken into consideration and we learn in what relation the number of inhabitants stands to the number of square miles ; in the same way the amounts of rain which fall at different periods become comparable when they are reduced to equal intervals of time.

These things would be too simple and too much a matter of course to be dealt with here, if it were not that we have to show the importance of AVERAGES.

5. The arithmetical concept of an average is the quotient obtained by dividing the sum of a number of given numbers by their number, and arithmetic shows that the sum of the positive and negative differences between all these numbers and the average equals zero, *i.e.*, that of the numbers those which are smaller than the average are together just as much less as those which are greater are more. Thus the average serves as a measure for the magnitude of the numbers when we disregard their differences among themselves and think of them as supplementing each other. More especially when we are dealing, not with the magnitudes of the particular addenda, but with the whole result, the averages of two different groups, each containing as many numbers as the other, tell us the relation between their sums and fix the point about which their particular addenda vary with an equal rise and fall ; they afford further the means of comparing two groups consisting of unequal numbers of numbers with respect to the magnitude of their addenda.

In dealing with the average height of the adult male population of a country, it is the arithmetical mean of all the measurements which is taken; the sum of the differences by which the smaller men fall short of the average is equal to the sum of the differences by which the taller men surpass it. In comparing the peoples of two countries the average gives us at once a standard of magnitude, and although we disregard the differences between individuals, yet we know that the one people must contain more tall men than the other.

The mere statement of the average may then be supplemented by the statement of the limits within which the numbers vary, or further still by the number of individuals falling into the particular sections into which the interval between the limits is divided. Thus the statement of the average age of a people at a given time is supplemented by the statement of the maximum reached by some, and the statement of the absolute number, or the percentage of individuals who are below five years of age, between five and ten years, etc.; in this way we can survey both the magnitudes of the differences which are forthcoming, and the grouping of the individual cases above and below the average.

6. In its concrete application we must note an extended use of the term "average." According to the definition the *particular* numbers are given and the *sum* formed from them, but the relation may be reversed, and we may start from the *sum*, knowing only that it is formed from a certain number of addenda, without knowing exactly the magnitudes of the particular numbers and their differences; here, in place of these different addenda, of which the particular magnitudes are unknown, we take the average value as a standard of their magnitude and as a means of comparing one group with others. Thus from the sum and the number of terms we get the average value of the particular term, as it would appear if the values were equally distributed. If in 14 days I walk 280 miles, I shall average 20 miles daily; if in an hour I read 6,000 words, I shall read on the average 100 a minute; in this way we obtain a measure for the speed with which we read or travel.

7. We may apply this arithmetical concept of the average in two ways: we may state either the average relations between indivisible unities, or the average magnitudes of a class of objects.

In the former case we determine the numerical relation in which the representatives of the species within a genus stand to each other, or the relation in which the total number of a class of objects stands to

the magnitude of the space or time within which they have been observed; in the latter case we measure each one of a class of objects, and determine the mean value of all these measurements. It is in the former sense that we say that on the average 106 males are born to every 100 females; in the latter that we say that the average magnitude of the adult man is 168 centimetres, and the average length of the day twelve hours.

Our first use of the concept of the average in the former sense is in the reduction of instances which are distributed over a large area of space or time to a common measure; we start from the total number, think of it as equally distributed, *i.e.*, regard it as a sum of equal component sums, and thus obtain a measure for the relation between the number of instances and the extent of the space or time in which they are found. The statement in statistics of population that in Europe there is an average of 1,550 people to the square mile, or that on an average so many people are struck by lightning in the year in Germany, merely gives us a comprehensible expression for a relation between two numbers, of which one enumerates the frequency of the instances, while the other expresses a spatial or temporal magnitude. Uniform distribution is a mere fiction, but it is one which is harmless so long as there is no question as to what are the particular groups, and how they are distributed, of which the whole sum is composed. The fiction becomes necessary when it is impossible to localize the particular instances in space and time, or to find units which are not arbitrary. The population of a country cannot be assigned to particular square miles, as it would be an entirely arbitrary matter how these limits should be drawn; all we can do is to take as our smallest units certain districts for which there is no common measure. In the same way we cannot ascertain how much salt is used annually by each individual in the nation, although we may determine how much each one uses on an average.

It is similar with the numbers expressing the relative magnitudes of the differences which occur in a genus. We say that on an average 106 boys are born for 100 girls; this expresses the relation between the totals which have been established, but we do not trouble ourselves about the particular components of the total, about the families, parishes, etc. That this relation is equally distributed in space or time is a mere fiction; the expression is only intended to state the relation between the total numbers found throughout the whole sphere of observation; it is in the

same sense that we say there is one case of twins amongst 87 births; all differences between particular regions or times disappear, just as when it is ascertained that upon a given railway there has been a daily average of x passengers travelling y kilometres.

From this point of view, then, there is no mystery in the average as such; it is an abbreviation, and it has so much in common with the ordinary logical abstract concept that it neglects all differences, and we cannot tell from it how far the numbers from which it is obtained, or which it has to represent, may differ from each other. It is, however, inferior to the general concept in so far as the latter is a statement of what is the same in all the particular instances, while the average is a merely fictitious value which may never actually occur in any particular case, and which by itself does not even justify us in expecting that the majority of the particular instances in a region will approximate to it.

It is the same with average measurements; these again give only a general result, in which concrete instances with their differences have vanished, and which in itself is as much a matter of chance for us as the number and magnitude of the particular objects from which the average is drawn. The average of the morning temperatures in the month of May in a given year in any place is as much a fortuitous number as are its particular components; it becomes important as expressing the magnitude of a sum when we are interested, say in the total amount of heat present in the month. So far as concerns a large part of the vegetable world, it is much more important, for example, to know whether and how far the minima have fallen below 0, than whether the average was a degree higher or lower.

8. The calculation of averages serves primarily only to enable us to survey and to compare sum-totals; but the comparison leads to the possibility of establishing uniformities of a peculiar kind, which, while they are allied to the merely descriptive laws, differ from them in that they do not state what takes place, constantly or under certain conditions, in particular objects, but merely express the invariable numbers and numerical relations of objects or events of some kind within a conceptual, or spatial and temporal region.

In saying that the average temperature of any place is so many degrees, we do not mean to convey merely the fact that the sum of our observations of the temperature has shown this average for a number of years, but to state a constant attribute of this place, that is a *rule* according to which

this average will be found in the future also. This extension of the actual average to be the expression of a rule is based either upon the fact that the average of the particular years from which the observations were gathered varied little, hence that in spite of particular deviations, and in spite of the different distribution of heat over the particular months, the total annual result has remained almost constant, and hence that we may expect the same value to appear in the future also ; or, at any rate, it must be based upon the fact that longer partial periods give similar results, hence that the variations of particular years have always compensated each other. Where this is not the case, an average can merely express a fact, it cannot be the ground for any expectation.

We may, therefore, always assume a rule where the average of a larger whole is repeated exactly, or with only small deviations in the smaller divisions which constitute its parts.

If, e.g., the preponderance of male births varied greatly in different lands, and in some was converted to a preponderance of female births, then the average ratio drawn from our sphere of observation would have only the value of a memorandum ; we could not expect to find the same relation confirmed elsewhere, and by taking in new districts we might essentially alter the average. But if the facts are such that we find the same preponderance in the various districts with only small variations, then, when we have extended our observations over a sufficiently large area, we are justified in assuming that it will be found everywhere. We may formulate a general proposition which is at first purely empirical, but which is certainly approximately true, and its lower limits are those totals in which the relation shows itself clearly ; by a simple inductive inference we infer that the same relation will be found elsewhere.

We are limited to the statement of these uniform averages where the differences between individuals and the variability of the objects falling under a concept do not admit of a general law applying to each particular one. This is especially true in physiology, which tells us the average size of the individuals belonging to a species, the average relative proportions of their limbs, the average rate of breathing or of pulse, the average number of seeds produced by a species of plant, the average duration of the vital periods in an animal, and so on.

The average size or number is frequently taken as that which is a *normal* one, and the assumption contained in this expression is justified in the sense of its being what occurs ordinarily, and as a rule, when the small

deviations from the average are much more numerous than the large ; we speak of the normal weight of brain of an adult man because the extreme cases are rare, and the larger number are found to approximate to the mean, and we may also fix the limits within which the deviations are not as yet abnormal. We always tend, however, to introduce the teleological idea that the average size is really that which is intended by nature, and which is appropriate to the tasks of the individual, while the deviations are consequences of disturbances which have prevented her from realizing her idea exactly.

9. The inference that those average numbers which have shown an actual uniformity so far will recur in the future and be confirmed is, as with the merely descriptive laws (§ 96, 4, p. 363), only justified if we tacitly assume that the conditions to which the uniformity has hitherto been due do not vary, and that the totality of the conditions from which the particular instances with their variations have proceeded remains in some way the same.

But however often an average may have been confirmed, we can never attribute to it the importance of being by itself the expression of any necessity. Every result is necessary when its conditions are given ; every particular instance was necessary in so far as from the given conditions it could only be such and no other ; all individual determinations and differences in the particular cases, which were neglected by the average, were necessary ; the most extreme deviations were necessary, and it will also be necessary, if all the particular conditions recur in exactly the same way, that they should again have the same results, and that therefore the sum of these results will be the same.

But this necessity, to which the particular instances are due, is concealed from us if we can do no more than group them together and count them ; and because they all differ and each one varies without regard to any law, because the particular instances of a group showing a given average do not generally correspond in any way with the particular instances of another group showing the same average, we cannot even speak of the exact repetition of the same conditions. The extraordinary thing is that where the conditions differ so in the particular cases we should yet regularly find almost the same averages when we take larger numbers. All that we can infer from it is that the manner in which the conditions of the particular instances have so far acted necessitated that larger numbers should always yield the same total ; that these conditions

themselves and their combinations must always be present does not follow, and no ground for such a statement can be found in any of the presuppositions of our investigations. It is the same as with the alternation of day and night. However often the same alternation may recur, it does not follow that the rotation of the earth and its illumination by the sun must continue; only so long as the alternation continues does it prove that a cause has been at work to produce it and that no other cause has counteracted its action.

It is often forgotten that what is elsewhere proved by the recurrence of instances is something quite different from this. The ordinary inference is, that because *B* always follows *A*, therefore a *B* will always appear whenever an *A* is present; here it is assumed that because the same number of *B*'s always recur in the same periods of time, therefore it will continue to be so *ad infinitum*. And when we look for the two elements to be combined by the necessity, we arrive ultimately at the absurdity that space and time, or the number of the things within which we are counting, produce the given events, or that the one specification is the cause of the others. According to our mode of procedure elsewhere, we should have to infer from the proposition that as often as a year passes 3% of the population die that there is a causal connection between the passing of the year and death, and that time itself acts as a poison; from the proposition that as often as 100 girls are born 106 boys open their eyes to the light it would follow that the girls have the boys for followers; or we should have to infer that the suicide who hangs himself thereby compels another to shoot himself, in order that the received relation between the different kinds of death may not be disturbed.

10. Such uniformities of numbers and averages are primarily mere descriptions of facts, which need explanation as much as the uniformity of the alternation between day and night; and the explanation can be found only where the actual conditions, the efficient causes, are forthcoming. But these are the concrete conditions of the particular instances counted, they are not directly causes of the numbers; it is only the nature of the concrete causes which can show it to be necessary for the effects to appear in certain numbers and numerical relations. From the way in which the stamps work in the Mint we can see the necessity that one should produce a thousand shillings every day, and the other a thousand pennies, and that if both work at once the number of the two coins must be the same; but without knowing whether both continue to work we

cannot infer from the fact that one thousand pennies have been produced to-day that one thousand shillings have also been produced, because yesterday the number of the two sorts of coin was the same.

Thus the question is : How must the conditions of a number of instances be constituted, in order that, whatever irregularity there may be in the particular cases, the average of large numbers may yet show regularity in time, or within equal areas, or in the relations between the specifications of a general concept ?

11. We should have an answer to this question if we could make a probable assumption concerning the conditions upon which the phenomena depend which are numerated and measured in their relative frequency or varying magnitude. If all the objects numerated are ruled by a fixed number of conditions which are partly constant, partly variable, and upon which depend both their specific differences and their magnitude and the occurrence of various changes in them ; if the variable conditions are so constituted as to pass through a certain limited circle of values ; and if the effect of the variable conditions is both to increase and diminish in equal proportion that of the constant conditions ; then, when the variable conditions have run through the whole circle of their values, they will be neutralized in the sum of all the particular effects, and this sum will be the same as if only the constant conditions had acted ; the total effect will therefore be constant. Or if we had none but conditions which varied within certain limits, then if we assume again that they have passed through the whole circle of their values, the total result will be the same as if it had been the mean value of the variable conditions which was active in all the instances. This is true again when variable conditions, which do not annul each other in their effects, combine with constant conditions ; here also there will be a mean value of the variable conditions around which the variations neutralize each other, and this, when all the values have been exhausted, plays the part of a constant factor in the total of all the particular effects. So long as the same conditions persist the same averages must follow, and we shall be able to assume that even a single average drawn from a considerable number of instances may be the measure of the mean around which the conditions vary.

12. We may illustrate this first by the most simple cases. In the oscillation of a pendulum with a metallic rod all the conditions affecting it may be constant with the exception of the temperature ; as this rises the pendulum lengthens and its movement slackens, as it falls the movement

becomes more rapid. If we suppose that the temperature, starting at 10° , first rises 10° , then falls 20° , then rises 10° again, and that this happens uniformly in such a way that all the values up and down succeed each other with the same rapidity, and make their effects felt in an equal degree, then the retardations due to the higher temperature and the accelerations due to the lower will counterbalance each other; the average time of oscillation will correspond to the mean temperature of 10° .

Or to take the favourite example: when a coin is thrown it will fall either heads up or tails up; if it is always laid in the hand with the head up, whether it will afterwards show heads or tails will depend upon whether it turns an equal or an unequal number of times; but the number of times it will turn depends upon the strength with which it is thrown. Suppose this varies from the strength which makes it turn once to that which makes it turn ten times; when we have passed through all these values there will have been the same number of heads and tails. The same result would follow if either heads or tails should precede the throw indiscriminately; it would not, however, follow in the first series of throws, but only after a lengthened repetition, when we had exhausted all the combinations of variation in the original position with the stronger or weaker throws.¹

The result is not changed by varying at random the stronger and weaker throws, so long as no particular strength predominates and always tells in the same direction. But there is no ground for such a coincidence as this; and since we know that the one side is as likely to fall up as the other, and the strength of the throws varies without reference to the result, we are justified in expecting that gradually all the combinations will be realized an equal number of times. Because of the irregularity of variation some one combination will slightly predominate at any given number of throws; but the greater the number the less will this predominance affect the average; and therefore, the greater the number of throws,

¹ If we assume that the throws vary in strength regularly up and down the scale from 1-10, and that the coin is always picked up as it falls, then, if we begin with heads, we get: 1st throw, one revolution—tails; 2nd throw, two revolutions—tails; 3rd throw, three revolutions—heads; 4th throw, four revolutions—heads; 5th throw, five revolutions—tails; 6th throw, six revolutions—tails; 7th throw, seven revolutions—heads; 8th throw, eight revolutions—heads; 9th throw, nine revolutions—tails; 10th throw, ten revolutions—tails; that is six tails to four heads. But if we continue in the same way, beginning again with 1, then heads will fall first, and we shall get six times heads to four times tails; after the second series, therefore, there will have been ten times heads and ten times tails.

the nearer will the average approximate to that equality in the numbers of the two alternatives which is to be inferred from the conditions.

If we are dealing, not with successive events, but with co-existent things, we may illustrate the case by throwing a number of grains upon a chess board ; the various directions in which they fall, and their various amounts of motion after falling, must combine as easily with the black as with the white squares ; and if we could assume that no one direction and no one velocity in falling predominated, we should necessarily find the sum of those lying upon the black squares to be equal to the sum of those lying upon the white squares. For the same reason we should expect that on a chequered board of different colours we should find the number of grains lying on each colour to be proportional to the space occupied by the colour.

To express it in a general form : If a given number of a 's can combine with a given number of b 's and c 's, and no circumstances are present to favour the one combination rather than the others, then the combination ab will occur as often as the combination ac , if there are an equal number of b 's and c 's ; but if there are different numbers of b 's and c 's, then the combinations ab and ac will stand in the same numerical relation as b and c ; and the relation which we find between ab and ac , or between their necessary consequences β and γ , will enable us to infer the relation between b and c .

13. These deductions differ from the simple calculations of probabilities in that they make definite assumptions ; they do not merely operate with disjunctive judgments with reference to which we know no ground why the one alternative should take place rather than the other ; but with disjunctive judgments with reference to which we can assume that no such ground is present, and that the existing causes, when they have passed through all their values and realized all their combinations, must have the consequence which is calculated according to the theory of probabilities (cf. § 85. 8. p. 224).

14. Such hypothetical deductions enable us to *draw inferences from* the merely numerical results of counting and measurement, more especially of average relations and average magnitudes referring to the conditions of numerated phenomena ; but never, unless we are justified upon other grounds in making assumptions with reference to the causes upon which the phenomena in question depend.

The inferences which we make here are like all inductive inferences in

that they compare the given results of observation with the consequences which follow from certain hypothetical assumptions, and conclude that where they agree the assumptions are true.

On the other hand, there is an essential difference in the assumptions which can be tested in this way. The assumptions to be confirmed by those methods of discovering causal laws which we developed previously are hypothetical judgments saying: if A happens, B happens. The assumptions upon which is based the use of statistical results contain not only the law which makes B depend upon A , but also the actual constitution of A , the number or magnitude in which it takes effect; they have to find therefore two unknowns. If the law were known, we need only find the actual constitution of the causes, or if this were known, only their law; but such favourable instances are comparatively scarce. In the mere result of the numeration there is no distinction made between what is due to the necessity of the law and what is due to the mere actual constitution of the conditions, and these cannot be isolated and investigated singly.

15. We may start from the illustration we have already used. Observation gives us a certain numerical relation in the frequency of β and γ , as differences appearing in a common α . We assume that β is conditioned by a combination ab , γ by a combination ac ; we assume that a is equally likely to combine with b and c , both by its own nature and by the circumstances, and we then infer that b and c are present in the same ratio as β and γ . Here we have four assumptions: first a law according to which β is the consequence of ab , and γ of ac ; then a law according to which the nature of a together with other circumstances is equally conducive to the combination ab and ac ; then the presence of these circumstances; and finally the ratio in which b and c are present. Or take another case: we had assumed on the ground of certain conjectures that a combines with equal facility with b and c , and that there are the same number of b 's and c 's, and we expected that the consequences of ab and bc , β and γ , would occur the same number of times; observation, however, showed a preponderance of β 's over γ 's. From that it follows that our assumptions are not quite correct; either there is in addition to ab a further combination ax , which also has β for its consequence, or it is natural for a to combine more easily with b than with c , or the circumstances present are of such a kind as to be more conducive to the combination ab than to the combination ac , or there are more b 's than c 's within the range of our observation.

For instance, the preponderance of male over female births may be

due, according to the hypothesis which makes sex depend on the degree of maturity of the ovulum, to the surplus of the time in which conceptions yield boys over the time in which they yield girls, which would necessarily give rise to the preponderance of males if the conceptions were indifferently distributed in time. In this case the fact would be grounded upon a physiological law true of all instances without exception, in consequence of which a would combine more easily with b than with c . But the same result would follow if the differences concerned were attached merely to the individual, and a disposition of the female organism to develop the male sex should merely occur more frequently than the reverse, there would be more b 's than c 's forthcoming. Ultimately this frequency also must have its causes; but these again may consist in some special combination of facts, and the same general laws from which this proceeds would under other circumstances have given rise to the opposite result.

16. We could only succeed in inferring the reality of one or the other of these or other assumed possibilities if it were possible, within the statistical method of averages, to isolate the particular factors assumed.

If, that is, it were possible, in dealing with a large number of instances depending upon many unknown conditions, to select a part which is connected with a given circumstance, then the question whether this circumstance belongs to the conditions affecting the phenomenon may be determined by comparing the average of this part with the average of the whole. If there is any divergency, we must assume that the circumstance has some influence, and when we are dealing with average magnitudes we may even determine the amount of that influence. For if we take a sufficient number of cases, we may assume that the remaining unknown conditions with their variations compensate each other even for the specially selected part of the instances, and therefore if these conditions alone were active we might expect the same average; if it does not appear, that points to the presence of a condition not present in the remaining instances, and the fact that the circumstance in question is common to all these instances leads us to regard it as this condition.

By carrying out this investigation in different directions we may succeed in obtaining a number of partial results.

It is by these methods alone, for example, that we can establish with sufficient certainty the favourable or unfavourable action of medicines and methods of healing; there is so great a complication of unknown and subjectively varying causes upon which depends the recovery or death

of a patient suffering from any given illness that individual instances prove nothing, we can never ascertain how the particular agent has acted, whether it was indifferent, whether it acted favourably, or whether its unfavourable action was counteracted by other causes. But by taking the average of many cases in which the remaining causes are varied in every way we eliminate their differences.

But by means of such observations alone we can never directly prove a causal law, according to which a certain result would depend infallibly upon certain conditions.

In the example adduced a change in the average of mortality would leave it uncertain whether a given treatment acted in the same way with all individuals, or whether it acted favourably upon some only and not upon others, or whether it was even injurious to some. At the most we can only infer that it acted favourably upon the majority, and in order to ascertain whether certain contra-indications were present it would be necessary to draw new averages according to different symptoms. If, for example, it has been established that the treatment of typhus by reduction of the temperature diminishes the average mortality amongst typhus patients by so much per cent., then all we can infer with certainty is that in a considerable number of cases this treatment has had a favourable result, and has co-operated with other conditions of recovery. But if it should also be established that the average of a part, *e.g.*, of typhus patients suffering from affections of the lungs, showed on the contrary a greater mortality, then it would be proved that in these cases the conditions of an unfavourable issue were strengthened, and that it is due only to the smaller number of these cases that the total average was notwithstanding favourable.

From what point of view we are to take these partial averages we cannot learn from the numbers themselves, but only from particular observations or more general considerations; so that here again we see the hypothetical nature of the process by which we try to confirm or refute certain assumptions by comparing their consequences with the matter of observation.

17. It is only a special application of this method which we employ when we are dealing with periodically occurring, or periodically changing, influences, which we have to distinguish from among a number of other conditions.

The position of the barometer in a given locality passes from day to day, and from month to month, up and down through all possible variations, in

which we can at first find absolutely no rule. An average taken over a considerable time gives a mean value about which the particular positions rise and fall, and this sums up the result of the constant conditions together with the mean value of those which vary. The constancy of this mean value over long intervals of time enables us to infer that nothing new has occurred within the circle of these conditions.

But if we calculate the averages for the particular hours of the day over a considerable time, we find a periodical variation between two maxima and two minima with respect to the general average. This period cannot be recognised in the single day because it is mingled with other influences which occur irregularly or in different periods; it appears in the average over a longer time because here the other variable elements for each hour neutralize each other, and it points to a condition, or a complex of conditions, which periodically raise or depress the position of the barometer. That the period is daily points to the influence of the sun; that it does not stand in any simple relation to the position of the sun shows that the influence is not simple and direct, but is affected by concomitant conditions. But unless we had conjectured that the different positions of the sun, and the changes brought about by them, had some influence, we could not have thought of summing up the particular hours of the day apart from each other.

In a similar way the existence of an ebb and flow in the atmosphere has been proved. If we calculate the averages according to the periods which are marked by the culmination of the moon, we find a variation between a maximum and a minimum the times of which correspond with those of the upper (or lower) culmination of the moon and of the positions 90° removed. As these culminations pass within one lunar month through all hours of the day, the differences in the time of the day and the other variations are neutralized, and we obtain the amount of influence exerted by the moon.

Here, again, it was to be expected from general laws that ebb and flow did take place in the atmosphere, and we had to investigate whether it was perceptible. It was this expectation alone which caused us to take the average of the hours of culmination of the moon, and to compare it with the average of the times a quarter of a lunar day later; and here again the calculation served only to confirm a hypothesis which the mere sight of the numbers could not have produced.

That we must then accept the moon as the cause of these variations

was to be inferred from known relations. But even if no such inference had been possible, the evident coincidence of the periods of the one phenomenon with the periods of the other would have pointed to a connection, and enabled us to conjecture either the dependence of the one phenomenon upon the other, or their joint dependence upon a third factor (cf. § 96, 9, p. 367).

18. The inferences from statistical statements, which aim primarily at the description of collective wholes (§ 92, 7, p. 285),¹ follow essentially the same principles. Every collective whole is characterized by the absolute and relative number of the units of which it is composed; and here again the first process consists in reducing the numbers to comprehensible relations. This reduction allows, moreover, the comparison of different collections falling under the same concept, of which the simplest example is the quantitative analysis reduced to percentages of a mixture of different substances, such as a kind of soil, a mineral spring, etc. The first object in this is to ascertain the actual manner in which a whole is compounded from its different elements; and from this it may perhaps be inferred, if the actions of the particular components are known, what will be the nature of the total effect to be expected from a unity so constituted.

A large part of social statistics is concerned with this characterization of a whole by the numerical relations between its different components, and with the collective actions proceeding from them. The wholes which are here depicted are held together not only by the most complicated interaction between their parts, but also by unity in the end they have in view (§ 78, 12, p. 178); it is this which justifies us in not merely referring the activities of each particular member to the individual unit, but in regarding the sums of their activities as the actions of the collective unity. The enumeration of the whole population of a state at a given point of time, the relation between the sexes or ages at the same point of time, the relation between the different occupations due to the human will, between married and single, and so on; all natural and obtained commodities, the amount of commodities produced within a given period of time—all this expresses the special peculiarities of this particular whole, and any difference in these relations and values at once characterizes emphatically different communities.

In so far as these wholes continue and change in time, the successive

¹ Cf. with what follows Rümelin's *Reden und Aufsätze: über den Begriff eines sozialen Gesetzes*, p. 1 sq., and *Zur Theorie der Statistik*, p. 208 sq.

alterations in the numbers expressing their component parts and the totality of their actions represent the history of their changes, which deals not with the individual fates of particular units but with change in the relations of whole classes of similar parts. The increase and decrease of population, the change in the proportion of ages, of occupations, etc., the increase of poverty or wealth, the accelerated or diminished production of commodities, tell us the direction and amount of the changes which are undergone by the whole as such, and which are continually modifying the conditions under which takes place the interaction of the members amongst themselves and their collective influence outwards.

The only ground which determines us in drawing the limits within which the number of births, deaths and marriages are registered, lies in the fact of belonging to such a whole. When we calculate the ratio of the number of births in a year to the total number, or of children to adults, for Germany, France, England, etc., our reason for making just this selection of individual cases is our desire to characterize the conditions of the nations as wholes.

19. But these same numbers, which were originally intended to subserve a purely historical description, afford upon closer consideration the most surprising revelations as to laws to which the life of the whole is subjected,—laws which are concealed from us when we regard the facts of the particular members in isolation, but which become obvious as soon as we compare the statistical results which register from year to year the whole of certain events occurring within the community. The percentage of deaths and of births within a large population remains approximately constant from year to year, as also the number of marriages and divorces; the number of crimes of certain kinds, of suicide, etc., shows an alarming uniformity. It is no wonder that in the first surprise it was thought that the great natural laws of society had been discovered to which the individual is subjected without knowing it; and that these numbers were regarded as proof of an inexorable necessity which year by year not only delivers over to an inevitable death its victims from all classes of society and ages, but also constrains a predestined number to commit murder or to slay themselves.

However worthy of consideration this uniformity of numbers may be—we do not aim at dealing exhaustively with the subject here—still more careful examination will show, that to infer the existence of a law holding sway over the particular members of a community and concerned only with

completing the number, no matter by whom, is premature. If even the uniform repetition of the same event in one and the same subject cannot, by itself, express any necessity, still less can the mere constancy of numbers in which the instances included refer to constantly changing subjects, and are indeed instances which agree only in general categories, and while abstraction is made from their concrete individual differences.

20. The constancy, or more accurately the approximate constancy, of these statistical numbers is primarily no more than a fact which stands in as much need of explanation as any particular event which is merely described. But explanation can only be satisfactory when it explains that which really happens, when it can state the causes from which proceed the individual cases included in the enumeration, and can deduce from the nature of these causes that their effects in a section of the members of a community are equally distributed in time.

These uniformly recurring numbers after all express no more than the fact that, *e.g.*, the many causes of death to which sooner or later every living being is subject act sometimes quickly, sometimes slowly, in such a way that within a large population about the same number of men die every year. This result of a uniform distribution in time is just what is to be expected when a large number of independent variable causes, acting according to the most various laws, exercise their influence upon a multitude of objects which remains the same in number and composition. Suppose that of fifty million men living unaffected by change for fifty years each should write letters or take railway journeys from mere caprice, and not according to any fixed occasion or rule, then just because of the complete absence of regularity it would be most probable that in every year there would be approximately the same number of letters written and of journeys made. The fact that everything went by chance, and nothing favoured any special time, would lead us to expect in a large average the indifferent distribution of these activities over the time taken, and if a cause were to be sought, it would be rather for the accumulation of such events in any given period. Or, to put it in a different way, suppose it were determined that every one died before he was 100 years old, but a mere matter of chance whether he died in the first, second, third, or ninety-ninth year, then if we take a very large number, it will again be most probable that the cases of deaths of new-born infants would be so distributed over the ninety-nine years that the same number would occur in each. Thus there is little ground for inferring directly that, because the number

of instances which occur in perpetually different persons is constant, there are certain constant and invariable causes acting upon the whole community, and effecting these uniform percentages.

21. If, nevertheless, we are forced to give some further significance to the figures, we shall find it in the fact that we have to regard the particular cases enumerated as the necessary consequences of certain causes, and that we have special knowledge concerning these causes themselves derived from the observation of the particular concrete instances, and are thus in a position to form hypotheses concerning the ground of uniformities which by themselves point to no cause and no necessity. From the consideration of concrete instances we learn that every generation is composed of individuals who have different power of resistance, that the general conditions of life and widely spread dangers, as well as the various fates of individuals, tend to shorten life, whilst other conditions help us to guard against those dangers. The uniformity of the numbers enables us to infer that the relations upon which depends the duration of life for the particular individuals remain as constant as the proportion between weak and strong constitutions, but from the constancy of the numbers we can learn nothing about these relations in detail.

22. When it is possible to infer from statistics to causal laws, the ground lies not in the constancy of the numbers, but, on the contrary, in their deviations. When the annual number of deaths increases or diminishes in proportion to the population, we are justified in looking for the explanation of the difference in corresponding changes. These will most probably be such as affect a large number of individuals together, *e.g.* the increase or absence of epidemics and favourable or unfavourable states of weather; they will not consist in the improbable coincidence of isolated and individual circumstances.

Thus it is the application of the method of difference to averages which leads to the explanation; and it leads especially to explanation when it proceeds by breaking up the whole number into special groups of a specific nature, and comparing the averages resulting from these groups with the total average. When the mortality of one month deviates regularly from the year's average, this suggests to us that that wherein this month is distinguished from the others intensifies the conditions favourable or unfavourable to the duration of life. When the number of births in a year rises above the average, we shall not assume that the difference is grounded upon a large number of individual and indeterminable causes, all working

in one direction, but that some far-reaching influence has been acting upon many at once ; and if a comparison with other numbers shows that the number of births varies inversely as the price of corn in the preceding year, then, although it is not proved that the lower price of corn is the direct cause of more births, yet we know from psychological and physiological considerations in particular instances that deficient nourishment tends to limit production, and statistical comparison shows us the amount of influence exercised by this condition in combination with those which are constant and those which vary in the ordinary manner.

So it is throughout. Where correspondences appear between variations among different sets of facts we may conjecture that some connection exists ; but to prove the presence of a causal connection from the mere numbers is only possible by means of a most comprehensive observation of many wholes in which the same events are repeated, while knowledge from other sources of actual causal connections, which are true for the particular individual, frequently enables us to decide at once that the correspondence between the variations is based upon a causal connection. When we find that an increase in the proportion of married to single people, or a fall in the average age of those who marry, is accompanied by a fall in the percentage of illegitimate as compared with legitimate children, we do not hesitate to say there is a causal connection, not because it is revealed by the numbers, but because we expected it beforehand ; if the numbers had been contrary to our expectation, we should have looked for other causes to have counteracted those known to us. All that statistics corroborate in such cases is that causes which we know in some other way have taken effect, and have not been checked by others, and they afford a measure for the relation between their efficacy and that of all the others. There has seldom been a more senseless statement propounded than that it has been proved by statistics that marriages are not based, according to the ordinary belief, upon individual inclination, etc., but are regulated by a law which, regardless of the heads and hearts of individuals, makes marriages depend upon the price of corn.

23. If the variations of averages justify us in inferring to general and wide-spread circumstances, we have also a right to regard their constancy, not as the result of the random combinations of a number of independent conditions, but as the expression of constancy in general conditions which, acting in varying combinations upon many individuals, have these uniformities for their result. We shall then be able to interpret the fact that the

number of crimes of a given kind remains much the same, as signifying that men from generation to generation are constituted in the same way as regards their inclinations and temperaments, and that the temptations to crime due to the social relations remain as numerous, while, on the other hand, legislature, morality, and religious conviction continue to counteract criminal inclinations with the same force. In other words, we accept that interpretation of the numbers which enables us to explain their constancy by the simplest assumptions.

But even these assumptions can only be confirmed by breaking up the total averages according to different points of view, and in our numeration combining so far as possible what is similar and distinguishing what is dissimilar. When we analyse the number of criminals according to certain points of view, and compare the numbers so obtained with the general averages, we find that in the special modifications of human nature are contained certain constant conditions which lead to violation of the law. When we find, *e.g.*, that by far the greater number of criminals are men, while the number of men and women in the whole population is almost equal, then it is evident that the nature and position of the men contains conditions favourable to crime, or is wanting in checks; when we find the number of young people concerned in offences against the person is more than proportional to the total number of people of that age, then it follows that the tendency to assaults diminishes with age.

In this way a wide field is opened for inferences in which we attempt on the one hand to connect together permanent relations, on the other to ascertain the effects of changes.

Exactly the same method by which we distinguished the influence of the moon upon the position of the barometer from positions which were irregular in their particular variations, enables us to say of every partial average which is taken from some particular point of view, that, by reason of its deviation from the total average, whatever is common to the selected instances stands in some direct or indirect relation to the magnitude of the result which is measured by the average. It is thus that we ascertain the influence of certain occupations upon the duration of life, the influence of town and country upon infant mortality, the influence of change in legislation upon production and trade, etc.

We cannot, of course, even by comparing different communities, construct really causal laws which would determine every particular case. All we can infer is that in a larger or smaller number of the instances enume-

rated the factors present throughout are modified in a given direction by another one. Any formulation of real laws can refer only to the activities of the efficient units, of individuals, and must be based upon psychology. A real *explanation* of social phenomena (p. 445 sq.) must start from the laws according to which human nature develops either generally or in its individual differentiations, and is influenced in its developments by natural conditions and social intercourse. The collective results which we obtain from statistics by neglecting individual differences appear as the sum-totals of the particular activities concerned, and in their turn afford a measure for the average energy of the particular factors.

24. The same reasons which forbid us to regard statistical numbers as the expression of a necessity dominating the particular instances enumerated also prevent us from drawing any argument from moral statistics for psychological determinism, or against the assumption of a real freedom of the will. No one has seriously maintained that if a man is free he must therefore act quite disconnectedly, and be able at any moment to resolve to do anything whatever. It is a matter of course that his inducements to certain actions are due to his natural inclinations together with external relations, his temptations to certain crimes to his individual nature, to his social position, and to the circumstances which excite his emotions; the question is whether what is thus imposed upon him is a disjunction of different possibilities between which he has to decide, or a completely determining cause. If the former were the case, then even if he should approach the omission or commission of a suggested action in a state of complete indeterminateness, and his decision should be purely fortuitous so far as concerned the complex of circumstances, the probabilities would lead us to expect that the decision in a large number of instances would be as often for as against; in this case the constant numbers would only show that the temptations to crime recur with approximately the same frequency.

§ 102.

In inferences from statistical uniformities to particular instances, deduction takes the form of the CALCULATION OF PROBABILITIES.

It is *justified* so far as it can be assumed that the same, or at any rate equivalent, combinations of causes affect every particular case, *unjustified* where we cannot assume that every particular case falls under some disjunct specification of a predicate belonging to all.

1. The deductive inferences which can be made from statistical uniformities to particular instances necessarily assume the character of a calculation of probabilities, based upon the empirically determined numerical relations of the disjunct instances (cf. § 85, 11, p. 227 sq.).

When these instances cover a sufficiently large range, they admit of the interpretation that the constant and variable causes active within that range combine in such a way that, assuming them to continue unaltered, the effects already observed will continue to be produced in the same relations. In this way we obtain a basis for the expectation of the relative frequency of different cases, and find the fraction of probability which measures the expectation of a given determination of the particular instance; when, that is, we are forced in the particular instance to have recourse to the indefinite knowledge of the totality of the possibly active conditions, and do not know the individual causes.

The purely logical disjunction that every human being who is born is either male or female, together with our complete ignorance of the conditions by which the sex is determined in the particular instance, would lead us to estimate the probability for either sex in the particular instance at $\frac{1}{2}$. But statistical enumeration shows that the conditions actually forthcoming do not give rise to an equal number of male and female births; the assumption of equal possibility is corrected, and in consequence of our experience we shall take as the basis of the calculation of probability the fact that of 33 combinations of causes 16 will result in a female birth, 17 in a male, and shall estimate the probabilities at $\frac{1}{3}\frac{1}{3}$ and $\frac{1}{3}\frac{2}{3}$.

2. For the particular case, nevertheless, this probability remains a purely subjective one, and affected by our ignorance of the actually determining grounds; we are not in a position to say that all the active causes are affecting any one case, and the varying combination favours the one issue more than the other. According to the registers of death, the probability that a new-born child will die in the first year is $\frac{1}{3}$, that it will reach the second year $\frac{2}{3}$; but this calculation is not based upon any knowledge of the collective conditions upon which the life of this particular child depends; it is not supposed that this one individual is exposed to all the active and opposed influences which favour or injure life, and that all depends upon whether the friendly or the hostile forces get the upper hand in the arena of combinations; many conditions which in other cases bring about death or protect life have nothing to do with the particular life. To this extent the distribution of the probability taken from the

general average amongst individuals is a pure fiction ; the calculation of the really existing chances would generally give another result, but we are unable to discriminate these. The probability expressed in the fractions represents reality only in the total numbers ; if a large number of individuals were subject to the same circle of conditions in an analogous distribution, the total result would conform to the probability : we shall find that next year in the same large population about a third of the new-born infants will die before completing their first year.

3. The distribution of the total relation amongst individuals in the form of their individual chances would be more justifiable in proportion as such averages were confirmed over smaller ranges. We could then assume that the results depend upon conditions which either are alike everywhere, and affect every one equally, or at any rate, though they may be different for different individuals, are equivalent to each other, the absence of one danger or advantage being compensated for by the presence of another, so that each individual really is affected by conditions of which the import is expressed in the fraction representing the probability.

If, for example, the *classification of deaths* which is calculated from a population of millions, and according to which from every 10,000 born in a year so many die in the first year, so many before the fifth, and so many before the tenth—if this classification is confirmed in a small area, a small parish, or any small number included in one enumeration, it does not follow that there is some natural law inexorably demanding the numbers ; but it does follow that the conditions contained in the relation between the various bodily constitution and the external influences, the manner of life, occupation, etc., however manifold they may be on the whole, yet concur for smaller sections to form combinations having the same result, and that favourable and unfavourable conditions are mingled in the same proportion even over smaller areas.

4. Under such conditions we may allow that there is some ground for the distribution of probability amongst individuals, but it is meaningless where we have not for a basis some predicate belonging to all individuals, and having differences which can be developed into a disjunction. Because all men die, but at very different ages, the disjunction is that every one will die in the first or second, etc., year, and the probability can be stated for each year. But to calculate for the individual the probability that he will be a railway porter or a millionaire, because within a given population we always find so many per cent. railway porters and so many per hundred

thousand millionaires, would be as unreasonable as to calculate the probability that he will next year commit a murder or a theft. The general element which must appear in some special determination is wanting, and we know that the conditions which lead to crime are unequally distributed, and cannot be divided out amongst individuals; here again statistics might give us very useful clues towards their investigation, but ultimately it is only the analysis of the particular and the application of the inductive methods upon which we can rely for the discovery of real laws.

CHAPTER VI.

SYSTEMATIZATION IN THE DEDUCTIVE AND CLASSIFICATORY FORMS.

§ 103.

IT is the work of systematization to represent all the knowledge attained to at any given time as a whole of which the parts are all connected in logical relations.

There are two forms of it, according as the relation determining the arrangement is that of propositions or that of concepts. The former is SYSTEMATIC DEDUCTION, the latter SYSTEMATIC CLASSIFICATION. In the former classification is a subsidiary operation, in the latter deduction.

Classification takes the shape of a logical division of concepts, which by a process of determination by opposed characteristics proceeds from a highest concept down to the lowest species, as the fully determined concepts which are accepted upon the ground of perception as exhaustively expressing reality.

The usefulness of a classification is determined by two points: in the first place, it must give expression to the natural relationship of things, and, in the second place, it must enable us to subsume the particular with ease and certainty.

Where there is a difficulty in forming the lowest species themselves, as in the organic world, we must either look for a certain criterion for specific differences, or where this is not to be found select certain forms as types round which the proximate groups range themselves. The selection of these groups is guided, even under the Darwinian theory, by teleological considerations; the application of statistical methods to determine normal types by averages is itself governed by the idea of the end, as well as the arrangement of the classification of the organic world in the form of a gradual evolution.

1. In the induction of general propositions from particular perceptions we were obliged to assume provisional concepts, by means of which alone

we were able to obtain both the specific laws and their generalizations (§ 97). The inductive processes served on the one hand to confirm and enrich the assumed concepts, on the other hand to correct them when it was found that their attributes were not necessarily connected according to the original assumption. In so far as the inductions were successful they led to the establishment of laws of evolution and causation, and so to the determination of the essential concepts of substances, or where this was not possible, to the formulation of the laws according to which phenomenal unities are related under certain conditions.

These inductive processes start necessarily from particular points upon which the preliminary hypotheses are grounded, describing gradually widening circles and including an increasing manifold of phenomena, partly by subsuming them under known propositions, partly by combining them to form new laws. The result of extending the inductive methods over wider fields consists on the one hand in the increasing number of specific concepts and specific laws which follow, on the other hand in the progressive comprehension of these concepts and laws in generic propositions, in which general predicates are made to depend upon characteristics which are common to a large number of different things.

When the totality of knowledge thus obtained is regarded at any given time as relatively complete, and perception has covered the universe accessible as completely as its present limitations will allow, then the need arises of surveying the whole, of arranging the results of knowledge in a comprehensive inventory, of representing them as parts of a comprehensive whole, and of expressing the relation of the parts to the whole by means of logical relations. Such an arrangement of our knowledge into a whole is called a *SYSTEM*.

2. This logical arrangement may be guided, and the form of the whole determined, from two points of view: by taking as its fundamental form either the logical relation of propositions which is expressed by combining them syllogistically in the forms of deduction, or the logical relation of concepts which is represented in division.

3. The first arrangement aims at explaining specific propositions from the smallest possible number of first principles, at representing them as their simple or composite consequences. The specification of concepts subserves the development of general propositions by enabling us to construct the various minor propositions which are combined with the major propositions to produce the conclusions; and the whole complex

of propositions included in this arrangement assumes the character of a deductive science, with the difference that its first principles are not axioms, but hypotheses obtained and confirmed by induction. We have such an arrangement in mechanics, the propositions of which deal with the most general properties of bodies ; it does not need to descend into the whole manifold of the given, inasmuch as it deals only with events which take place in a similar manner in bodies differing in many other respects ; it lays down propositions which are true in the same way of all fluids, all gases, etc., and represents them as the consequences of general presuppositions. That this or that phenomenon falls under these laws is a matter for subsumption in dealing with the particular ; it is not necessary to the completeness of the arrangement of the whole, which need go no further than the predicates are specified.

All such propositions are by nature hypothetical ; they state what under certain conditions follows according to law ; they do not directly state *that* the conditions take place, or that all possible variations in the conditions are realized. The mechanical theory of gases disregards their chemical differences in so far as they do not affect its special province by giving rise to differences of specific gravity ; it is no part of its task to enumerate how many sorts of gases there are : it is enough to say that if a body is a gas it conforms to certain laws of compressibility, of expansion by heat, or of capacity for heat, etc.

For this reason this arrangement of knowledge finds its limits where the specific laws cannot be derived from more general laws, and where the modification of a general predicate cannot be represented as the consequence of a modification in the concept of the subject ; from that point it must pass into an empirical enumeration. In the theory of heat we may be able to establish the general proposition that heat expands bodies and alters their aggregate state, and to derive therefrom a series of consequences ; but we are unable to deduce the coefficients of expansion of the particular substances, or their melting and boiling points, in such a way that they may appear as the necessary consequences of certain conceptually determined differences. In optics, similarly, we are able to express the laws of refraction in general formulæ, and from the constancy of the quotient which obtains between the sinus of the angles of incidence and of refraction to deduce a number of specific phenomena ; but we are unable to find any invariable general formula by which to determine whether, in passing from one medium to another, the ray is

broken towards or away from the perpendicular, for difference in density is not invariably a guide. Even if it were a rule that in passing from a medium of less density to one of greater density the ray of light is broken towards the perpendicular, there is still no possibility of deducing the magnitude of the coefficients of refraction from any general differences, and of stating the standard by which it may be determined for any two substances. Here again deduction comes to an end in the enumeration of empirically given constants.

4. That arrangement of the whole of our knowledge which takes the form of a division of concepts—*i.e.*, of systematic classification—starts from another point of view. While the deductive arrangement is of a hypothetical nature, this starts from the given, from the empirically real, and moves in categorical propositions which always assume the existence of their subjects; it generally takes the form of divisive judgments, stating that a general concept which comprehends a given plurality of distinct realities falls into such and such specific differences, or includes such and such species. The definitions which include both general and special concepts are no longer mere verbal explanations of logical concepts, for which we have to find an application; they express the concept of that which is empirically realized, and for this reason they extend no further than their application to the given is certain, or at any rate probable.

5. We must here distinguish between a wider and a narrower sense in which we may speak of classification. In the wider sense every arrangement in the form of conceptual division, of all those concepts expressing the given which fall under one general concept and may be regarded as exhausting its empirical extension, is called classification; in this sense we speak of a classification of visible colours, of audible notes, of shapes of leaves, of illnesses, etc. The assumption is, that the empirical extension of whatever falls under the general concept is determined by it, and the problem is to arrange the manifold of whatever it includes in the form of a progressive division in such a way that all the differences which go to make up the concrete determination of the particular are as fully recognized as the partial resemblances; the problem is solved by producing a system of divisions which combines together in its lowest specific concepts that which is similar in most characteristics, and distributes that which differs most in opposite highest classes.

6. Over against this classification of predicative determinations we have classification in the narrower sense, the classification of substances according

to their essential concepts, or at any rate the classification of those unities which, though they may not satisfy all that is demanded by the concept of substance, must nevertheless serve us as subjects of all judgments of perception, as the *things* to which we refer our predicates and of which we speak in our laws.

Since all judgments of perception dealing with the particular and all laws obtained by deduction ultimately refer to such unities, and serve to determine their concepts, it becomes incumbent upon us to set out the whole of what we know of the universe in an exhaustive statement and arrangement of the essential concepts of things. This classification of the totality contained in the universe would be, if we imagine it complete, the final and perfect result of all empirical investigation, the conclusion of all the processes we have been considering, the all-embracing completion and logical perfection of knowledge. Since all events of which the necessity is contained in our laws must be explained by the nature of the existent, any deductive form of science which aims at being the expression of the actual universe must presuppose a knowledge of the actual constitution of the universe, in a form which connects the hypothetical necessity of laws with the existence of fixed forms; it is a one-sided view to regard the deductively systematic forms as in themselves higher and more excellent than the classificatory. The latter do not exclude, but include, the knowledge and investigation of laws, while they also add richness of content to the general formulæ from which no one has ever succeeded in deducing the actual in its concrete constitution.¹ On the other hand, the laws obtained by induction cannot be regarded as well grounded and invariable until we have tested them by employing them to guide us in carrying out a classification in all directions.

7. The assumption involved in such an all-embracing classificatory system is, that the universe of things given to us in perception must be capable of being arranged in a comprehensible logical division, in which we proceed from one most general and highest concept, by way of determination by opposite characteristics. This comprehensibility would be attained if the more general concepts themselves contained the ground of specification, if the grounds of division were not only introduced from without, but were already contained in their definitions (cf. § 43, 5. I. p. 280). In the merely logical treatment of the concept, which we dealt with in Part II., there is the general possibility of descending by different deter-

¹ Cf. Schleiermacher's *Dialektik*, § 197.

minations from one higher concept to lower ones, and of ascending from one lower concept in different directions to higher ones; whether the arrangement of essential concepts allows us the same freedom, or whether their higher concepts can be constructed in only one way, we must learn from a consideration of their content; it is not absolutely necessary that there should be only one correct arrangement even here.

8. From what we have said in § 77, 6 (p. 163), and § 94, 5 sq. (p. 320), it follows that the complete expressions of the essential concepts of things must include their causal relations; these must take the form of laws, according to which invariable things act under certain conditions by virtue of their mutual forces, or the development proceeding from the nature of the things themselves is modified by their relation to other things. Because of their hypothetical nature, these laws can only tell us that a subject assumes a certain determination *if* it stands in certain relations to other things; whether this or that relation actually exists depends, not upon the concept of the thing, but upon the actual constitution of the universe, in which there is never realized more than a part of these hypothetical relations. It is this which always, even when we think of our concepts as complete, distinguishes the logical division which describes the extension of the possible from the empirical division which deals with those determinations only which are conditioned by the actually existing circumstances; otherwise we must assume that from the essential concepts of things there follows with necessity the actual number of individuals falling under the concept, and their distribution in space, hence that nothing is possible but the actual. This has been affirmed in the philosophy of Spinoza and of Hegel, but neither the one nor the other has been able to carry it out. According to Spinoza, the existence of the particular remains unexplained by the essential concepts of attributes, inasmuch as it is always determined by other particulars in an endless regression; and Hegel leaves a wide field open to empirical chance in that his method aims at deducing according to logical principles only the system of concepts, and not the particular manner in which they are realized. So far, therefore, as concerns the presuppositions of method, the possibility of the distinction between logical and empirical division remains open to us.

9. The ideal of every classification of all the manifold things of the universe is therefore an arrangement of the concepts expressing their essence in the form of a deductive analytical development (§ 79, 5, p. 190)

from one highest concept, or in the form of a deduction showing what specifications of general concepts must become actual under the given relations; and in order that such a development should be possible it is necessary that the highest concept, that of the existent or of substance, should contain the ground of a differentiation. Leibnitz indeed, with his peculiar logical acuteness, constructed the most general concept of substance in such a way that it should contain in itself the possibility of difference in relations of action and passion and in the degrees of perception; he saw that further development is not possible from an absolutely simple attribute, but only from a determination which contains a plurality in a unity, and that any division which should start from an absolutely simple concept would have to introduce differences from without. But not even Leibnitz in his theory of monads has succeeded in actually carrying out the classification of the given in the form of pure conceptual development down to the concrete.

10. The state of our actual knowledge prevents us from regarding this ideal stage as attained, and the attempts of speculative natural philosophy to deduce from the general concept of the real, or of the otherness of the idea, say nitrogen and oxygen, hydrogen and carbon, have been rightly ridiculed. Although we must not hesitate to maintain the logical rule that specific concepts, which present themselves as the species of a genus, are not to be simply accepted, but that we are to look for the ground which combines the attributes of the genus now with one specific difference and now with another, yet we must not delude ourselves into thinking that the given means suffice to the fulfilment of this ideal claim, and that we have obtained in any province a final knowledge of the essential concepts and of the grounds of specification contained in them, such that we could base upon it a complete deductive development of concepts.

Where the establishment of *infimæ species* has been most successful,—*i.e.*, in chemistry,—the concepts of the simple elements cannot be represented as specifications of general concepts in such a way that we can formulate a law according to which variation in one attribute gives rise to variation in others, and which constructs the different concepts of the particular elements, as the general equation of a curve of the second degree constructs circle, ellipse, parabola, and hyperbola, by variation in the relation of the constant values; we are confronted, on the contrary, by a plurality which is quite beyond our grasp. Nor has the Darwinian theory of evolution as yet fulfilled its promises of revealing the laws

according to which variations must follow from any special stock, let alone the common stock of all organisms ; in this direction the theory has done much more to raise problems—no doubt fruitful ones—than to attain certain results which would show that internal interaction of all parts of an organic form, according to which the variation of one part causally conditions the variation of others, and the necessity whereby certain variations should follow certain external conditions. The confidence of the statement that it is so is not yet supported by the convincing proof of the laws according to which it is so.

11. Matters being so, we are driven in our methodical classification to find a substitute for that complete understanding of the development of essential concepts which is as yet beyond our reach, and, while keeping the final end in view, to begin by aiming at the most practical logical arrangement which is attainable with the means at our disposal.

From this point of view we must relinquish the attempt to include all concepts in one system designed upon one plan. This is especially opposed by the different meanings in which we form and determine the concepts of things. In § 78 we saw how our concepts of things are grounded upon different forms of unity ; the concepts of substances and the concepts of individual forms cannot be brought within one framework of complete classification because of the different way in which the attributes are synthesized. But it is not only by this consideration that different spheres are distinguished within which we may hope to carry out a classification. Within the sphere of individual forms itself, we find departments marked off in which a different meaning attaches to the individual forms, the departments of the organic and the inorganic ; and yet another break separates the totality of the beings which we regard as individual subjects of psychical activities from externally perceptible things. So long as we have no clear insight into the relation between, *e.g.*, the concepts of organic individuals and the nature of the substances from which their forms are built up, so long as the relations between psychical and organic activities remain obscure, there is nothing for it but to hold these spheres apart, although the philosophical systematization of the universe will always make hypothetical attempts to bring them into comprehensible relations. It is fortunate that on the whole these different provinces of objects which are to be logically arranged can be distinguished easily and with certainty.

12. The other consideration by which we are guided in methods of

systematic classification is connected with the fact that the things which are given to perception are completely determined and concrete, that our most certain knowledge refers to the nature and behaviour of particular observed things, and that we must therefore start from these data as being the most firmly established. A simple process of abstraction first reveals the elements common to all which is contained within the wide spheres, which are naturally and easily separated in their most general attributes; between these most general concepts of matter, of animated beings, etc., and the lowest specific concepts under which we assume completely similar concrete phenomena to be comprehended, it is most important to construct the middle concepts, which serve on the one hand as specializations of the common element, on the other as comprehensive generic concepts for the manifold of particular species.

13. Whether or not the construction of these intermediary concepts is adapted to its purpose must be determined from two points of view.

In the first place, the superordinate generic concepts must be so constructed as to enable us to make the greatest possible number of universal judgments, hence to comprehend that which falls as far as possible under common propositions and laws having their predicates as fully determined as possible; subsumption under such a concept would enable us to make the greatest number of inferences, which by applying these laws to the subsumed subject would lead to the fullest and most definite knowledge. If, to take an extreme instance, I should try to form a general concept of plants with white blossoms, this would not enable me to make any further general propositions than are involved in the fact of blossoming in general; the white colour of the blossoms is neither ground nor sign of any predicates which are common to plants with white blossoms and distinguish them from those with red blossoms; in this respect the concept would be unfruitful. But by forming the concept of monocotyledons I am enabled to establish a number of predicates which are common to the plants showing this form of development, and which also distinguish them from dicotyledons.

It is because these concepts are chiefly valuable as adding a number of definite predicates to every subject, whether particular thing or specific concept, which is subsumed under them, that the second need arises of forming the classification in such a way as to enable the particular to be subsumed under its framework easily and with certainty. For this it is necessary that the subsumption, both under the higher genera and the

specific concepts, should be guided by as few and as easily recognisable characteristics as possible. From this point of view we must arrange our concepts in such a way that they may admit of those abbreviated formulæ of definition which we described in § 77, 7, p. 164, as diagnostic definitions, hence of comprehension in concepts of such a constitution that one easily recognised characteristic is the certain sign of a number of others by which the concept is distinguished in different directions from others, and which contain the ground of further predicates or modes of action. To express it differently, classification must subserve on the one hand the interests of generalization, on the other those of specialization. For subsumption attains its completion in subordination under the most specific concept, while the syllogistic process opens out into major premises of the greatest possible generality.

14. If we begin with the first rule, that of so arranging the classification that the intermediary concepts combine whatever is similar in most respects and hold apart whatever is dissimilar in most respects, and that they therefore express the degrees of natural relationship between things, we shall find that we again have to notice two points. On the one hand, what occurs to us first is to proceed upwards, combining those species which have most determinations in common into genera, and these again from the same point of view into higher genera, by means of a general survey and comparison of things in all their aspects. But there is also the other problem of arranging these concepts in such a way as to represent an exhaustive division which includes the whole extent of the department in question, and in which all the concepts must stand in simple and clear relations of subordination and disjunct co-ordination, in which therefore each concept takes a given place, and stands in simple opposition to co-ordinate, in simple subordination to higher concepts. All division must proceed according to certain grounds of division, which admit of disjunct members; thus the problem is not merely to combine whatever is similar, but to combine in such a way that the concepts thus formed may be represented as disjunctly co-ordinate according to certain grounds of division. For this reason we must always notice both the common attributes and those which are different and opposed, and as a preliminary to every classification we must not only discover whatever agrees in many characteristics, but must also survey the many disjunct characteristics which are opposed on definite grounds of division.

15. Thus the process may begin in two ways. In the first place, we must break up the total extension by taking a characteristic which is common to all, by finding differences in it which exhaust the whole extension, by again dividing in the same way the classes thus obtained, and then we must see whether in this way we obtain concepts which combine whatever is most similar. For instance, a general attribute of animals is motion; division according to the different kinds of motion or the different conformation of the locomotive organs is a primary division which is complete when it includes all existing varieties. A general attribute of plants is propagation, and we may get a first division by stating the different kinds of propagation. We shall naturally endeavour to form the subdivisions also from the same ground of division, as, for example, in the Linnæan system; but ultimately we shall always have to pass to other grounds, which will differ for the different classes. The test of such a classification would be that as we descend the concepts we thus form of orders, families, and genera should be found to combine more and more common characteristics; in other words, agreement in the distinguishing attributes employed for the division should be the ground or the sign of agreement in a number of other attributes, and opposition in those distinguishing attributes the ground or sign of opposition in other respects also.

Division thus proceeding downwards, would be met by the process of abstraction, which, mounting upwards, begins by combining the smallest group of species in the larger circles of genera. If we had begun with the latter the success of our progress from this starting point would be known first by the fact that the whole extension would be exhausted by the larger as well as the smaller circles, hence that no isolated species were left out, then by the fact that as we proceeded further groups of higher concepts would arise, which from their opposition in certain attributes and similarity in others would naturally fall into disjunct co-ordination under one common higher concept. To take a simple instance, in which the species are, for the sake of clearness, replaced by individuals: if a large community of men should naturally fall into two groups of similar individuals, the one set having dark eyes, dark complexion, and dark hair, the other being fair, blue-eyed, and light-haired; if it should then be found that all the members of the first group spoke a Latin tongue, and all those of the second a Teutonic, or if the former were lively and excitable, the latter more thoughtful and calm, then we should have the

type of an easy classification, in which the groups obtained by the ascending process are distinguished by a certain attribute which subsequently proves to be connected with a number of other differences in habits, customs, etc. If, on the other hand, we had proceeded downwards, say by dividing according to language, and had thus obtained by division just those groups which we were led to form in our first process by preponderating similarity, we should again have found a proof of the success of the process in the fact that it led us to groups within which everything was similar.

16. This easy coincidence of the lines drawn downwards by division, with the boundaries shown by combining the most similar things, is just what is generally denied to us by the manifoldness of actual things; and we are always in danger either of obtaining by division, as Linnæus did, class concepts of which some members are much more similar to members of other classes than to those of their own, or if we proceed upwards of missing the clear oppositions which distinguish co-ordinate concepts.

In combining from below, though we often come upon groups of forms which undoubtedly belong together, yet we are often confronted by numbers of forms which, from different points of view, would give different genera, and have therefore to choose which point of view shall be preferred. Here we shall be determined in the first instance by reference to the division, and those things will be held to be similar which agree in differing from others, and admit of a clear disjunction into as few members as possible;¹ if this fails, we must avoid the choice, and must represent the given concept as the result of a division from a twofold ground, having cross-members. The elements of the ancients may be combined according to similarity in two ways: the dry are earth and fire, the moist water and air; on the other hand, fire and air are

¹ If we had before us the combination (1) *abce*, (2) *bcd*, (3) *bcd*, (4) *abcf*, and wanted to combine the most similar, we might combine (1) and (2) in *bce*, (3) and (4) in *bcd*, or (1) and (4) in *abce*, (2) and (3) in *bcd*. But if *e* and *f* were opposed subordinates to a general attribute *E*, while *a* and *d* were disparate, then the first combination is preferable, as giving us *bce* and *bcd* as disjunct co-ordinate concepts of a higher *bcE*.

But if both *e* and *f* and *a* and *d* were opposites, then it would be correct not to give the preference to either combination, but to represent their relation as the result of a combined division of *bc* according to two grounds of division:

$$\begin{array}{c} e \quad f \\ a: bcae-bcaf \\ d: bcde-bcdf. \end{array}$$

warm, water and earth cold. Neither similarity can claim precedence over the other; but they are easily represented in their logical relation by means of a composite division, which, without giving the preference to either ground of division, divides according to both.

17. But there are natural groups of allied and similar things, which nevertheless admit of no generic concept expressing their connection, and which by their peculiar constitution refuse to conform to the orderliness of the logical system. If we had a group of concepts represented as follows (*a* and *a'*, *b* and *b'*, being taken as contradictory attributes),—

$$\begin{array}{ccccc}
 & abcdef' & & abcde'f & \\
 abc'def & & abcdef & & abcd'ef \\
 & ab'cdef & & a'bcd'ef &
 \end{array}$$

then each of these concepts would be like the middle concept in five attributes, and different in only one, while it would be like each of the others in at least four characteristics; but the deviations from the middle form are always in different directions. In constructing our concept by abstraction we ought to be able to select the element common to all, but all we should be able to find here would be a general concept *ABCDEF*, containing the general attributes *A*, *B*, etc., in which *a* and *a'*, *b* and *b'*, would appear as contradictories. But this general concept would contain the very different concepts *a'b'c'd'ef*, etc., and would not be adapted to express in any way adequately those concepts which in our instance form a connected natural group when the other members which would occupy the same rank with equal differences are wanting. Such a group justifies a common name, but there is no concept corresponding to it which can be fixed in a definition; the statement that the things belonging to the group generally have the attribute *a*, but sometimes *a'*, that they generally have the attribute *b*, but sometimes *b'*, cannot be a substitute for a definition, which must be equally true of everything included in it. Such a group is represented by the central form, which has most similarity with all the others, so that these others may be regarded as the variations in different directions of a central type.

In proportion as we are unable to find within the extension of a higher concept far-reaching and fundamental differences which would enable us to make our division safely, and in proportion as the differences themselves are of a quantitative and changeable nature, we are obliged to have recourse to the formation of such groups in our survey of the manifold of

things. In the classification of men, *e.g.* according to their structure, we have been forced, in proportion as our knowledge of the different races became more extensive, to relinquish the idea of a strict logical division and confine ourselves to finding such groups as form themselves around certain types; we cannot, however, avoid the difficulty of intermediary forms, which leave us doubtful to which group they belong.¹

In our endeavour, therefore, to make the classification as far as possible the expression of natural relationship, we are obliged to admit such logical anomalies throughout the lowest stages, and are limited to arranging the higher orders according to certain characteristics of agreement and opposition, by discovering the most far-reaching differences.

18. The second question now arises, whether the arrangement thus obtained enables us to carry on our subsumption with ease and certainty. If the latter end should be pursued apart from the former, we might imagine a group of given objects as classified in two ways, or a classification subservient merely to diagnostic interests preceding that which starts from the other point of view. For what is most needed in subsumption is to find the lowest specific concept to which any object can be subordinated, because that gives us the predicate which is richest in content, that upon which the most numerous, and the most definite statements depend. If the lowest specific concepts were known and named, all that would remain would be to subsume an object under its appropriate specific concept in the quickest way; and this would be for every species to have its easily recognised characteristic attribute distinguishing it from all others, so that the subsumption would be determined by the presence of this one attribute. Diagnostic definitions would then consist entirely of these attributes; they would be like a simple index to the complete concepts, and all we should have to do would be to learn the characteristic attribute for the name of every *infima species*, so as to be able to name every object, and by means of the name to recall or look up the other attributes.

¹ "The different groups of lower organisms, such as flagellatæ, dinoflagellatæ, rhizopods, etc., cannot be distinguished from each other by any complete difference. . . . The special characteristics of any one of these groups are only found in the main part of the forms, not in the outlying members. Each group centres itself in a characteristic type, which is sharply distinguished from that of the neighbouring groups. . . . It is well to review all the lower groups of organisms from a certain height, and see them as a great connected field; but it is not less necessary to look closer and to see that we are dealing with a hilly country. Flagellatæ or volvocineæ represent different types or hills of this kind."—G. Klebs, "Flagellaten Studien" in *Zeitschr. für wiss. Zoologie*, lv., 2, p. 266.

19. No method of classification can avoid this prosaic consideration, that as soon as the region to which it refers becomes extensive the totality of concepts which it is to arrange will be present to, and remembered by, few or none; the fact that only by fixing the definitions in writing can we obtain a survey of the whole manifold makes it necessary, in determining the process of classification, to keep in mind the practical interest of facilitating the acquirement of knowledge which can only be communicated by writing. If we could remember all the concepts with their attributes in fixed combinations, we should not need to resort to the expedient of particular characteristic attributes for our subsumption; the inferences which serve partly to guard against subsumption under false concepts, and partly to bring about the right diagnosis, would follow easily, and without consciousness of each step, from perceived complexes, so long as the things were at all distinguished by external characteristics. In determining a plant of the flora with which he is familiar, the practised botanist does not go through detailed processes of thought; but the beginner, if he has at hand only the systematic arrangement of the text-book, is confronted by the problem of discovering the name of the species of a given plant. Since the particular species are so numerous that it is impossible to give characteristic attributes for all, it would be at any rate the shortest way for him if there were easily remembered characteristic attributes for the lowest concepts, or smallest classes possible, from which he might start to compare further particulars. In this respect the Linnæan system of classifying plants is acknowledged to have been most fortunate, although its class attributes are only to some extent signs of further similarities amongst the plants combined in a class; it acts as an index, much in the same way as the directions to test the chemical nature of some substance by specific reactions give us the means of at once subsuming it under a special concept.

20. If we were unable to find easily recognised characteristic attributes for the lowest species themselves, or for the smallest classes possible, if we had to deal entirely with attributes which are common to many and different objects, so that the only difference which could be expressed in definitions was based upon different combinations of widespread attributes, we might still, with a view to facilitating diagnosis, constitute our classification in such a way as to afford the most easily comprehensible plan.

This will obviously be the case when the division proceeds by simple

contradictory opposition, where it may be that an attribute is absent in one subdivision which is present in another, or that two positive but clearly opposed attributes are employed for the division. Then we need only remember the sequence in which the division proceeds through these opposites, and we shall be able to subsume every object under first the higher and then the lower genera; the most convenient form of division is therefore dichotomy, unless we can substitute for it such simple polytomies as are afforded by the successive numbers.

The most perfect classification would be one in which the purely theoretical interest of arranging the essential concepts according to their natural relationship could be combined with easy diagnosis, and the concepts so constituted that they might be always represented by easily recognised, certain signs. Where this ideal is not attainable we must have a twofold classification adapted to the different ends, or effect a compromise between the claims on either side.

21. So far we have tacitly assumed that the formation of *infimæ* species by combining completely similar things has already been carried out before we began the arrangement of these concepts; and to a large extent, not merely where these concepts are as fully determined as those of the chemical elements, but also in a large part of the organic universe, this first step of forming concepts has been unanimously carried out, although it was necessary in doing so to overlook individual differences of more or less importance. It has at any rate been easy to mark off groups of which the individuals agree in a number of attributes, and are distinguished from proximate groups by definite points of difference; the differences between individuals of the same group were found to be insignificant in comparison with the agreement, and to consist in variations in particular attributes which do not involve variations in any others, or else in merely quantitative differences. But other relations less favourable to classification have been found, which do not admit of such demarcation, where a number of individuals are connected by gradual transitions in such a way that their extreme differences are as great or greater than those of the individuals which elsewhere belong to separate groups. Even if we did not hesitate to treat the former groups as *infimæ species*, the question arises whether these groups, in which are involved larger and wider differences, stand upon the same level; and in this way we are led to the necessity of finding a criterion by which to decide what differences are to be regarded as specific, in other words of determining

the concept of the species. It is well known that when the merely morphological comparison was found to be insufficient to yield a certain criterion, recourse was had to genealogy, and common descent was fixed as the criterion of the species. We include in one species individuals which have a common descent, and individuals which are as similar as those which have a common descent. And since common descent often cannot be directly proved, the further principle has been employed that only individuals belonging to the same species can produce permanently fertile descendants; thus those individuals belong to the same species which have permanently fertile descendants. Where this was accepted as fixing the concept of species, the attempt was made to account for the differences still remaining within the species by the introduction of the concepts of varieties and of races, differences of variety being differences which are common to a large number of instances and which there was a tendency to refer to external conditions, and differences of races being inherited variations.

22. We cannot enter into the question how far it is true of the species hitherto accepted that members of the same species have a common descent and fertile descendants, while the members of different species have a different descent and unfertile hybrids; nor can we decide how far the doctrine of evolution, which ascribes an ultimate common descent to even the most different forms and thus nullifies the old criteria of the specific concept, is likely to become a proved scientific theory, or has any claim to be regarded as such.

After the account we have given (§ 94, 13—18, pp. 326—332) of the logical bearing of the Darwinian theory, we can only raise hypothetically the question as to what position, if we assume the validity of the theory, is to be taken by classification, of which the object always is to find concepts under which to arrange the whole manifold of the given. For at first sight the Darwinian theory seems to do away with all possibility of a classification which presupposes fixed and distinct forms; all that it recognises is a history of individuals in successive generations which, if it were fully known, would show none but imperceptibly small differences, and within the flux of which it would be impossible, and in any case quite arbitrary, to draw definite boundaries. At best it allows us, as we showed on p. 329, to construct at any given moment groups which fall apart through the failure of intermediate forms; the unlimited complexity of the external conditions, affecting the variation and maintenance of

particular individuals, makes it absolutely impossible to construct anything like formulæ, according to which certain conditions would give rise to one form, and others to another, or by introducing causal laws into the divisions to produce the closed system of concepts which is needed for an exhaustive survey of the whole.

But the Darwinian theory, by its own point of view, forces us to a basis of classification which at first sight seems diametrically opposed to its whole tendency. If the existence and reproductiveness of certain forms depend upon the variations by which they are distinguished from others being *useful* to them, upon their being in accordance with the conditions of life and ensuring advantages to them in the struggle for existence ; if those forms must become stable which attain the maximum of adaptation, because every variation must then be a disadvantage, while a tendency to vary in all directions is regarded as a seeking after the most favourable form, then the only way of finding fixed forms which have any counterpart in reality is from the point of view of the end, and those forms stand out amongst the chaos of differences which are the most *perfect* when the end is taken as the standard by which to measure them. And because the organization is the more useful to its possessor in proportion as it is independent of external conditions and elastic in its power of accommodating itself, we are led by the same point of view away from the external conditions to the internal organization, to the relation between the organs and their functions, to the system of compensatory arrangements which enables the machinery of life to work on undisturbed by change in external conditions, and we must give the most important place to internal adaptation to ends.

Thus the basis of classification which, when the old one has been destroyed by the theory of evolution, proves to be not only compatible with the theory, but necessitated by it, is the teleological. The forms around which the others group themselves are those which are most perfectly adapted to an end, in which we can see from the point of view of the end how every special modification of an organ co-operates in the most favourable way with special modifications and functions in other organs.

If we look backwards from forms which have developed in this way, the whole complex of varying forms and all the conflict among the conditions of life appear as a means towards attaining this end ; and, indeed, this is what we mean when we always speak of evolution from lower to

higher forms. The standard we erect is not that of mechanical causation, according to which evolution means only that invariable forces successively come into action according to general laws of nature; nor is it merely that of the *causa immanens*, according to which a series of states proceeds from the nature of the subject itself, and not from external influences alone. Contrary to this concept, we have to deal, at any rate according to Darwin's own view, not with internal development, but mainly with the influence of external conditions. What is here meant by evolution is that the lower forms are merely the conditions for those which are higher when measured by the ideal of adaptation to the end, and the whole is represented as if the whole complex of original dispositions and external conditions had been so arranged as to produce, by a gradual development and successive progress, the highest forms. When thus looked at retrospectively our types arrange themselves into a graduated course of development from the lower to the higher, from the imperfect to the perfect; and in this aspect also the doctrine of descent coincides with the natural philosophy of Schelling and Hegel, in which, though empirical history was disregarded, the classification of forms was represented as a teleological development determined by concepts.

23. To construct entirely from the point of view of the end the ideal types which are to serve as points of crystallization round which to form connected groups would indeed be a task needing an insight into the laws of organic life which we do not possess. But even from this point of view we may have recourse to external considerations, as supplementing and guiding the classification. Since the struggle for existence destroys the less adapted forms, and encourages the best adapted, it follows that number is a criterion of the most perfect constitution within a group of nearly allied forms, and we are thus enabled to apply statistical methods here also. If within a group of allied individuals, varying by small differences, we find a certain combination of attributes to be the most frequent, while of the forms grouped around it those which deviate from it least are the more numerous and those which deviate most the less numerous, then it is probable that this combination is the most favourable and represents the equilibrium best adapted to the end. Thus the average would rank as the normal determination of the concept even from the teleological point of view; and the attributes found by the average would constitute the concept which the classification presents as the type. If the medium height, the medium weight, the medium size of the

brain, are the most frequent in any community, while the great majority of cases do not exceed narrow limits above and below the medium, and giants or dwarfs, very heavy or very light people, are rare exceptions, then we are justified in regarding the average as that size which gives the most favourable relation between internal organization and external conditions, as the form which is most stable in face of the constant and variable causes and conditions of life. Thus the most external point of view, that of number, coincides directly with that which is taken from the highest metaphysical meaning of the concept.

24. If there is any object in making further distinctions within a group which lies in this way around a central normal type, it can only be done by contrasting with the means certain extremes, which show the direction of deviation; each individual is then assigned its position upon the radius which connects the mean with a given extreme, and finally, where the number of differences is too great to be fixed in concepts, we may represent the classification graphically.

It has been impossible, for instance, to carry out the classification of the various shapes of skulls into separate groups of brachycephalous and dolichocephalous: the arrangement has no meaning unless we start from a mean from which the extremes deviate in opposite directions; dolichocephalous and brachycephalous denote opposite maxima of relative diameters of length and breadth. It must be a matter for arbitrary decision what numerical relations separate dolichocephalous skulls from mesocephalous, and these from brachycephalous. Whilst these differences are represented by a line, if the extremes are determined from two points of view, they will group themselves in a plane. If, *e.g.*, human skulls were grouped on the one hand according to absolute size or capacity, on the other according to the proportion of the two diameters, then the position of each one might be determined by its distance from two axes perpendicular to each other, one representing the direction from the longest to the shortest, the other from the largest to the smallest, the mean value in both respects being found at the point of intersection.

25. If we review the results to which we have been led in considering the problem of systematically arranging our knowledge, it appears that the conditions essential to carrying it out consist in the possibility of deductive processes, of analytical development of concepts, and of syllogistic derivation from general laws. In so far as these have originally been obtained inductively, the hypothetical nature of all positive propositions

reached by induction affects also the whole system of classification ; on the other hand, the formulation of the highest concepts, from which the division starts, attains more and more to the character of construction as they become further removed from empirically given, concrete, particular phenomena. The general concept of matter, of which the particular substances are specializations, whether we regard it as filling space continuously, or as consisting of atoms, can only be determined by means of a construction the essential parts of which are of a purely mathematical nature, and which employs besides the ultimately *a priori* elements of substance, force, impenetrability, etc. In the concept of the organic, again, in any fixed sense, we must have recourse to construction in proportion as it becomes more difficult from the material point of view to draw a strict limit between the organic and the inorganic, and the distinction between organic and inorganic processes is disputed. There remains only the form in which such processes are combined, for which we can find a certain basis only in the concept of the end. Thus it is that the highest and most general concepts finally relinquish all sensuous content ; they are logico-mathematical schemata, in which we try to grasp the essence of the existent, and have their origin in the nature of our thought. Just as we are able to arrange and understand the infinite manifold of spatial forms only by the aid of our geometrical concepts, so we endeavour to master the given by the analogous ideal forms of substances and forms of unity among things.

CHAPTER VII.

THE METHODOLOGICAL PRINCIPLES OF ETHICS.

§ 104.

IN considering what man ought to do we need first of all an analysis of the activity of volition itself, and this must be carried out psychologically.

In this analysis we always find already present certain ENDS of action affirmed by the will, and recognised convictions as to the RULES of action ; and thus the first task which presents itself is that of logically elaborating the rules and ends which are assumed as valid.

On the one hand, we have to apply them by way of deduction to particular cases ; on the other hand, we have to find one principle of unity for all by way of reduction.

This necessity of logical elaboration involves the necessity of rising above the mere actual given consciousness to an ideal consciousness which is one with itself and all-embracing.

The higher problem of practical thought calls for the establishment of unconditionally valid normal laws of volition, and it can only be solved by assuming that there is an immediate and self-evident certainty with reference to the "ought," just as logic assumes an immediate self-evident certainty with reference to objectively necessary thought. These unconditionally valid principles are discovered by analysis.

If this analysis leads merely to formal principles of the unity and harmony of the will, as logical analysis leads to the principles of agreement and contradiction, we cannot derive from it the system of concrete ends to be realized by the given means, but can only establish negative canons. The positive idea of the highest good must take in wider considerations which are derived from the actual and natural volitions of men, and must have recourse to the values of ends as expressed in feeling.

This, however, can only be on the assumption that the natural volitions of men tend towards the realization of morality. This is the postulate of every system of ethics which is not merely negative and critical.¹

¹ Cf. my *Vorfragen der Ethik* (Freiburg, 1886).

1. The two main problems which our thought sets before itself are knowledge of the universe given to us in perception, and reflection as to the ultimate ends of our will; and from the first we have been obliged to separate knowledge of what man *ought* to do from knowledge of what is actual, because the former is never a part or a simple consequence of the latter.¹

When, however, we endeavour to develop the methods leading to this ethical science, methodology, which is firmly established only when it can prove the practicability and success of its instructions, finds itself in a difficulty like that encountered in psychology. No sure science, certain of its own principles, has ever revealed itself from the chaos of endeavours to give form to ethics, and thus our first task is not to give definite instructions, but only to formulate the problems, to guard against their confusion, and to consider the possibilities of methods which might be employed for their solution.

2. The question as to the ways which lead to certain ethical knowledge is complicated in a peculiar manner by the fact that the will, of which the rules are to be discovered, is already active in knowledge, and acts as the motive power in the effort towards truth and certainty.

In knowledge of the external world the subject and object of knowledge fall apart clearly and distinctly. The sensations from which we construct the external world are primarily independent of our will, and offer mere material to our endeavours to arrange them and to form them into a consistent and comprehensible whole; the activities of thought which are directed towards them are the means of attaining the ideal of truth or adequate knowledge. Will and thought stand together confronting an externally given material.

But when thought turns to the will itself, then the object of thought is that which is also its ultimate subject, and we find ourselves in a circle which is not to be broken through. In the sphere of ethics also the will to think remains the ultimate, unanalysable presupposition of all scientific endeavour, and reflection upon our own volition cannot but realize that the will can never be objectified in the whole of its extent.

3. There is, no doubt, one way of knowing our voluntary acts which

¹ In the attempts to regard ethics as only an applied science which employs empirical laws to learn by what modes of action the general welfare may be attained, it is forgotten that the "general welfare" means many things, and that we are first concerned to know what the general welfare is which is worthy of our efforts. It is nowhere empirically realized.

is comparable to the mere receptivity of sensation. The momentary immediate consciousness of particular volitions which take place in time and develop consequences, the consciousness that something is just being willed by me, seems to be as capable of being the simple object of mental elaboration as the consciousness that this or that is just being seen and heard. But even if we grant this—though it might well be asked whether anything could reach consciousness in such a way as to be grasped as its object without the co-operation of the will—yet in this way we should only get a manifold of particular perceptions, to the effect that I will this or that at any given moment. Any attempt to comprehend these facts, to analyse them, and refer them to their grounds, is the expression of a will which sets before itself knowledge as its conscious end, to be realized by means of thought.

Nor should we be able to succeed in disentangling the will which is directed towards knowledge, and in setting it over against the will which is directed towards other ends, so that the end of knowledge and the will which is directed towards it should, as it were, stand outside of and above the rest, and the will which is directed to other ends become the pure object of the will to know. Wherever there is will, wherever an end is affirmed as mine and the means to it are sought, the cognitive activities are at work, and constitute a part of the processes by which the will is developed. On the other hand, the ends which we set before us in the sphere of action are some of the most potent incentives to thought, and the question: what ought I to do? always demands an answer which contains an element of material truth, *i.e.* of the end at which pure knowledge aims; for any end which I can rationally accept must be one that is *realizable*. Where this question is absent there might indeed be immediate desire, but no conscious will in the proper sense. Just as the assured judgment is distinguished from blind association by the question which holds subject and predicate apart in order that they may be consciously united, so the will which proceeds by reflection upon ourselves is distinguished from the momentary and involuntary excitement of desire.

Such is the circle which seems from the first to make it impossible to find any safe starting point for an investigation of method. But it merely forces us to begin with a provisional distinction to be afterwards put aside, to isolate artificially elements which we know to be never separate in reality, to disregard one factor at first in order to study the others.

We begin by taking the will to think as something given, a matter of course, which is not itself the object of reflection, when we investigate the human will; by means of artificial abstraction we place ourselves, not indeed outside the circle, but at one fixed point of it, from which we may move forward in one fixed direction, with the expectation of being brought back to the starting point.

Over against the will which is directed towards knowledge, and which first manifests itself as the will to think, we set the will which leads to action, *i.e.* to influence upon the external world, and we take this by itself as the object of our investigation. The distinction is justified, for if there are any general, ultimate, and highest rules for the will, they must be discoverable in every kind of will. It was thus that in logic we analysed thought, although the analysis itself was only possible by means of thought; it was however necessary that the results of the analysis should themselves be applicable to the thought which investigated them.

4. Thus from the beginning of the investigation we can only accept as fact, that there is a consciousness of the "ought"; that a need exists and is felt to subject our will to certain rules and to divide the possible ends which it may set before itself into such as are in accordance with these rules and such as are opposed to them. As the fact that we distinguish between true and false, the fact of error and dispute, caused us to distinguish between that part of our actual thought which attains its end and that which misses it, between the subjective psychological necessity which we think of as producing actual thought and the necessity which is objective and logical, so the human consciousness always distinguishes between actual will and action as they take place and proceed from the existing psychological conditions and the will and action which would be in accordance with an objectively necessary rule. The same difference between actual will and will which conforms to a rule is expressed in the facts of repentance and blame.

5. To this parallelism it is due that in our investigation of method we shall have an arrangement of problems similar to that in logic itself.

When we ask: how ought we to act in order to act rightly? we need first an analysis of voluntary action in general, partly according to its form, partly according to its actual conditions. For the first analysis the rules of method can only be those with which we have become familiar in the analysis of concepts, nor is it in any way contrary to the relations found to exist in the purely theoretical sphere that the result obtained

by an analysis of the form of actual will consists in a plurality of simple acts which are interconnected and mutually conditioning.

Without attempting to undertake this analysis in detail,¹ we must nevertheless presuppose so much as is necessary to the consideration of the methods of ethics. The object of every actual will is a state of myself and of other things which is thought of as real, or at any rate realizable, in the future, and which is conceived in a particular determination or in a more general concept; where all the processes are clearly differentiated this object first suggests itself as a *possible end*, and after due consideration is affirmed or rejected as *my end*. The consideration which precedes the decision of the will is, on the one hand, purely theoretical, and refers to the material possibility of the end, the adequacy of the means at my disposal to carrying it out; on the other hand, it refers to the practical question whether I ought to make this possible end really mine, to direct my action to its realization, and this question requires a consideration of the relation in which it stands to the whole of my personality, my inclinations, my interests, my principles, etc. The affirmative or negative reply to this question is the inward decision of the will, which accepts or rejects the possible end as mine, and which finds utterance in "I will" or "I will not"; if the decision is affirmative there follows from it the motor impulse to the actions through which my will becomes causal.

This analysis of the mere form of voluntary action is enough to show one of the main difficulties which obscure this chapter in psychology; the relation, that is, between comprehensive determinations of the will and stable tendencies of will, and more special acts, the particular concrete impulses. The determinations of the will, as they take place in time, do not succeed each other in a chain of which the links can be counted like beads in a necklace. They penetrate each other in the most manifold, and often in the most complicated, manner; sometimes simply subordinated to each other, as in the series of particular actions which are willed as the combined means of realizing a desired end, and which follow from the end as logically necessary consequences; sometimes thwarting and disturbing each other, and conflicting in many ways, where a previous act of will is disturbed, modified, or annulled by a later one. It is as when in the theoretical sphere doubt attaches to inferences from an established judgment, or we are confronted by contradictory alterna-

¹ Cf. my essay, "Über den Begriff des Wollens und sein Verhältniss zum Begriff der Ursache," *Kleine Schriften*, ii. 115 sq.

tives without finding a solution. These disturbances are of a more superficial nature where they refer only to the conditions of success, and force us to relinquish an end because it cannot be realized; but they are more deeply rooted when the affirmation of one end conflicts with the affirmation of another, and contradiction enters into the will itself.

We will note only what is most important in this analysis: the subordination of special to general will, and the possibility of conflict. The relations with which we have to deal here are logical; that which determines subordination or conflict is the logical relation in the one case of the general to the special, in the other of the incompatibility of different ends, which, when we think of them together, are in conflict, or which if realized together would annul each other by the existing causal relations.¹

¹ We must also mention here the relation which holds between the general rules which I lay down for my action and the ends towards which my action is directed. Language seems to insist upon a distinction between the two. The end is something concrete, the rule something abstract; by the word "end" we generally mean first a definite particular result, which is to be attained in a given time; the rule only prescribes the general nature of the action, and leaves it quite undetermined where, and when, and under what circumstances, and with reference to what objects it will take place. The rule is thought hypothetically, the end categorically; the rule says: if you act, act in such and such a way; the end says: do this and that. But the distinction disappears before closer observation; every end may be regarded as a specific rule, every rule as a general end. In § 75, 5, p. 148 sq., we have shown that the end, even when fully determined and directed towards a particular satisfaction, nevertheless remains general in so far as what is directly willed is ordinarily not a given particular thing as such, but its quality as the means of my satisfaction, and that it is a matter of chance whether the same quality is offered by only one or by several objects; also that general ends are accepted which are realizable by many different means, so that the general end requires us to make every appropriate and attainable thing the means to this end. The end of earning our living may also appear as a rule commanding us to make use of every opportunity of remunerative work, the end of forming a herbarium as a rule to collect every species of plant within reach. Inasmuch as the complete attainment of the end depends upon opportunities which cannot be foreseen, the end itself becomes in practice a hypothetical rule, even when its complete realization would, as in the last example, present a whole of concrete things. On the other hand, every hypothetical rule implies the assumption that the cases to which it applies will actually occur; no one lays down principles and maxims for wholly improbable cases, unless in the play of imagination; the rules which he seriously prescribes to himself refer to cases which he expects to occur, and the rules are to these just what the general final concept is to the special means by which it is realized. It is not all the individual particularities which are willed, but the aspect common to all which corresponds to the will. And the result which ultimately follows from the application of the rule may also be represented as a concrete whole, to the realization of which conformity to the rule has been the means; the prosperity earned by the man who makes work his rule is a state present *in concreto* which follows as total result from particular instances of the rule.

But does there not still remain a difference which forbids us to identify the two concepts—to regard them only as different perspective views of the same content? Are

6. Our analysis moves in a far more obscure region when we ask about the presuppositions of the will, and endeavour to investigate them empirically by observation and by inductively inferred connections. Whence arise the ideas of ends which hover before us as possible resolutions, and upon what depends the Yes or the No by means of which we accept or reject an end which thus attracts and tempts us? Fortunately ethics has not to wait until psychology throws light upon the labyrinth of confused human action and shows in unbroken causal connections the clue which enables us to assign to everything, even to the most startling events, their place. For its inquiry is not as to what happens, but what *ought to happen*. Just as logic does not undertake to explain every error and superstition, every lie and sophistry, but only to estimate them and to show them for what they are by telling us how we must think in order that our thought may be true and universally valid, and accompanied by the consciousness of objective necessity, so also in the region of ethics thought endeavours to find out how we ought to act in order to act well and with the consciousness of objective necessity.

7. In investigating this question the first fact which confronts us is that this "ought" is recognised everywhere, and that even the content

there not limiting cases to be regarded only as ends, and others only as rules? Can the willing of a completely determined end be conceived as the laying down of a rule, or a universal maxim as the content of an end?

If the human will were quite disconnected, then indeed an end might be thought as absolutely particular. But then we have not yet reached the stage of rational conscious will, have not yet reflected upon the relation between the particular thing which tempts me as object of my desire and the unity of my personality; this reflection is accompanied by the subordination of the particular to the more general directions of the will, and more careful consideration now shows the particular object to be an end only in so far as it is also the means to a more general end. With this restriction the most concrete is no doubt a particular, but it points to a more general end by which the willing of it is determined; it stands in the same relation as the individual and fully determined stands to the general concept, and the determination by virtue of which it is particular is imposed upon it by the external constitution of the object.

On the other hand, there are, in a certain sense, general rules which no longer constitute the content of an end if we apply the concept of the end primarily to the external result to be attained by the action. If the rules refer, like Kant's categorical imperative, only to the subjective nature of the will, which may be the same in every action independently of variation in the objects, then they determine nothing with respect to the objects of the action, and there seems to be no content whatever for a final concept. But this is only apparent; we have at any rate the subjective unity of the will and the consequent satisfaction as the one end which follows from the rules, and all particular action appears as the casual and indifferent means, which has no essential reference to the abstract general end, but is merely subordinated to it from without. We lose the reference to an external end that we may make it entirely internal.

of the moral laws is largely the same as the object of general conviction, thus having an at any rate relatively universal validity, and is active amongst the factors which determine the action of particular individuals. In any attempt to explain the action of the individual by the actual content of his consciousness, we find amongst the conditions of the actual will rules which are recognised as valid within a community. The fact that they have a determining influence on the action of the individual is due partly to his having accepted and affirmed them as objects of his own will, and being therefore guided by them in the particular instance from simple consistency, partly to their acting as standards of praise and blame and arousing ambition even where their content cannot in itself be regarded as the conscious and fixed will of the individual, partly also to the fact that the constitution of the community has, by virtue of its institutions, the power of connecting material consequences with their observation or infringement. We find in history that all morality in the widest sense of the term, whether it is of a worldly or religious nature, and all valid law, has this variously grounded actual authority over the individual.

8. It is one of the most difficult problems of historical and psychological analysis to investigate even the way in which these universal convictions *exist* in the consciousness of the individual, and co-operate with individual impulses in particular actions to form actual life. It is only the results, and these for the most part only in broad outlines, which are accessible to observation and to historical knowledge (§ 99, p. 450 sq.). To investigate further how these convictions *have grown up* and obtained their power would be the last and highest problem in the explanation of comprehensive historical facts; the only result of all our philosophy of history which we can accept as certain is, that they cannot have arisen by the comprehensible empirical way of sensuous motives of utility alone, and that we must therefore acknowledge the independence of moral ideas of the mere course of fact. The facility with which we generally receive what is handed down by tradition, and voluntarily affirm rules which are already authoritative without further reflection, may indeed explain their continuance, but neither their genesis nor their alteration.

9. But even if this explanation could be successfully carried out, it could not get rid of the fact that the universal validity of the propositions stating certain rules of action to which experience is said to testify is,

even apart from their frequent actual infringement, a mere illusion, and that it is certainly impossible to present them as the simple result of human nature, or as psychological products which can be deduced with uniform necessity from tendencies common to all, as the perception of the sense-given world is a uniform result of psychological activities. They are not conceived by every one in the same way, nor are they affirmed by every one in the same way; to those for whom they are authoritative in the fullest sense they are so because accepted by their will; while to others they are as an inconvenient external power, against which their individual inclination and actual will rebel. Such people, if they act in accordance with them, do so for the sake of other ends which have nothing in common with the rules of morality and law. At best it is the harmonious will of the majority which finds utterance in them, and even here there are many degrees in the sincerity and unreservedness of assent. If we try to explain this actual relation by analogy with other methods, we must regard those convictions of the majority which make themselves felt as an average result, in which the many conflicting efforts of individuals have counter-balanced each other, as the common resultant of all conflicting forces in which that tendency which is common to most preponderates. This interpretation may be so far extended as to regard the rules as containing an element common to all and free from subjective and disturbing subsidiary influences; and then the attempt to derive these convictions, which claim to be universally valid without actually being so, from a rational or moral tendency which is common to human beings and merely combined with other tendencies in a different proportion for every human being, would be perfectly justified.

10. Any treatment of ethics which aims merely at the description of practical reason, and which takes as its starting point the consideration of existing morality in the actual totality of human will and action, must refer to considerations like these from this point of view. But a treatment of ethics which overlooks the historical way in which moral convictions exist and make themselves felt in the particular individual, and ignores the consequent distinction between the actual will of the individuals, in which consists the whole life of the community, and the rules which constitute their openly acknowledged conviction; a treatment which recognises a universal reason or a universal spirit as the only subject of rational action, and states the aims and ends of this action only in general

concepts, as in the ethics of Schleiermacher, is from the point of view of method a hybrid creation. By an arbitrary abstraction it systematizes one portion of what actually happens, selecting it from the whole of the rest as the moral part according to a tacitly assumed standard; and while appearing only to state a formula of history, it cannot hide its real intention of teaching us rather what history ought to be than what it is. Only by assuming that history tends of itself by continual progress to realize the concept of morality can it find any connection between its general concept and what actually happens. It is as if in logic we should aim only at describing the forms in which human thought moves, and should live in faith that it would produce truth by virtue of its nature, and indeed that it is well aware of the right way even in its blind tendency, as if we should disregard the fact that when the end is attained the consciousness of the normal laws of thought is an essential factor in the progress itself, and should attempt to ignore the errors and mistakes through which actual thought has passed and the labour we have expended in considering the nature of knowledge and the criteria of truth. It is the same vague personification of a universal reason which we found being employed to account for the growth of law, whilst ignoring conscious, voluntary work.

We cannot accept this mixture of empirical investigation of the actual and its estimation according to a presupposed standard as the right method; but we find that, starting from the same standpoint of given convictions, there is still another task than that of explaining them and deriving them from their psychological grounds, the task, that is, of their *logical* treatment.

11. It follows from the nature of the case that all rules which are intended to regulate conduct, whether in the conviction of the individual or in the expression which they find in generally recognised commands, are general and to a large extent hypothetical. No end could be set before particular individuals or before the community which would determine the sequence of their actions in an unbroken connection, nor any programme devised which would claim their whole will for its simple execution. In realizing his ends every one is dependent upon a thousand incalculable interruptions, which must be met by him, and it is impossible to foresee what varied incitements to definite action will present themselves. Ethical convictions involve a willing of universal ends, which specialize themselves in the most varied manner according to circum-

stances, and they prescribe how we ought to act when certain conditions occur; prohibitions especially, though they are in themselves unconditional, and are always complied with when the forbidden action remains undone, have no significance for the will unless the temptation to violate them is present.

12. There is needed, therefore, a continual application of the general rules to the concrete cases occurring in time, and a continual specification of general ends according to the means at our disposal, which require the logical operations of deduction. The application of the existing law is the clearest and most obvious example of this; and we need not do more than point out that the difficulties in this process of deduction lie in the subsumption of concrete instances under the authoritative rules. Every general rule contains a general concept of the relations to which it is to be applied; these general concepts are obtained empirically by abstraction, and contain the general attributes of that in reference to which the legal ordinance expresses some command. But, on the one hand, the concrete instances are never completely exhausted by these attributes, while, on the other hand, we cannot assume that the legislator whose will is to have authority (or the person whose will the law commands us to carry out) has actually thought of all the modifications of which those general attributes are capable. If we refer to the conscious will of the legislator, we are at a loss whenever a case occurs of which we cannot assume that it was meant to be included in the general concept; if nevertheless such a case is subsumed under the concept by the judge, we can indeed say from the purely logical point of view that what takes place is a simple application of the law, for the syllogism is correct, but when we consider that the connection between these general characteristics and the legal consequence is not logically necessary, but only created by the will of the legislator, and that it is possible to doubt whether the ground of that consequence has found its true and complete expression in the formulation of the general concept, then we see that the decision of the judge does not simply apply, but supplements and continues the legislation. The nature of the process of abstraction, which starts from an empirically limited group of objects, necessarily involves that a concept may express the given empirical extension of objects correctly and so as to distinguish them from all other known objects, and yet be an incomplete formulation of all that is common to these objects; any extended application of that which is established in such a concept beyond the limits within which it

was obtained is then no longer a syllogism, but only an inference from analogy. It would cease to be such only if it could be shown that the attributes by themselves contain the ground of the legal consequences which the legislator assigns to them, and that no conceivable species of the concept contains further attributes which would modify those consequences.

It is this which constitutes the other difficulty in deduction from general rules, that one and the same fact may fall under different general concepts, and that according as it is subsumed in one way or the other the consequences will be different. Generally every legislature foresees such cases, and establishes rules which correspond to the laws of the composition of effects (p. 350); but here again the complication of real relations is wont to exceed what is foreseen, and here again the application of the rule passes over without fixed limits into its supplementation. Nothing but the complete classification of all specialities would make it possible to proceed entirely according to the syllogism.

We must content ourselves with showing here where the rules of juristic interpretation find a place in methodology. An exactly similar process takes place in other provinces. The maxims which the individual forms for himself from his subjective conviction, or accepts from the rules prevalent in the community, are just as insufficient to determine beforehand how he is to act in all concrete cases; the individual will cannot avoid extending its legislation, and passing beyond simple logical necessity to make its own decision.

13. We come upon difficulties also in the deduction of the means from the ends. In order that we may say that a given action is the right means to a given end, and must therefore be willed because the end is willed, we need the certainty not merely that the end is, generally speaking, the necessary effect of the means, but also that this means will produce the end under the whole of the given circumstances. In the simplest cases this knowledge is present with all the certainty attainable, but in most cases only with more or less probability; we have not before us all the premises which would enable us to deduce from the end the most appropriate means, and hence the only rule for practice is to do that which will most probably bring about the result. It is the most trying limitation of our action that frequently we do not merely fail of our end, but bring about by our action what we did not will; the impossibility of always reaching necessary conclusions when the ends are given

as premises calls for the supplementation of thought by the will in the resolution to act even upon mere probability, and to risk the consequences.

But even where the relation of the means to the ends lies clearly before us, the end does not generally determine the means so completely that it can be attained in one way only; what is necessary here also is in most cases only a general element which may become real in different ways. It is here that we find room for the choice between different modes of action, for imaginative construction, for that moulding of circumstances by the individual which is in the widest sense a work of art; it follows necessarily from the logical relations obtaining in the sphere of ends that it is impossible to determine the concrete from the general in such a way as to prevent the will from continuing to be freely formative in the particular action.

14. In addition to this need of supplementing our general rules because of the incompleteness of the concepts upon which they are based, and bringing the ends, which also are wont to appear first in a general form, into the full determinateness in which alone they can be realized, we find ourselves called upon to undertake an opposite logical operation when we start from the assumption of an authoritative ethical conviction. For while the rules, which are the expression of this conviction, have from one point of view the character of generality, and require activity of thought and will to develop them into the particular, they generally appear as a plurality of commands and prohibitions, which regulate different courses of action and refer to our relation to different classes of objects, or, in another form, as a plurality of judgments as to what is good and bad, right and wrong, praiseworthy or shameful. Here what we have to do is to generalize, and to reduce the plurality of co-existent propositions to their final and most general principles. We have given the general form of such a reduction (§ 82, p. 203), and have shown what are the conditions under which general propositions obtained in this way may be regarded as absolutely valid.

15. It is not this branch of reduction which we wish to develop at present; what we must first do is to point out a presupposition which is involved in the unquestioning application of these logical processes to the propositions in which general rules find expression. When we assume that whoever wills these rules, and acknowledges them to be binding for himself, also wills their necessary consequences and their necessary condi-

tions which he has not explicitly received into his consciousness, we have already substituted for the actual empirical consciousness, in which these rules are first thought as the content of will, an *ideal* consciousness in which everything is connected according to strict rules of logical necessity; the question is not what is actually thought and imagined as object of the will and affirmed by the will, but what under certain conditions and according to logical rules ought to be thought and affirmed. Even logical legislation ultimately presupposes such an ideal, all-embracing consciousness, which consistently thinks every thought in its right connection with others; if that is to be authoritative which follows with logical necessity from an authoritative rule, then the legislating will is not the empirical, but an ideal will, and the supplementation of which we have spoken above is ultimately the supplementation of the empirical by an ideal will.

16. The development of the logical consequences of any actually recognised principle may be also treated in the way explained in § 99 (p. 457). We expect that the deviating opinions of individuals will ultimately counterbalance each other, and that whatever is necessary to common human nature will be left, that in consequence of this what actually is authoritative will be identical with what ought to be, and that any basis of ethical or legal judgment will in this way be imperceptibly, but consistently and systematically, developed. It is this which justifies us in treating the historical development of ethical convictions from the point of view of a logically necessary process, in regarding the actual as the reasonable; to carry out such a treatment, we must select from the totality of errors whatever falls in the line of objectively necessary progress, and reject as fortuitous and irrational whatever is in conflict with it: that is, we must regard history as *teleological*.

17. But the same considerations extend further, beyond the half-historical, half-teleological treatment. For the same reason which makes us regard anything which follows according to logical rules from an actually assumed ethical conviction as established by that conviction, thus depending upon the certainty with which the logical normal laws make themselves felt, we must also apply the same standard to the assumed principle, to see whether it contains anything which can be recognised as an unconditional "ought," independent of all subjective opinion. The process of reduction itself can only be carried out when the principles presupposed do not merely possess the problematic validity which belongs to them as possible premises of known propositions, but can be known as necessary

in themselves ; and thus every ethical conviction requires a final criterion of its truth.

18. Such a criterion can only be if there is something in our actual will which is accompanied by the consciousness of unconditional necessity, and if there are objects of the will which are not merely to be willed because something else is willed, but must be absolutely and unconditionally willed wherever anything is willed, or wherever there is a consciously willing subject. Just as the feeling of certainty, which in the logical sphere distinguishes objectively necessary thought from that which is peculiar to the individual and determined by varying psychological motives, is an ultimate fact beyond which we cannot pass, so, if there is any ethical knowledge at all, the conviction of an "ought" must find its ultimate fact in the certainty of its unconditional necessity, and what we have to do is merely to become aware of the conditions which give rise to this certainty, and to reduce them to their general expression (cf. § 3, 2. I, p. 15).

19. The process of discovering ethical principles can be none other than that which is employed in obtaining logical or mathematical axioms, and which we described in § 82, 5 (p. 208). We are generally immediately conscious of the necessity under the guise of a concrete case, and we have to find out by analysis what serves as a ground for the necessity and what merely applies to a casually given object. Thus there is no other way than that of reduction, or of reduction mediated by induction, by which to obtain the ultimate and absolutely certain ethical principles. This reduction may start either from the immediate consciousness which accompanies the particular act of will, or from the standards by which we estimate our own and others' actions, and which present themselves with unconditional certainty ; but the principles are not discovered until they appear as self-evident and accompanied by the consciousness of necessity. In this respect Kant employed the only method possible. Herbert followed him in this, but stopped half-way in the attainment of his end, by ascribing only an empirical validity to his standards of judgment and by being satisfied to have more than one.

20. We must also agree with Kant that in this way only an abstract formal principle can be found. For if the ends which are significant for human action cannot be willed unless they are practicable, while their practicability depends upon external conditions which do not exist by our merely willing them, then any given action can never be an uncon-

ditional duty for every one, and all imperatives commanding us to bring about definite changes in the world must be hypothetical. On the other hand, an unconditional principle for the will, even with respect to mental conditions, can only demand what lies in the nature of the will, and its form must therefore be determined by the nature of the will. As the normal laws of logic are determined by the nature of judgment, because they command us to carry out this function correctly, so also must the ethical laws of action be determined by the nature of the will as directed towards actions, for they command us to will in the right way. And we may find here a relation similar to that between the natural and the normal laws of judgment: just as the principle of contradiction is a natural law of thought in so far as we *cannot* both affirm and deny at the same moment, and becomes a normal law by being extended to the whole comprehensive unity of consciousness, and commands us to think everything in such a way that all may be combined in one ideal and all-embracing consciousness, so a normal law of the will would be one controlling the particular act of will as a natural law, and obtaining authority as a normal law by the idea of an all-embracing willing self-consciousness of absolute unity; and just as the principle of contradiction is purely formal in that it does not say *what* is to be affirmed and denied, but only that *if* a proposition is affirmed it cannot be also denied, so we may conceive of an ethical first principle which would not itself say *what* must be willed, but only that *if* one thing is willed another must be willed, and a third cannot be willed; and just as the universal authority of logic rests upon the assumption of a human reason which is common to all, and by virtue of which it is possible for every one to recognise its rules and to regulate his thoughts according to them, so also ethical legislation rests upon the same assumption with respect to the highest and unconditional rules of our will; in the last instance upon the idea of a self-consciousness which is completely one, but also, and for the same reason, upon the recognition of the equal right belonging to the reasonable ends of all willing individuals.

21. If we are right in assuming that the analysis of the will can only lead to purely formal laws, and that the ideal to which it leads is nothing more than the complete unity and harmony of the will in each particular subject, and of the will of all the subjects in a community, then it also follows that it is impossible by way of pure deduction to arrive at the particular ends which ought to form the content of our actual and concrete

will. For what we actually will is ultimately something particular and completely determined; our general ends are realized only in actions which are directed towards particular things and their changes. But what this particular something is can be no more derived from formal principles than the whole content of knowledge, although it conforms to logical rules, can be deduced from them.

22. We must here note an essential difference between the conditions of theoretical and ethical knowledge. In our theoretical knowledge there are given to us with immediate certainty not only these highest rules, but also the particular propositions which express our immediate consciousness, and the process of theoretical knowledge consists in constructing the presuppositions from which these data follow, which have their independent validity. The material principles are hypothetical, their consequences immediately certain. Exactly the reverse takes place in the sphere of ethics. Here the particular determinate action has no independent necessity of its own; it follows from the nature of the will and from the ideal of an all-embracing end that particular actions can derive their ethical necessity only from a general end or from a system of ends. Any system of ethics which is applied to real action, and, passing beyond mere formalism, aims at stating what is the content of the ends which men must set before them, meets with difficulties arising from the fact that an ethical system cannot be built up from below, but can only be sketched out from above, while, on the other hand, the conditions of actual action consist in empirical particular states, and a general end of which the possibility is not contained in these conditions has no basis in reality.

The material problem for ethics is therefore to construct a comprehensive, harmonious end as the object of human action, in such a way that it may be possible under the given conditions.

23. But the problem as thus expressed is indefinite. Starting from any given state, different courses appear to be possible, for since the conviction of what ought to be willed is itself a factor in moulding the future, this future will depend to a large extent, even from the merely causal point of view, upon the way the ends are determined. On the other hand, by insisting upon the unity of the end, upon the absence of contradiction in the will, we get only a negative canon, not rules for what ought to be.

Here we find the basis for two branches of ethics. If we start merely from the formal principles, then their consequences can only be negatively applicable to that which happens independently of them, and does not

conform to their requirements. Ethics becomes merely critical and repressive, but has the advantage of standing upon firm ground and of being demonstrative ; it is able to refute with certainty whatever does not agree with the idea of a harmonious legislature for the will. But it is unable to say what ought to be willed, what state ought to be brought about by the action of the individual and of the community ; it is obliged to leave concrete ends to chance impulses.

The positive content of the general end which we seek for the totality of human action, the idea of the highest good as of a state to be realized by concrete actions, cannot be derived from these elements alone ; and every system of ethics has accordingly either openly or tacitly had recourse to other data in definitely formulating its ideal of life for the individual and for the community, to data which are taken from that human nature which is present before ethical reflection begins, and from the natural, empirically given tendencies of the will.

This psychological basis is necessary from two points of view. On the one hand, ethics, like logic, cannot proceed to erect an entirely new edifice ; any action which it calls for must be carried out under given relations and with given means, and all common action within a community of given convictions and tendencies, which can only gradually be remoulded, corrected and brought into harmony. How we are to progress from this point towards the end is a question which cannot be answered from the unyielding standpoint of the absolute ideal ; the continuity of human action makes its claims felt in the sphere of ethics, as of logic. For this reason ethics, in its immediate application, passes into the arts of education and politics, which have to employ given forces to the best advantage under given relations.

In considering the rules of our will, we have therefore to look to the psychological conditions in order to find out how progress is possible, and how the existing course of action is to be guided towards the end ; and we are also obliged to understand human nature in order to give concrete form to the end itself, to that which is not merely relatively the best of what is immediately attainable, but is to be the absolute final end of all action, towards which it has gradually to approximate. For it is ultimately only the estimation of the value possessed by the various natural tendencies of our actions and the benefits arising from them (a value which makes itself known in feeling) that renders it possible to make a systematic arrangement of human action. At this point ethics and æsthetics, the

ideal of the good and the ideal of the beautiful, meet; the highest good attains its concrete and individual form only through the harmonious satisfaction which it ensures to the totality of the natural human desires, through the right relation in which that which is valuable only because physically indispensable stands as subservient means to the higher forms and products of activity which are valuable in themselves. If æsthetics could succeed in finding an absolute standard for the felt values of ends, it would then determine the matter which, in the ruling form of the will, constitutes the whole content of the end in the establishment of which we should have completed our consideration of the ends of human action. In the last instance every attempt to find an unconditional standard of value points beyond the subjectively varying and historically conditioned feeling of value to the thought of an objective universal end, and can reach its highest aim only by having recourse to the principles of metaphysics and theology.

24. We cannot follow further these brief hints concerning the connection of ethical problems and the application of methods to their solution; from the point of view of method it is more necessary to point out the presupposition necessary to a systematic unity of ethics. This presupposition is, that the nature of man is so constituted that a system of ends is possible which is both harmonious and consistent in itself, and satisfying to those who accept the ends, and which is therefore able to produce the volitional impulse according to natural laws. This presupposition corresponds to that from which all our effort towards knowledge starts, *i.e.*, that the perceptions given to us and their logical elaboration must lead to a system according to logical forms, to a system of concepts and a system of laws.

25. One part of the highest good is knowledge itself, for which we seek the rules in logic, the elevation of the individual consciousness to the unity and clearness of that ideal consciousness which is one with itself and harmonious, and the ideal of which, though nowhere empirically realized, is the living force in all the rational activity of men, a self-consciousness in which is realized not only the formal unity of the relation of all the particular to the knowing subject, but also the unity of the merely given content as independent of conscious and voluntary thought.

FINAL RESULTS.

§ 105.

The presuppositions from which all methods must start which do not refer merely to the development of our ideas according to subjective laws include an agreement between that which is demanded by our conscious thought and will as aiming at unity in our ends and that which is due to involuntary and externally conditioned activities.

This agreement between two spheres, which at first and from a causal point of view appear to be independent of each other, can only be made comprehensible by teleological considerations.

If we are to find a real explanation in these, it must be by assuming one ground both for conscious thought and its laws and for the objects confronting it and independent of it, a ground which, as the ultimate ground of explanation of the relation between subject and object, must also be unconditioned.

Thus the principles of methodology point to the idea of God, which can only be more definitely apprehended by means of the ideal which hovers before our thought and will as the end at which they aim.

Thus metaphysics appears as a *problem*: that of obtaining a consistent view of the final ground of the relation between the subjective laws and ideals of thought and will and the objective content of knowledge, of comprehending under it on the one hand the ultimate presuppositions from which proceeds all rational thought, on the other hand the results at which it arrives.

The highest and most difficult problem of metaphysics consists in determining the relation in which the necessity which guides all our knowledge of the existent stands to the freedom which is the subjective postulate of the conscious will. Inasmuch as logic itself presupposes such a will, and as the principles of its methods are postulates, it prescribes itself the limits to its demands for knowledge of all pervading necessity.

1. When we survey the results to which the investigation of the presuppositions and aims of logical methods has led us, we find that they all of them ultimately point in one and the same direction.

In the first place, it has appeared that the ultimate basis of all the mental activities, for the right conduct of which we seek a clue in methodology, is a will which sets before itself definite ends; and that to this will is due the motive force which impels us to investigation, while the most

general principles of the investigation are derived from the ends pursued by it. In the choice of means this will is confined to the natural forms and laws of ideation and thought, and thus the first task of logic is to bring these into consciousness, in order that in our action, so far as it is determined by the nature of mental activity, we may be certain of necessity and universal validity. It is in this way that we obtain a firm and immovable scaffolding for all knowledge. Geometry and logic, with its development in arithmetic, give us the outlines to which all further content must conform. They themselves are produced by a spontaneous and in its higher branches constructive action, which not only finds its laws in the consciousness of our own action and its necessity, but also creates its objects according to these laws, not merely copying the given, but forestalling experience by a network of relations in which the concrete content of sensations takes its place only gradually. The syntheses, again, which are contained in the concepts of substance and of causality, also have their origin in the subject, and in the way in which, by a natural necessity combining the plurality of sensations, it maintains its unity throughout change, and refers the continuous flux of events to a single and comprehensible ground. Here, indeed, our thought is not in a position to construct freely and arbitrarily, but is confined for the application of these concepts to the material given; still even here it must recur to the proper meaning of its action by moulding these concepts so as to be more clear and distinct and by finding general rules for their application. In the last instance it is by construction again that we complete the task of comprehending the ultimate grounds of given phenomena in concepts.

2. Thus we find the thinking and knowing subject to be an independent source of activities, productive of those forms of its action by which everything is controlled and determined, but limited so far as concerns absolute certainty of judgment to the sphere within which it merely brings its own functions into consciousness. When we pass beyond this sphere to knowledge of the universe given to us in perception, and turn our attention to the ends which guide our conscious will and our material activities, we find that no systematic movement of thought, sure of what it aims at and of the way of attaining it, is possible except under presuppositions which do not possess the absolutely axiomatic certainty of the principles of our own action, and to which is also denied the guarantee from matter of fact of complete empirical confirmation.

No logical arrangement of the phenomena entering into consciousness

by way of sensation and perception can be undertaken with any prospect of success except where there is confidence that they will conform to the forms of thought, that their content is such that they can be arranged in a complete system of concepts which is determined by the fundamental functions of thought, by the recognition of agreement and difference. Moreover, our work upon the varying material of our sensations must start from the belief that the syntheses which—impelled by an inner necessity of thought in combining the many to unity and reducing it to one ground—we impose upon all given content, are met by no insuperable resistance in the material of sensations in space and time; that, on the contrary, the flow of phenomena which is represented by the succession of sensations will crystallize by the aid of those forms into fixed concepts, and that their connection may be expressed in unvarying laws. The processes of induction can only aim at constructing a science in universal propositions, or be more than subjective associations of doubtful value, if the particular phenomena are based upon invariable necessity, and what our thought expresses in its universal propositions is only this necessity which is grounded in the object, and which binds together the attributes of an essential concept and makes the activities of a substance the unfailing consequence of its nature and its relations to others. It is by this presupposition that we are always guided in constructing the hypotheses by which we attempt to explain the plurality of given phenomena in a comprehensible manner.

In the same way we cannot attempt the systematic arrangement of our action, which extends itself through time and spreads itself over a number of various objects, under one single final concept, unless we assume, on the one hand, that there are fixed relations between means and ends, and that it is possible to foresee and calculate results, so that our attempts to influence the course of things may not proceed casually in all directions in the absence of any plan, and work against our intentions, to make a sport of our blindness and weakness. This assumption is none other than that upon which our theoretical knowledge is also based, *i.e.* the immutability of the causal connections which offers the only basis for purposive will. On the other hand, we cannot conceive of any serious willing of an end which we recognise as authoritative, unless we believe that the whole nature of man, even when acting independently of or before our conscious will, is controllable by single ends, and that the practical anarchy of momentary and particular impulses and desires does not deny all obedience to the authority of reason, while a self-consciousness which is one only in

theory holds together in unceasing torment volitions which are necessarily broken into contradictory and conflicting fragments. In the same way human society is only conceivable if it is possible for all to unite in directing their will towards common ends, and that no inevitable conflict between opposed wills should destroy the will of one by that of another; there must be some ground for the hope that the battle for the right will lead to the right.

3. Looked at from the subjective side, the end aimed at by the cognitive activities combines with the belief in its attainability to form the ideal of an all-embracing universal consciousness, which would possess the totality of the universe as extended in space and time in intuitive knowledge, while it penetrated with its thoughts the connection of the particular into one unity. The conceptual system of such a consciousness would be the adequate expression of the nature of substances, and it would be able to derive from them as obvious consequences the course of events at all times and places, penetrating at once into everything which now we can only calculate in a fragmentary manner.

In the practical sphere the end towards which we strive and the assumption of its attainability also combine to form the ideal of an *ego* which is harmonious in its will, directing all its activities towards an end of unconditional value, controlling its own activities by its will and nature by its activities, successful in realizing the highest good, and nowhere prohibited by insuperable resistance from ordering the actual in accordance with its end.

The thought of this ideal is the motive force both in our effort towards knowledge and in the will which we direct towards action. All reasonable activity which is conscious of its end is the rising towards this ideal, the complete reference of all particular action to this perfect unity. Community of reasonable action both in science and in practice is possible in so far as this ideal is harmoniously active in all, as the motive force of reason; and we approximate towards the ideal from the given state of individual limitations in such a way that our action is a means of realizing the same end for ourselves and for others. In the form of a community of action, in which strife and sin have disappeared, it denotes the final end of the common rational activity of mankind.

4. When we are conscious of the full significance of these presuppositions of the voluntary thought which directs itself with the purpose of knowing towards that which is and that which ought to be, then their full

bearing and the whole content of the belief upon which they rest become obvious.

In the first place, and for our immediate apprehension, our thought, with its search after unity and connection, with its connecting forms of concept and judgment, and with its categories of substance and causality, on the one hand, and the plurality of sensations on the other, are quite independent of each other. Neither on the ground of pure subjectivity, nor on the assumption of a real external world, can it be proved that they must correspond to each other; for our immediate consciousness the one function seems independent of the other: thought is one with itself and guided by the will, sensation absolutely involuntary and given. We see no possibility of determining one function by the other, of knowing it simply as the consequence of the other; in the actual course of knowledge they always stand to some extent apart, sensations being incomprehensible for thought and presenting it with the problem of arranging them in logical connection, and thought in its constructions passing beyond and seeking to be supplemented by sensation. The idea, however, from which we always start is, that the complete reference of the two spheres to each other must be capable of being realized, that all the given is thinkable.

It is the same in the sphere of the will. Over against the self-confidence with which we demand the complete conformity of our activities to the rule of a clearly conceived end stand the varying impulses and inclinations of our nature which drive us in the most various directions to the satisfaction of our desire for momentary pleasure, and manifest their independence of the single moral will by the conflict which we feel so deep as internal discord. Moreover, when we attempt to realize our ends in the external world, we are confronted by the uniform order of nature, which follows its mechanical laws blindly and without concern for our ideals, and only permits us to influence it on condition that we obey those laws. Here again it often happens that, whatever progress we may have made in the control of our own activities and in our power over nature, our reasonable will and the actual event are opposed, opposed not merely, as in the theoretical sphere, by being external to each other, but in a strife in which the real causality of nature refuses obedience to the real causality of our will. Nevertheless we still demand the realization of the highest good within and without, and believe in the possibility of overcoming the resistance opposed to us and of a harmony between the laws of nature and the claims of duty.

This involves an assumption as to the relation between our will and thought on the one hand, and on the other the independent forces within and without, which in its consequences necessarily leads to teleological ideas. In what other way than in the thought of an end controlling their relation can we explain and understand the harmony between two spheres, each of which appears from the causal point of view to be original, and not to be derived from the other? What does the belief that the universe given in perception is knowable ultimately mean, if not that our thought and its object are mutually determined for each other, that nature is ruled by thoughts, and our minds so organized as to think these thoughts? That doctrine which, while hostile to all teleology, yet maintains the complete uniformity of the universe and is confident of grasping it in formulæ, forgets that laws have a real existence only in the form of a comprehending thought, that the invariable reference of a plurality of elements to each other is possible only in the form of a thought, that the concept of law itself had its origin in human actions, and presupposes a general concept which, when thought as such, controls a plurality of activities by conscious will, and is therefore prior to its particular realizations. To make the uniformity of the laws of nature responsible for what actually happens is to affirm that it realizes thoughts, and to be teleological without knowing it; it is to compare nature to a kingdom, in which the members carry out ordinances which have been thought out beforehand, and which prescribe the appropriate activities for every position. Even a purely mechanical natural science cannot dispense with a teleology which forms its ultimate presupposition, and upon which all its endeavours are based; the effort which is most its own, that of endeavouring to understand the given as necessary, can be really concluded and completed in no other form than that of the idea of the unity of the end. The inductive methods lead ultimately to the hypothetical necessity of the results which follow from a plurality of efficient substances, when there is a certain distribution of them in space and certain relations between their states; neither their number nor their arrangement can be derived from their concepts, even if these should be arranged in a complete classification. When we assume one state of the universe we can only show it to be the necessary consequence of another, and the cause of a subsequent one; our demand for causality never gets beyond mere hypothetical necessity to the knowledge that the given as it is, in its whole causally connected course, is necessary. If we try to assert this, there remains only the empty statement that it is

necessary, that it is a part of the concept of the existent to be such as it is ; but this statement itself accepts the concept as the ground of being, and if it is to be made comprehensible, and to explain the plurality of existence by one ground, there is no way but that of teleology. If we can regard the whole existence of the universe, or the result to be ultimately realized by its organization, as an end in itself worthy to be realized, then the plurality of elements falls into subordination to this end, as the means to its realization ; the merely hypothetical necessity of causality, which says that if certain things are present in this order, this result must follow, makes way for the unconditional necessity which belongs to the highest end, and which requires the existence of these causes as the means of its realization. Our limited range of view may force us to relinquish the hope of understanding the common end of the universe and of explaining it as unconditionally necessary, but that does not affect the purely logical relation, according to which the plurality of elements which actually are causally connected can only be subordinated to one common necessity from the point of view of the end, and it remains true, as Leibnitz held, that the system of laws finds its only ground in the system of ends.

5. But it is impossible to stop at this purely formal application of the final concept. If it is to be the ground for the totality of efficient causes, it must be conceived as a real force, itself determining the nature of things and their actual organization ; but we cannot think of it as such except as the object of an efficient will which it brings into activity ; the rule that only a *vera causa* must be taken as a ground of explanation forces us to the only analogy within reach, that of our own action. Thus the ultimate ground, to which we are driven by all hypothetical necessity as the ultimate and unconditioned, appears as the real force of a will which aims at an end, and by these considerations the idea of God receives its natural content from the ideal towards which our own will is striving in both the theoretical and the practical spheres. That end, the thought of which moves us as a living force, is justified if it is already realized as the ultimate and unconditioned ground of the universe, if the universe is known in its innermost nature by an absolute intelligence, and is known because it only realizes the end conceived by that intelligence, and is determined by it as a whole as well as in detail. It is to an absolutely single ground such as this, which dominates the plurality of things and their whole course, that we are led by the final consequences of the pre-suppositions of method. For if we should cease in our investigation of

causal laws at the plurality of substances which are by their nature related, then this inner relation would itself be the greatest problem of all, unless we could assume some ground for it, binding one to the other; and as even from a mechanical point of view these causal relations, themselves eternal and immutable, determine the varying events, it is but one step further to the one ground in which all these relations, realizing themselves in logical sequence in the course of time, are thought and willed.

6. The methodological right of metaphysics to find in the idea of God the one keystone to theoretical knowledge and to practical certainty is to be found in the principles of all scientific methods, all of which contain ideal presuppositions. So long as we are concerned only with carrying out these principles with respect to given objects, so long as our standpoint is that of our subjective need to explain the Given and to be certain of our human ends, we remain within the finite; but as soon as we begin to ask about the actual justification of these needs, and take for the object of our investigation the relation of the subjective to the objective, of the ideal to reality, the necessity arises for an ultimate and unconditional ground. Nor is it as if this ground appeared only upon the horizon of our finite knowledge, as the Kantian division between understanding and reason teaches, as if we could have a science of the finite complete and self-contained in its principles, the idea of God, meanwhile, merely reflecting in the theoretical sphere our insatiable craving for complete knowledge of the universe. Its significance lies not merely in extending our knowledge in this way, but in being the presupposition without which no desire for knowledge in the true and strict sense is conceivable at all. It transcends given experience in no other sense than every attempt to explain the Given transcends it; by the same right by which we construct the particular substances and their forces as an intelligible kingdom which is the ground of phenomena, driven by the same instinct to bring together what is scattered into unity, our thought calls upon us to take this further step to the final explanation of the universe; in neither case is any proof in the strictest logical sense possible, because external reality can never be proved. Here again our only guarantee lies in the harmony amongst our thoughts, and in the fulfilment of our requirement that the Given shall be explicable. It is not by its method that metaphysics is distinguished from other science (this must ultimately be absolutely the same for all knowledge), but by the universality of its problem, a problem as necessary

as that of knowledge in general. In that it brings to light the principles presupposed in all scientific effort, metaphysics stands at the beginning of all science ; and in that its assumptions can only be verified by the result, by the complete concurrence of all knowledge, it stands at the end of science. Hence it will remain fragmentary, just as all knowledge is fragmentary, until finite thought is extended and elevated into Divine thought.

Thus metaphysics, in which we aim at grasping in concepts the ultimate ground of the relation between subject and object, thought and being, ideal and reality, appears as a problem both underlying the analysis by which we set forth the postulates presupposed in all voluntary thought, and crowning the process of thought by which we press forward in widening circles to seek at last the final explanation of the existent. In either case it appears as the fulfilment of the deepest and most comprehensive effort.

7. But this universality of metaphysics involves also a problem which passes beyond the sphere of the purely logical methods, and for which a solution must be sought before another tribunal than that of logic : the problem of the relation between the necessity which is the element in which thought moves and the freedom which is the postulate of the will. Our cognitive thought, taken alone, would find completion in the certainty of the all-pervading necessity of all that is and happens ; in a system of metaphysics which could deduce the specializations of the existent and the whole series of its developments, with logical infallibility from one ground, and which should comprehend God and the universe in one formula, in such a way that all reality would merely represent what had been implicitly contained from all eternity in the existence of the ultimate ground. Our will, with its conviction of an "ought" to which the Given does not correspond, refuses to acknowledge this infallible necessity, and opposes to the course of nature ideals which are only to be realized by free action. Neither the idea of the good, nor that of the true, can be represented as a self-realizing natural force, controlling the course of Nature without resistance, for the actual course of events brings forth error as well as evil ; nevertheless when conceived of as ends for our will these ideas have unconditional authority, though they can only attain to real causality by being accepted and willed as ends. The genesis of these ideas, and their affirmation by the will, forms an insoluble problem for the science which seeks for merely natural laws.

It is just because knowledge itself is only realized by a will which

accepts the idea of truth as an end that there is a limit to the claims to be made by the principles followed in our knowledge of the existent upon an all-inclusive and conclusive metaphysic ; it cannot be called upon in the name of logical method to tear up the roots from which logic itself has grown, by destroying the independence of the will. There is nothing in our treatment of logic to prohibit a view of the universe according to which the most fundamental fact of self-consciousness is the will, and which has therefore the task of solving the most profound problem of philosophy, that of determining the relation between ethical principles and the principles of knowledge. If the latter are what we have represented them to be, POSTULATES, then they leave open a possibility from which we should be cut off if we were obliged to regard them as axioms.

APPENDICES.

APPENDIX A

My attention has been called by the thorough and careful way in which E. G. Husserl has treated this subject in his *Philosophie der Arithmetik* (vol. i., Halle, 1891) to an incompleteness in § 65, 7 of the first edition, which might give rise to misunderstanding. In the present form of statement I hope to have remedied this defect; I believe that in the subject-matter itself I do not differ much from Husserl. It is true that Husserl rejects what he calls the "Unterschiedstheorie" of number, and attaches most importance to colligation. But in saying that every colligated content must be noticed by itself, that (when the plurality is given simultaneously) an analysis takes place, that a plurality is noticed, he assumes as the condition of the colligation a process by which the unities to be colligated are separated, are grasped independently, and their complex analysed; and this process I can only understand as distinguishing, not in the sense in which we judge of difference or likeness of content, but only in the sense that different contents which are apprehended by different acts of consciousness are spatially and temporally distinguished. That this is only a psychical *event*, and not a psychical *act*, that no one can be inwardly aware of an activity of analysis, I cannot allow; when the analysis is difficult, as in the case of insufficient illumination, we are expressly conscious of it, and from that we may infer that it takes place in a similar manner under more favourable circumstances, where no doubt we are not generally expressly conscious of it. But after all this is a subordinate question so long as it is allowed that a separation, an analysis, a "für sich Bemerken," must take place before we can colligate. On the other hand, I am far from wishing to base the idea of number merely upon distinguishing; I have always mentioned distinguishing and the comprehension of what is distinguished together, as the functions which are active in counting. In the description of the whole activity and its presuppositions Husserl has regard especially to those cases in which the plurality of objects is given *simultaneously*; here alone we can speak of analysis. But

where we have a discrete temporal series—successive flashes of light, successive reports, which we apprehend and count as a plurality, then there is no analysis at all, but only a successive noticing, and a distinguishing of the particular separate successive acts and their successive comprehension; the colligation proceeds step by step as the unities to be colligated come to be presented one after the other. This process takes place also, however, when we *count* what is given simultaneously; it is only in the indefinite apprehension of a multitude—as when we see a flock of birds or a snowstorm—that the collective appearance predominates, because the confusion of what is seen makes it difficult to be conscious of the particular acts by which we distinguish the one from the other; but even here we do distinguish in different ways, as otherwise we could not notice a plurality; it is only that we cannot count because the particular unities cannot be fixed. In the latter instance (the snowstorm) there is a difficulty even in colligating, because no definite limit is given; we do not get the idea of a whole, of a sum total, but both the colligation and the distinguishing remain incomplete; we have in the strictest sense a confused idea of plurality.

Apart from these considerations, I can agree in essential points with most of Husserl's views; to discuss them in detail would take us too far.

APPENDIX B.

THE acute attempt of Helmholtz and Kronecker (*Philos. Aufsätze zum fünfzigj. Doctorjubiläum E. Zellers*) to base arithmetic upon a series of symbols, which have been once and for all arranged and maintained in a fixed order, seems to me to overlook the fact that etymologically the ordinal numbers are entirely derived from the cardinal numbers, and that numerical terms are not originally employed to arrange the particular objects of a group each with a symbol, one after the other, to mark them as number 1, 2, 3 and so on, but that their fundamental significance is that of a certain number of things, and that for this reason they are grammatically connected with the plural. This points to the fact that at every step to a further unit there is a comprehension of the units so far distinguished to a definite plurality, that 3 *contains* 2 and 1, and does not merely follow them. If it is acknowledged that the psychological origin of the idea of number consists in our ability to retain successive acts in

the memory, etymology certainly points to the course taken by development from this origin, and here there can be no doubt that the meaning of numerical terms is to express the number of successive apprehensions of units. According to this theory, in passing from ordinal to cardinal numbers we do not, properly speaking, count objects, but the units counted are the numerical symbols themselves. Still the original concept of number must be surreptitiously conveyed into these concepts. "When I need the complete numerical series from 1 to n , to assign a number to each element of the group, then I call the number of the members of the group n ." This transition from n , as the symbol of a place in the series to n as the symbol of the total number would be incomprehensible if the symbols had not from the first possessed the significance of number. I can remember the letters of the alphabet in their sequence with as much certainty as the series of numbers, and may use them to denote a series, *e.g.*, of pages; but it is useless to say, I have needed the letters from A to W , assigning a letter to each page of a book, and the number of the pages is W . That would tell me nothing about their number if the particular letters had no meaning in themselves and were determined merely by their position in the series; the number is found only if the meaning of each particular symbol includes the comprehension of the steps already taken into the idea of their definite number, *i.e.*, when they are not merely symbols of position, but symbols of sums of units.

APPENDIX C.

IN certain profound and subtle enquiries as to the conceivability of a non-Euclidean geometry, the space of our intuition has been represented as only a special instance of a general concept, of a variously extended manifold, and its relations of magnitude as only empirically valid, others being in themselves conceivable; but I cannot help thinking that such enquiries have overstepped the safe interpretation of analytical formulæ, or have confused purely mathematical conditions with others which refer to the physical constitution of bodies in space, although in themselves they are valuable as bringing into clear consciousness the peculiarities of our space intuition, and completing the series of problems which must be answered by an exhaustive philosophy of space.

Riemann, in his lecture on the hypotheses upon which geometry is

based, starts from the general concept of a plurality of modes in which a concept may be determined, between which modes there either is or is not a continuous transition ; and he then represents colours as in this respect precisely similar to positions in space. But he seems to me to have overlooked that the parts of space are logically related to each other quite otherwise than are the various colours. The latter are no doubt related as different determinations of a concept ; and this is evident from the fact that each can be thought of by itself, and that a plurality of them is at first thought of as divided by perceptible differences, the manifold being only ultimately represented as a continuum in consequence of inferences, and by continually inserting intervening colours which can no longer be distinctly perceived as different. But the parts of space cannot from the first be thought of as independent of each other, but only as within a space which is given to intuition ; the continuum is not gradually formed, but is the primary datum, within which alone the particular points can be distinguished. Thus they are not different determinations of a concept in the sense in which colours are ; and if we should call the different values of a distance also different determinations of a concept, we must remember that the distance of one point from various others always presupposes space, and that the concept of a distance which is twice as great as another contains not merely the numerical concept two, but also continuity of the units of measurement in the same direction. In the former case (that of colours) the differences are qualitative, and depend upon comparison of content ; in the latter they are spatial, and depend upon comparison of directions and distances. Hence the numerical series by which the magnitude of the differences is measured in the two cases must necessarily differ radically in meaning ; formulæ expressing relations between the variable can only be applied to spatial differences, where the spatial ideas, which are fixed before we measure them, will admit of it. Again, in order to lend plausibility to the spatial interpretation of other formulæ than those obtained from the consideration of *our* space, reference is made to the attributes of cylindrical surfaces which can be unwound without any extension of the lines in them, to surfaces of constant curvature in which all figures can be moved about without change of shape, to ellipsoids in which the shape depends upon the position, and the distinctions thus obtained are then applied to space itself, distinctions between space which is plane or not plane, which has or has not a constant curvature. But all this is based upon the assumption of Euclidean space, in which alone we

can get the spatial intuitions appropriate to the formulæ, and then only of surfaces which we think of as plane or curved in our space ; while intuition forbids us even to think of three-dimensional space otherwise than we now do.

This is true again of the possibility of a space of one or two dimensions ; we can trace the relations of magnitude within a plane or a spherical surface, but we cannot think of the plane or the spherical surface except as in Euclidean space. And when (in enquiring into the form which would be taken by the geometry of creatures so constituted as to be able to move only in a spherical or pseudo-spherical surface, and therefore necessitated by physical constraint to confine themselves to the measurement of the paths thus prescribed to them) the concept of the shortest line is substituted for the straight line of our space, it loses just that specific nature by virtue of which it is the vehicle of our space idea. The concept of the straight line is not exhausted by the mere determination of magnitude by which it is compared with other lines ; although some of the geometrical propositions of Euclid make use of no other determinations than that the straight line is completely determined by two points and can be moved in the plane in such a way that lines of equal length can be made to coincide, and although these attributes belong also to the shortest lines of the spherical surface, yet others are absent which distinguish straight lines in our space, such as all those which, in our space intuition, are connected with the revolution about two points. We cannot form any intuition of a space in which the shortest line would not be a straight line. And if, finally, appeal is made to the fact that the relations of a spherical or pseudo-spherical space can be brought, at any rate partially, into intuition, still this can only be done by considering how bodies conforming to the formulæ expressing that space would be changed in *our space* and for our space-intuition ; thus our space is not replaced by another. The actual measurement of bodies by bodies teaches us nothing concerning the nature of space, but only concerning the way in which the dimension of bodies in space are related to one another and to the conditions of our perception ; in practice no one has as yet been led astray by finding that a trigonometrical measurement gave more or less than 180° as the sum of the angles of the visual lines between three points, or that a distant vertical object proved larger upon direct measurement than followed from the sine of the angle of altitude ; instead of correcting Euclid we have corrected the assumption that light moves in straight lines.

Thus the result of these enquiries is not that it is left to experience to decide whether we are to assume the plane space of Euclid, or a space which is in some way curved ; but only that from the purely logical standpoint of analysis the quantitative relations of space are not to be derived as the necessary form of a manifold which varies in three directions, but that on the contrary they are actual, because based upon an unanalysable necessity of our space-intuition, which is essentially different from any law which can be expressed in numbers and numerical relations. They open up no possibility of extending our space intuition, or of representing a non-Euclidean geometry not merely in analytical formulæ, but also for actual intuition ; we remain subject to those laws of space according to which we first think of it, and it is as certain that Euclid will remain unrefuted in geometry, as it is that Aristotle in his principle of contradiction has outlived the Hegelian logic (cf. the pertinent remarks of O. Liebmann, *Raumcharacteristik u. Raumdeduction, zur Analysis der Wirklichkeit*, ed. 2, 72 sq. ; and Wundt, *Logik*, 1, p. 445 sq.).

APPENDIX D.

I CANNOT convince myself of the justice of the objections raised against the exposition in the text by Benno Kohn (*Untersuchungen über das Causalproblem*, etc., Wien, 1881), although I quite acknowledge the acuteness which he has shown, especially in the criticism of Mill's propositions. He calls the concept of the action of a cause a transcendental something to which we can attach no meaning. I allow, of course, that we are dealing with something transcendental in the sense that something is added in thought to the purely empirical, perceptible reality, by which to explain it ; it is a concept which is constructed with the object of explaining the given as necessary upon one ground. But I cannot allow that we can attach no meaning to it ; if it were so, we could attach no meaning to the concept of force either, of which the meaning is just this, that it acts and is regarded as the ground of a motion. What is expressed by the concept of action is that a motion or other change is to be referred, not to the subject in which it occurs, but to some other as its ground ; but then, to say that the action of the cause and the beginning of the effect are simultaneous is a purely identical proposition. What follows rests upon a misunderstanding. In the concussion of elastic balls, we may think as we

will of the change in their form, or in the position of the molecules, as beginning at the surface of the bodies and proceeding inwards, but the efficient action begins at the moment in which the most external molecules of the ball which is struck begin to change their position, and it ceases as soon as the struck ball ceases to have velocity imparted to it by the striking ball, and moves away from the latter. But when, as in the kinetic theory of gases, we are dealing with the concussion of atoms, we must, so far as I can see, either assume that motion is imparted instantaneously to the atom struck, or we must say that in an immeasurably short time the velocity of the atom struck increases to a definite value, while the velocity of the atom striking vanishes proportionately; this time would be the time occupied by the efficient action. We cannot infer from mere concepts that only the second assumption is possible. Kohn seems, moreover, to overlook the strict distinction between efficient action and effect which I emphasize. The example with which he thinks to perplex me on p. 121 belongs to another department than that under consideration here. "In the analogous case in which a stone is thrown away from the centre of gravity, we should have to call the highest turning point of the orbit the point of special causation, etc." So far we had dealt only with those cases of action in which a body in movement imparts its motion to another, not with persistent forces such as gravity. Gravity acts from the moment in which the body begins its motion to the moment in which it ceases to move, in every time-differential it imparts to it a velocity in the direction of the centre of the earth; so far as concerns the action of gravity there is nothing to distinguish the highest point of the orbit from any other, it is merely that point at which the imparted upward velocity of the thrown body is annulled by accelerations imparted in the opposite direction.

Of course, if we define as cause those conditions which must be present for the effect to begin, we must assert simple succession; my intention, on the other hand, is to show that all ideas of causality are ultimately based upon the thought that things are the efficient causes, that this thought leads to many modifications, but that it never quite vanishes, and that without these "popular" thoughts it would be impossible to have any uniform arrangement of the multiform concepts which have grown out of it. See Appendix E.

APPENDIX E.

I MUST decline to discuss in detail the many works on the causal concept which have appeared since the first edition of this book ; they show anew the difficulties in distinguishing between the different applications of the general causal idea and the specific concepts developed from it, and in placing them in any clear relation to each other ; we have scarcely come any nearer to agreement in our way of apprehending and naming them. More especially have I been unable to convince myself that the different aspects presented by the causal idea can be represented more easily in their connexion and mutual relation, and the resulting antinomies more easily solved, if we do not—as I have attempted to do—start from that concept of the action of things upon which the popular view is originally based, and trace its scientific changes ; but endeavour, with the object (quite justifiable in itself) of eliminating everything transcendental and hypothetical and of giving to the concepts concerned a content as far as possible merely empirical, to limit the causal concept to the succession of events, or at least to represent this succession as its true essence.

It is true that it will now be recognised by most that a merely empirical derivation of the causal concept and the causal law from perceived succession is impracticable, and that J. S. Mill's attempt may be regarded as a failure ; nevertheless the fundamental idea that the scientifically correct and appropriate concept of causality refers to the regular succession of events, and that the original " popular " idea of action must be eliminated so far as possible, seems to be predominant. It has been most carefully and fully treated by Wundt (*Logik*, 1, 525 sq., and *System der Philosophie* 292 sq.), and I should like therefore to discuss briefly some points in his treatment of the problem, with the object of rendering my own view more precise. I prefer to keep to the more detailed exposition of the *Logik*, the principles of which are repeated, though in a different order of development, by the *System of Philosophy* ; although with reference to certain special points I cannot be certain whether they have been tacitly withdrawn or retained.

Wundt's leading idea is the distinction between the concepts of substantial and of actual causality (to use the terms introduced in the *System*). The most obvious view is to mean by cause a substance, and this substantiation of the causal concept no doubt has its psychological origin in the acting personality, which, while itself persisting, gives rise to

the particular action as a passing event. From this scientific elaboration first proceeds to the concept of force, which belongs as permanent attribute to particular things, and is attached to their substance (the illuminating power of the sun, the attraction of the earth). But inasmuch as efficient actions can only occur when some change takes place in the arrangement of the bodies which are the subjects of the forces, the causal concept is gradually transformed so as to deprive the cause of its substantial character and to resolve it into an event which precedes the effect (*Log.* I. 528).

In this way a distinction is obtained between the empirical and metaphysical elements, which are originally interwoven in the concept of cause ; the concept of force as completing the concept of substance partakes of its metaphysical character ; the causal law (here substituted for the causal concept) which refers merely to the connexion of events, applies in itself only to the way in which phenomena are linked together for our experience.

This limitation of the causal concept is justified from the nature of the case on p. 536. We never have occasion to apply the concept of causality to the objects of our external experience so long as they remain unaltered in their spatial and temporal relations. "Change is therefore the condition of causality, which refers not to things but to events."

Now the first proposition—that it is change which affords the occasion for the development of the causal concept, and that what we first look for are the causes of changes, which causes we find in things themselves, showing an action or change—is no doubt correct. But it does not follow that the causal concept does not refer to things, but to events. Events or processes are changes in things ; the concept of change cannot be thought at all without that of the thing or object. If an "event" or "happening" is to be a cause, then it is certainly the motion or change of a thing ; we cannot entirely abstract from this, especially as it is not a matter of indifference in what thing an event takes place which is to be regarded as cause. Motion of a given velocity is an event ; in another and purely phoronomical sense we may perhaps leave out of consideration the magnitude of the mass moved ; but if the motion is to be regarded as a cause, it is not all the same whether what moves is a grain of sand or a rock. It is the same with effect : the change effected is change in a given thing, and it is not a matter of indifference in what thing the change takes place. Suppose the effect to consist in the "event" of melting : iron melts at another temperature than that at which ice melts. The proposition that

causality refers *only* to events or processes and not to things, does not hold good in this abstract form.

In the *System* Wundt is so far more guarded as to point out that natural science at least must retain the concept of substantial causality, though perhaps in an altered sense, and rightly indicates the mutual relation between force and mass; but here again (296 sq.) we are brought to the proposition that mechanics is able to subject all the motions in nature to an analysis which works in principle only with empirically measurable magnitudes; it is not necessary to include anything in the concept of force, but the magnitude of the acceleration by which it is measured. Nevertheless, it is measured by the magnitude of the acceleration of a given mass; we cannot simply exclude m from the mechanical formulæ—e.g. $\frac{1}{2} m v^2$. Only spatial and temporal magnitude (volumes and velocities) are empirically measurable; there still remains, in the concept of mass, the metaphysical surplus, which is not directly measurable (v. § 78), but can only be determined by an assumption, which—as Wundt himself rightly shows elsewhere—contains the concept of substance. The same is true of the proposition on p. 298, that energy is measured entirely by the event which is given to observation, and requires for its definition nothing which does not itself belong to this event. Even if we include in the event the mass, which is not an immediate object of observation, still the event itself is not identical with the concept of energy; the definition of the event is not *eo ipso* the definition of the energy; we need to add a law, by virtue of which this event has further consequences, before we can get from the concept of the observed event, that of energy.

On the same page (298) it is rightly pointed out that for mechanics there is a difference between the acceleration which actually occurs, and accelerating force. “The results obtained concerning force and mass are grounded upon numerous observations, guided by general assumptions, and mechanical force is not that which is actually observed in the particular case, but that acceleration which is *possible* in respect to a given unit of mass.” Here it is acknowledged that the concept of force, like that of mass, contains assumptions which go beyond what we can observe.

The difficulties to which the attempt to refer the actual causal concept only to events leads, can perhaps be best shown by the example with which Wundt illustrates his propositions in both works. What is the cause of the fall of bodies? The answer that the earth, or its hypothetically assumed power of attraction, is the cause, is denied; that would involve

the false introduction of the thing into the causal concept. The earth is only the permanent condition, which being given bodies *may* fall; the cause of the particular phenomenon of falling is elevation to a given height. This alone satisfies the demand that the cause, as well as the effect, must be thought of as an event. For a body to fall 10 feet, it must necessarily be first raised to a height of 10 feet; but how this is brought about, and how its supports may be removed, are conditions which may vary in many ways without the effect being thereby changed (cf. *System*, p. 301).

This example is subject to certain considerations, especially when taken in connexion with the adjoining proposition that the cause is that event which is connected invariably with the effect, and the proposition on p. 543, "that the form in which causality appears to us is the sequence of cause and effect"—where, according to the context, we must understand an immediate and empirically demonstrable sequence. (These two propositions, taken in this sense, are, it is true, absent in the *System*; but not only are they not withdrawn, they appear to be at any rate partially presupposed in subsequent passages, although on p. 299 sq. there is substituted for particular succession the more guarded idea of continuous connexion).

In the first place, we may ask whether the raising of the body to the height of 10 feet is invariably connected with the effect—with falling from this height? Certainly not within the sphere of our experience. Millions of bodies are raised to any height which, so far as our experience goes, never fall again. The stones of a building which is blocked by rubbish from without have been raised, but they can fall no further unless room is cleared below them. Building would be a futile undertaking if every stone raised must fall again as far as it has been raised; nor can the necessity be affirmed that all volcanic masses which have once been raised must fall again. Hence, if we look from the cause to the effect, there is no invariable connexion between elevation and fall.

But the example fails also in the reverse direction. In the above propositions Wundt suddenly substitutes for the cause the *conditio sine qua non*, when he says: "for a body to fall 10 feet it is necessary that it should previously have been raised 10 feet." But here again there is no invariable connexion. An elevation to the height from which the body falls does not precede every phenomenon of falling. When a shaft is sunk in the ground, and a stone falls over the edge we cannot point to an elevation of the stone from the bottom of the shaft as a preceding event. What shall

we say moreover of the meteors which fall upon the earth? And if, according to the theory with which Wundt himself agrees, the moon is perpetually falling towards the earth, when did the "event" of its elevation to 240,000 miles take place? According to the form taken by the Kantian theory, its mass has only been checked in its preliminary fall towards the earth by the increase of centrifugal force; hence the preceding event was certainly not elevation but a checked fall. Wundt says on p. 542: the attraction which exists at every moment between the earth and the moon, is "occasioned" (why not caused?) by their immediately preceding motions, which have brought them into their present positions, and the effect itself can only be thought of in the form of a motion which needs a certain amount of time (how much?). But from amongst the "immediately preceding motions," which have gone on continuously for thousands of years, we can select no event which can be limited in time, and which would be qualitatively and quantitatively equivalent to those motions which are to be regarded as the effect, and which also persist continuously and indefinitely. From the position at any moment, as determined by the stage just reached by the motion, there follows only the particular amount of attraction, which varies within narrow limits with the distance, not the attraction itself, which has always existed and which helped to determine the preceding motion.

It is not possible to discover an event which is empirically perceptible, or even one which can only be constructed by inferences, to precede in the same way every actual case of a falling body, and to be in this sense its cause. Of course every motion downwards presupposes that at a given time a body is at a distance from some lower part of the earth's surface—otherwise the body could not move at all towards the centre of the earth either perpendicularly or slantingly—and the absence of any hindrance to this motion (whether some intervening body or upward motion). That which precedes downward motion in the region of terrestrial bodies, and so far appears as the regularly preceding event, is the cessation or removal of this hindrance; this is invariably connected with the event of the fall, and would therefore have to be called its cause, but then we could not speak of an equivalence between cause and effect. But in the case of meteors it is the direction of their orbit and their velocity which decides whether they may fall upon the earth; no definite event can be given. That which *causes* bodies to fall under certain circumstances is ultimately—according to the theory which is still almost universally accepted and which has been

replaced by nothing better—the attraction of the earth ; this is the constant and true cause which determines the motion of the body towards the centre of the earth ; all the others are variable conditions which being given, the cause takes effect, and it is absolutely impossible to select from amongst the “ events ” preceding the fall anything common to all instances of falling and invariably connected with them,

It is not difficult to see the motive which has led Wundt to the propositions illustrated by this example ; it has been the attempt on the one hand to make them fit in with the proposition of the conservation of energy ; on the other hand, his wish to distinguish between the empirical and metaphysical elements of the causal concept, and to present a concept of causality which would connect successive events in the sense of the empirical school ; in addition to this there has been—as we see from the *System*—the thought of questions of psychological causality which he has treated as parallel to these. But the principle of the conservation of energy is not in itself sufficient to serve as the ground for a causal concept in this sense. By the necessity of regarding mere spatial distance as potential energy, it presupposes as its basis the constant attraction of masses ; by the change of active energy into potential, and *vice versa*, it not merely connects events amongst each other, but it also connects events with states, for after all potential energy is not an event ; nor is it well to refer potential energy always to a previous event, in order to present a change like the fall of a body as the effect of some event which took place thousands of years ago. According to the theory of Kant and Laplace all the active energy of the present solar system has proceeded from the potential energy which was due to the merely spatial distribution of material elements, and it is impossible to form any idea of the event by which this spatial distribution was caused. But mere spatial distance in itself cannot be a cause in any sense, unless we assume the attraction of masses ; it is this therefore upon which alone the effect is grounded. (It is similar in the sphere of chemistry. According to the view which regards elevation to a given height as the cause of the fall, we should have to regard the previous separation of the combining substances as the cause of chemical combination.)

Wundt himself allows that “ substantial causality ” cannot be dispensed with, and must remain as an inevitable auxiliary concept ; but then we cannot separate the two, and try to establish a causal connexion which is to disregard it and to take place merely between events. We have in

events the phenomena of substantial causality; they cannot be separated from the substances in which they occur. It is, moreover, a subversion of the original meaning of the terms to speak of the force-endowed substances as only permanent *conditions*, and of their changing relations as *causes*; the older and opposite usage is that which conforms to the meaning of the terms. The distance of a body from the surface of the earth is one of the conditions under which it may fall, because where there is no distance no motion is possible, it is a *conditio sine qua non*; another condition is the absence of any hindrance, but the *cause* of the fall is "substantial causality," the earth by virtue of its attraction. The idea of the action of one thing upon another is no doubt a metaphysical element; but we cannot dispense with it if we wish to bring about connexion amongst events, and I hold it better therefore to confess it openly and to yield to it its fundamental importance, instead of putting it as much as possible on one side as a mere "auxiliary concept," which we cannot unfortunately quite do without.

ADDENDUM.

The following passage has been omitted on p. 350, end of paragraph 21:—

According to the same principle, when the motions are in opposite directions, we get their difference; and when they are in different directions the assumption leads to the so-called parallelogram of forces. These are the simplest examples of rules of the combination of causes.

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